The Stern-Gerlach experiment as a problem-situation to the learning of concepts and principles of quantum mechanics in secondary school

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Abstract

We analyze the responses of first and third-year students of secondary school to a problem-situation on the Stern-Gerlach experiment, presented to introduce the concepts of: state of a physical system and linear superposition of states and others, during a course about principles of quantum mechanics. Also, the course was related to other problem-situations that will not be discussed in this paper. Part of the students was evaluated through a test, at the end of one of the classes; and all of them through a written examination at the end of the course. We used the Meaningful Learning Theory of Ausubel and the Conceptual Fields Theory of Vergnaud, as theoretical framework, and for to design the course and to analyze the collected data. It was possible to verify (according to the assessment scale adopted) a lack of homogeneity between the groups of students, indicating that their understanding of concepts occurred in different ways. Nevertheless, more than half of them showed an adequate response to the proposed questions; only one of the groups presenting a statistically significant difference, according to Student's t-test results.

Keywords: Quantum mechanics, fundamental concepts, Stern-Gerlach experiment.

Resumen

Analizamos las respuestas de los estudiantes de primer y tercer año de la escuela secundaria a una situación-problema, con el experimento de Stern-Gerlach, el cual se presentó para introducir los conceptos de: estado de un sistema físico y superposición lineal de estados y otros conceptos, durante un curso sobre principios de la mecánica cuántica. Parte de los estudiantes se evaluó a través de una prueba, al final de una de las clases; y todos ellos a través de un examen escrito al final del curso. La teoría del aprendizaje significativo de Ausubel y la teoría de campos conceptuales de Vergnaud, se utilizaron como marco teórico, para diseñar el curso y para analizar los datos obtenidos. Fue posible verificar (de acuerdo con la escala de cuotas adoptada) la falta de homogeneidad entre los grupos de estudiantes, lo cual indica que su comprensión de los conceptos se produjo de diferentes maneras. Sin embargo, más de la mitad de ellos mostró una respuesta adecuada a las preguntas propuestas; tan sólo uno de los grupos presenta una diferencia estadísticamente significativa, según los resultados de la prueba t de Student.

Palabras clave: La mecánica cuántica, conceptos fundamentales, Stern-Gerlach.

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I. INTRODUCTION

Teaching Modern and Contemporary Physics (MCP) to non-university students is a matter of concern in many countries.

In Brazil several publications [1, 2, 3, 4, 5, 6] called attention to the need of introducing topics of MCP at secondary school (SSC)[1]. Master degree programs for SSC teachers of physics also emphasize the importance of the inclusion of such contents.

Nevertheless, due either to lack of knowledge and of available pedagogical materials to help the teachers, and also to lack of time, or institutional difficulties [7], the fact is that, such goal is far from being achieved. Lines of research of some graduate programs that promote work on and applications of themes of MCP at the SSC level, originated local experiences, but did not manage (up to now) to increase the participation of teachers and schools

[1] The formal duration of secondary school studies in Brazil, a curriculum that precedes university studies, is of three years

(particularly public) at the enrollment of the students, both in number and in time.

Furthermore, some projects attained mainly students of universities and were not tested at SSC. It is then very important that investigations concerning teaching MCP at the secondary level be performed directly inside the classroom to provide the contextualization of themes present in the daily environment of the student.

One interesting possibility is to insert topics of Quantum Mechanics (QM), as it is the basis of many physics modern subjects, several of them now-a-days strongly related to technological applications shared by the students (as in communication systems) or mentioned in the media (tunneling, tele-transportation, quantum cryptography, entanglement and others). A way to allow that the community be acquainted with the essentials of such achievements is to introduce to the numerous groups of SSC alumni at least the basic QM concepts responsible for such ideas. An introductory course at such level is not imagined to emphasize the mathematical developments of QM, but is expected to give a correct notion on aspects of this theory not present in classical physics.

With these perspectives in mind we constructed and applied to eight secondary school classes a project with several problem-situations of QM, such as the double-slit experiment and the Stern-Gerlach experiment (SG), polarization states of light, quantum entanglement and quantum cryptography. In the present paper we focus on the learning of concepts by SG, as this rich problem-situation is seldom used at the introductory level.

II. THEORETICAL BACKGROUND

A. Learning theories

The project construction and data analysis used elements of the Meaningful Learning Theory (MLT) of David Ausubel [8, 9], and of the Conceptual Fields Theory (CFT) of Gérard Vergnaud [10, 11].

According to Ausubel, if one could reduce educational psychology to one single principle, this would be that: the more important factor to contribute to the learning, is what the learner already knows. It is necessary to those who teach, to check what the student previously knows, and teach accordingly. In general, when a student is introduced to QM he/she has already an (at least) intuitive knowledge at least originated in classical mechanics. But as the two theories are incommensurables in the kuhnian sense of the term, it is recommended at the initial studies of QM to avoid that it gets support on classical foundations.

Meaningful learning as every learning process, is a negotiation (and acquisition) of meanings; and in order to promote it, there are some initial conditions to fulfill:

- The material must be related to an adequate part of the cognitive structure of the student in a non-arbitrary and non-literal way.
- The cognitive structure of the learner must contain relevant ideas to which the new contents may connect.

Another condition is the existence of a pre-disposition of the student to learn. This should not be understood as motivation, since the learner may be motivated to learn, but not be pre-disposed. It does not help to have a potentially meaningful material to promote the learning, if the student is not willing to relate it meaningfully to its subsumers.

According to CFT, the perception of concepts results from situations [12]. A concept should not be regarded as a mere definition; it refers to a whole set of situations, comprises a number of distinct operational invariants, and its properties may be expressed through distinct symbolic and linguistic representations. Not only situations but concepts are also composed of operational invariants and representations, the first ones defining the operative character and the meaning of concepts [13]. As to representations they indicate and symbolize the invariants allowing so to represent situations and proceedings to deal with them.

A situation is to be understood as a task or a combination of tasks. To consider the process of conceptualization as the principal stone of cognition, Vergnaud states that the analysis of this process is founded on the pair scheme-situation, which has a dialectic relation. A concept becomes meaningful through a variety of situations, but the meaning is not at the situation itself.

The description of elements of a situation must be performed essentially in terms of properties and functions [14], so that the didactical situations might be considered rich and interesting [15]. The organization of a situation implies the simultaneous analysis of the epistemological functions of a concept, of the social meaning of the experienced areas to which it refers, of the behavior of the actors of the situation and the results of their behavior, and also of the contract and the transposition.

A conceptual field may be defined as an informal and heterogeneous set of situations and problems demanding several classes of concepts for the analysis and treatment procedures, and symbolic representations, inter-connected along the process of acquisition.

Knowledge is then organized in conceptual fields that the learner masters, in general during an expressive period of time, thanks to maturity, experience and learning [16]. A few classes are not enough in most cases, to allow that a student really keeps the knowledge of certain content.

Some students may take even years to really master a certain conceptual field. So, an early contact of the SSC students with QM concepts, at least prepares them for a better understanding of the field later.

CFT considers the teacher as an important mediator on the process of mastering a conceptual field by the student.

The first action of the teacher in this sense is to choose convenient situations and also expressions to use as an expression is discovered by the student only in exceptional cases. The situations must be chosen to conduct the student to an understanding of the concepts, occurring inside the zