

Mathematical model of a separately excited DC motor powered by a solar array using external starter resistances



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Resumen

Este documento se centra en el modelo matemático de un motor de corriente continua que es alimentado por un panel solar. Nosotros involucramos diferentes tipos de ecuaciones para representar el sistema fotovoltaico, junto con la carga y luego analizamos el comportamiento de este acoplamiento. Además se procede a crear modelos de bloques en Simulink para poder compararlos y así finalmente se procede con el desarrollo de las conclusiones de acuerdo a las gráficas obtenidas en nuestra investigación.

Palabras clave: Motor de corriente continua, paneles solares, Simulink.

Abstract

This paper focuses on the mathematical model of a DC motor that is powered by a solar panel. We involve different kind of equations to represent the photovoltaic system, together with the load and then analyze the behavior of this coupling. In addition, we proceed to create blocks models in Simulink to compare it and thus finally we proceed with the development of the conclusions according to the graphs obtained in our research.

Keywords: DC motor, solar panels and Simulink.

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I. INTRODUCCIÓN

Nowadays the society uses energy that is basically not renewable. As the years pass the energy consumption increases and it reduces the availability that it has, until we reach a point in which we can not continue to exploit them, so this is the reason why we should think about alternative energies that are able to adapt to our needs.

Among the various machines currently used by man, engines are the most important due to its great versatility as almost we can find it anywhere from a toy car, up in large industries.

DC motors are the most common and inexpensive, and allow a wide range of speed and can provide high torque-controlled motor easier and cheaper than any AC motor.

In this paper we propose the use of photovoltaic as a power source for DC motors performing a feasibility analysis of this system using Simulink, for which we show the mathematical model of a solar array in combination with a DC motor separately excited, including the boot process using external resistors.

II. MODEL FEATURES

A. Solar array

Solar cells are devices that work using the photovoltaic effect in order to produce electricity, in which energy from the absorbed light is used to excite electrons to higher energy levels, so an assumption that we are considering in this paper is that the intensity of light is constant, it has the electrical representation as shown in the next figure:

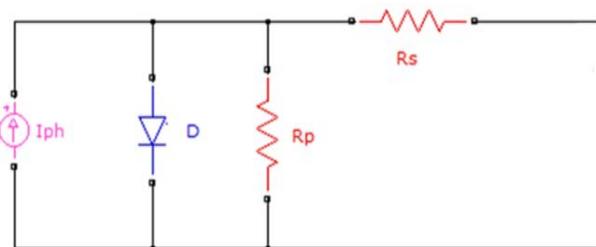


FIGURE 1. Electrical circuit of a solar cell.

This model consist of: I_{ph} the photovoltaic current that is a current source proportional to the amount of light the solar cell is receiving, in the case there is no light, no current will be generated, D the diode that is formed due to the p-n material by which the solar cell is formed, R_p the parallel resistance represents the losses because of the manufacturing defects and the highest its value, the best for the solar cell operation, R_s the series resistance represents the contact resistance between the metal and the semiconductor, the proper resistance of the conductors and the resistance presented in the semiconductor when a current is produced on it, the smallest its value, the best for the solar cell operation.

A solar array is formed by the union of solar cells in series (to increase voltage) and parallel (to increase current) and the voltage and current that this issue will depend on the load to be connected.

Another factor affecting the performance of the solar cell is the temperature, at higher temperatures the yield of the solar cell falls; in this modeling it was assumed that it was working under a constant temperature of 300 K.

The following figure shows the electrical circuit of a solar array [1]:

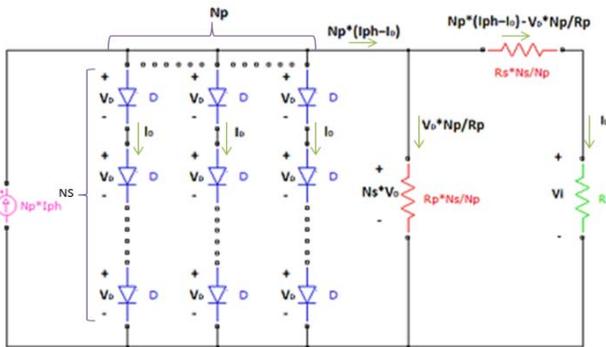


FIGURE 2. Electrical circuit of a solar array.

In this model, N_s represent the number of solar cells connected in series and N_p the ones connected in parallel.

In order to obtain the mathematical representation for the output voltage V_i , we need the load resistance. From the circuit and using the Kirchhoff nodal rule, the current equation can be defined as:

$$N_p * I_{ph} = N_p * I_D + \frac{N_s * V_D}{N_p * R_p} + I_L \quad (1)$$

Now by using the Shockley Equation [2]:

$$I_D = I_o * \left(e^{\frac{e * V_D}{k_B * T}} - 1 \right) \quad (2)$$

Where I_o is the dark saturation current whose value depends on the properties of the material, e is the elementary charge ($1.602176565 \times 10^{-19}$ C), T is the solar cell temperature in

Kelvin and k_B is the Boltzman constant $8.6173324 (78) \times 10^{-5} eV/K$.

By applying Ohm's law, the load current can be represented as:

$$I_L = \frac{V_i}{R_L} \quad (3)$$

Replacing Equations 2 and 3 in Equation 1:

$$N_p * I_{ph} = N_p * I_o * \left(e^{\frac{e * V_D}{k_B * T}} - 1 \right) + \frac{N_s * V_D}{N_p * R_p} + \frac{V_i}{R_L} \quad (4)$$

Using Kirchhoff voltage rule, the voltage across the series diodes is equal to the voltage in the equivalent series resistance plus the voltage in the load:

$$N_s * V_D = \frac{N_s * R_s}{N_p} * I_L + V_i = \frac{N_s * R_s}{N_p} * \frac{V_i}{R_L} + V_i \quad (5)$$

From Equation 5 we get the expression for V_D .

$$V_D = \frac{V_i}{N_s} * \left(1 + \frac{N_s * R_s}{N_p * R_L} \right) \quad (6)$$

Replacing Equation 6 in Equation 4:

$$N_p * I_{ph} = N_p * I_o * \left(e^{\left[\frac{e * \frac{V_i}{N_s} * \left(1 + \frac{N_s * R_s}{N_p * R_L} \right)}{k_B * T} \right]} - 1 \right) + \frac{N_p * V_i}{R_p * N_s} * \left(1 + \frac{N_s * R_s}{N_p * R_L} \right) + \frac{V_i}{R_L} \quad (7)$$

Solving for V_L :

$$V_i = \frac{N_p * I_{ph} - N_p * I_o * \left(e^{\left[\frac{e * \frac{V_i}{N_s} * \left(1 + \frac{N_s * R_s}{N_p * R_L} \right)}{k_B * T} \right]} - 1 \right)}{\frac{N_p}{R_p * N_s} * \left(1 + \frac{N_s * R_s}{N_p * R_L} \right) + \frac{1}{R_L}} \quad (8)$$

From Equation 8, V_i can not be totally solve because it still remains in the exponential factor, so this one will be found using iterative methods.

In this model we are working with a solar array that are in the common range of values such as $I_{ph}=3A$, $R_p=1000$ ohm, $R_s=0.01$ ohm, $N_s=410$, $N_p=20$, $I_o=1 \times 10^{-10}$ A.

The model is implemented in Simulink that represent the solar array is the one shown below:

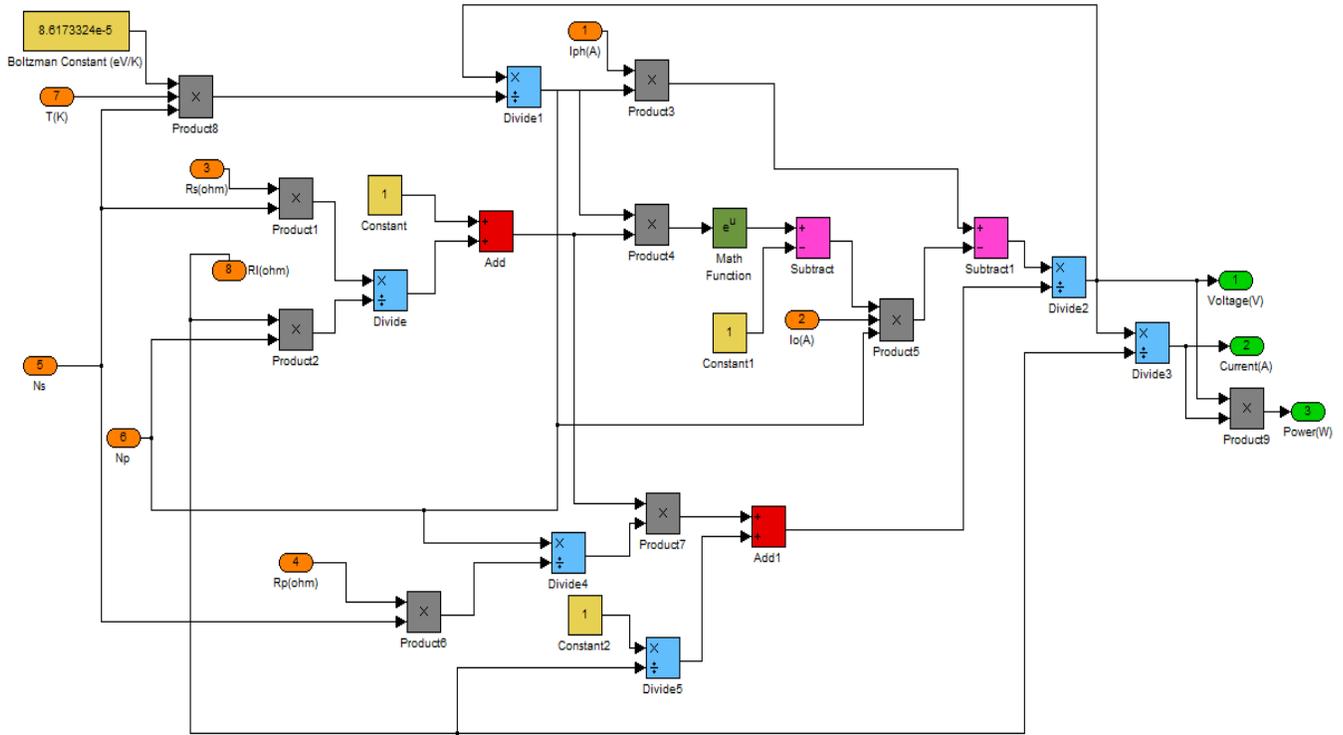


FIGURE 3. Solar array implemented in Simulink.

B. Separately excited DC motor

A DC motor is composed by a stator or inductor which is the fixed part of the engine and a rotor or armature which is the moving part. This is the load that we are going to use.

The motor to use is a separately excited motor, whose main feature is the coil (inductor) that generates the magnetic field, is not within the motor circuit, in simple words there is no electrical connection between the rotor and the stator as shown in the following figure:

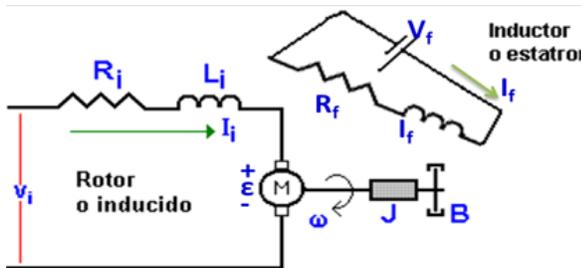


FIGURE 4. Scheme of separately excited DC motor.

Model shown has electrical characteristics consisting of: V_i the rotor supply voltage, I_i the current that flows through the rotor, R_i the rotor resistance, L_i the rotor inductance, ε the EMF, V_f the stator supply voltage, I_f the current that flows through the stator, R_f the stator resistance, L_f the stator inductance.

Normally on axis of the rotor we attach a load with mechanical characteristics such as: ω the angular speed, J moment of inertia respect the axis center of the rotor with the load, B viscous friction coefficient [3].

The equation that represents the behavior of the angular speed of this motor is [4]:

$$\omega(t) = \frac{V_i}{\beta} * \left(1 - e^{-\frac{\beta}{J}t}\right). \quad (9)$$

Where:

$$\gamma = \frac{R_i * J}{K_p}, \quad (10)$$

$$\beta = \frac{R_i * B}{K_p} + K_b. \quad (11)$$

The electromagnetic torque constant is represented by K_b and K_p represents the EFM constant.

In this model we are working with a DC motor that has values of $R_i = 0.6$ ohm, $J = 1.2$ kg.m², $B = 0.19$ N.m.s, $K_p = 0.4$ N.m/A and $K_b = 1.5$ V.s, so by doing the correct operation we get that γ and β are 1.8 for both.

The model is implemented in Simulink that represent the Equation 9 is the one shown below:

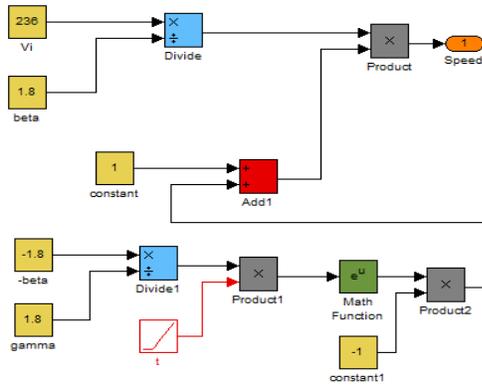


FIGURE 5. Speed equation represented in Simulink.

Finally, the current I_i is:

$$I_i(t) = \frac{J \ast \frac{d\omega(t)}{dt} + B \ast \omega(t)}{K_p} \quad (12)$$

The model is implemented in Simulink that represent the current is the one shown below:

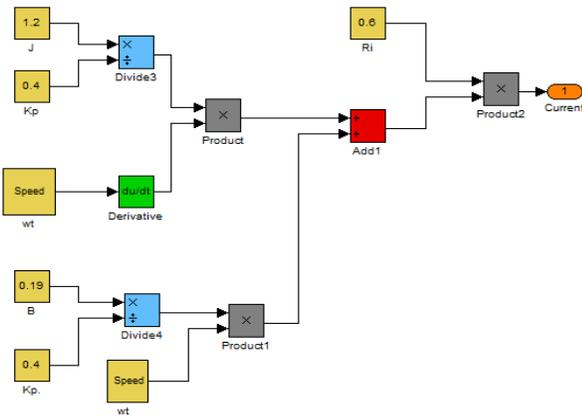


FIGURE 6. Current equation represented in Simulink.

C. External starter resistances

In order to reduce the starting current (usually is around 1.5 to 3 times the nominal current) we have different ways to do it, but the one which is used on this paper is the external resistance method. This consists in using resistances that do not belong to the motor, but these are short-circuited by using relays after a period of time in which the current reach it stationary state [5]. A simple model is shown below:

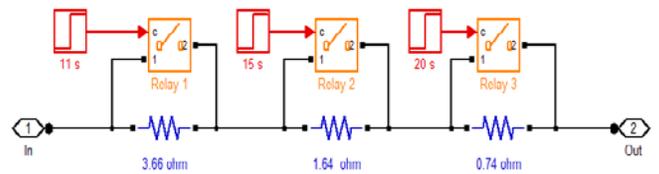


FIGURE 7. External resistance starter in Simulink.

In this model we will use three external resistances of 3.66 ohm, 1.64 ohm and 0.74 ohm, which will be short-circuited at 11 seconds, 15 seconds and 20 seconds respectively after the motor has been turned on.

The model is implemented in Simulink that represent the solar array is the one shown below:

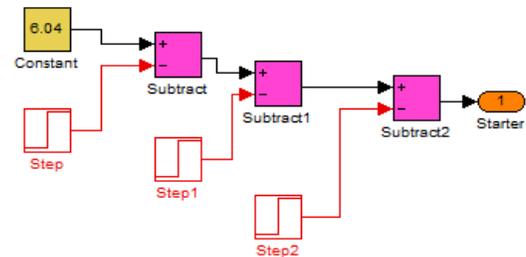


FIGURE 8. Speed equation represented in Simulink.

III. SYSTEM SIMULATION

In this paper the solar array is the voltage source that will supply the motor armor, so this will start moving until it reaches its stationary speed. The starter is inserted on the motor and only operates in during the motor boot.

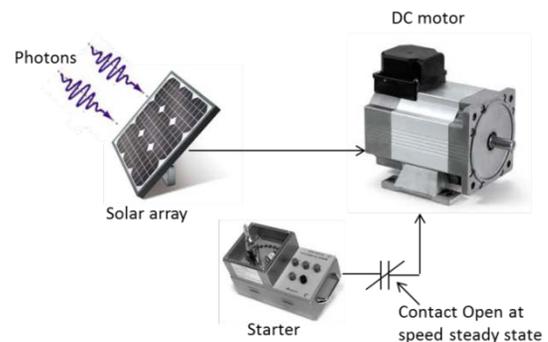


FIGURE 9. Solar array, DC motor and starter.

We implemented the whole model in Simulink by using blocks diagram.

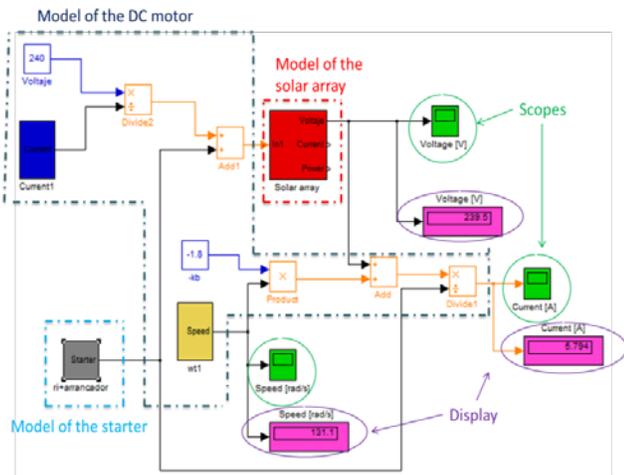


FIGURE 10. Solar array, DC motor and starter in Simulink.

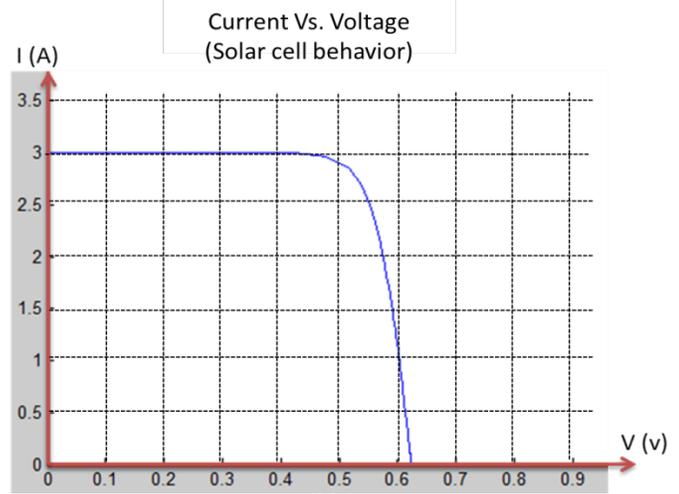


FIGURE 12. Graph of IV curve of a solar cell.

IV. ANALYSIS OF RESULTS

Once the simulation is done, it shows the behavior of the input voltage, arm current and speed versus time by using the different scopes.

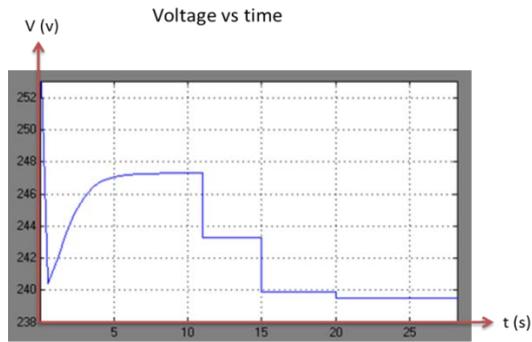


FIGURE 11. Graph of solar array, voltage vs. time.

The previous graph (Figure 11) shows the armature voltage (that is also the voltage produced by the solar array) behavior in time. We can observe that at the beginning occurs a pick of voltage that is produced by the connection of the solar array to the motor. In addition, there is a 3 starter resistances that are connected, nevertheless after 11, 15 and 20 seconds each of them are removed, thus the motor resistance is reduced, the current demanded is increased and the voltage provided by the solar array is decreased [6].

After all the resistances have being short-circuited, the system reaches its stationary state at 239.5 V.

In the graph is shown how while the starter current is decreasing, the voltage produced by the solar array is increasing [7], this is because when a solar array needs to supply a high current its voltage goes down, and when it need to supply a low current its voltage goes up as is shown in Figure 12.

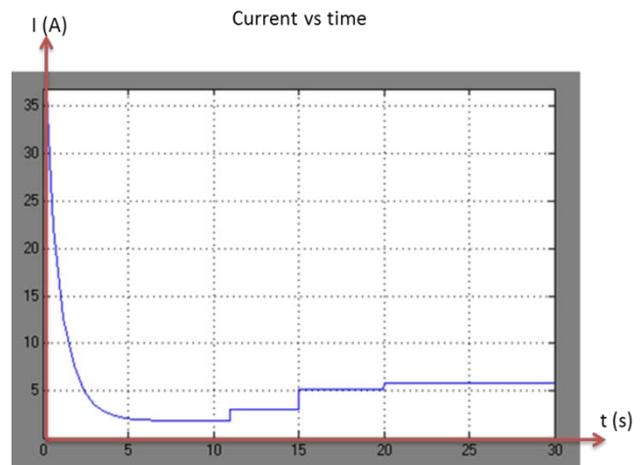


FIGURE 13. Graph of solar current arm vs. time.

As we can see in the Figure 13 there is no movement from the motor in the beginning, so there is no induced EMF and a big current is produced, notice that this current is the one in which the starter resistances are connected, otherwise the current will be even higher and in time will damage the motor. As the time passes, the motor starts to increase its velocity and a EMF is produced, because of this the current starts to decrease in time [8], and then the starter resistances are released after 11, 15 and 20 seconds, so the resistance of the system in these times is suddenly reduced thus the current is increased. After all the starter resistances are removed and the current reaches its stationary value that is 5.794 A.

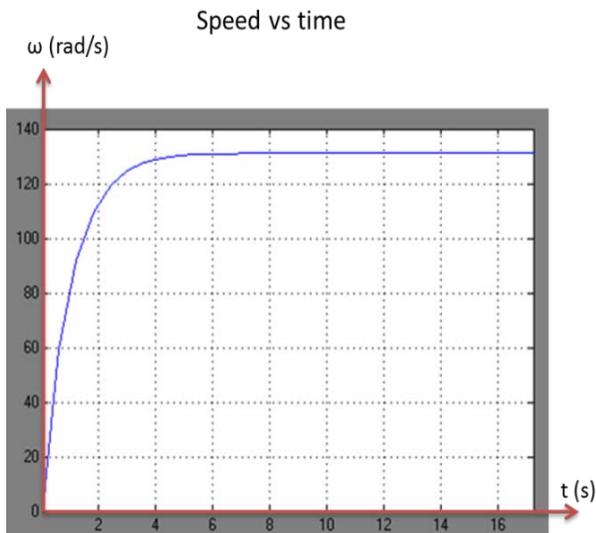


FIGURE 14. Graph of angular speed vs. time.

The Figure 14 shows that the motor is no moving at the beginning, when it is connect with the solar array, a current is generated on it that will start to make it move, we notice that it has an exponential behavior as seen in equation 9, and that it reaches an angular speed of 131.1 rad/s.

V. CONCLUSIONS

1. The starter system avoids a high current to be produced at the beginning of the motor operation and is acceptable because it will not force it.
2. A high number of solar cells were used to reach the motor nominal operation because they were not working at their maximum power output; this number could be reduced using a Maximum Power Point Tracking.
3. Due to the high combination of the arrangement of solar cells, they can provide a high range of voltage and current operation for different DC motors.
4. As seen from equation 8, the voltage value for the solar array can not be solved directly and it is necessary to use iterative methods, for this we propose the model block diagrams using Simulink, obtaining adequate results.

5. The mathematical model of motor speed is $f(t) = A(1 - e^{-\tau t})$ [9] and as we can see in the Fig. 14 and it does not change if we use a solar array as the source supply, which only influences in the magnitude of the velocity.

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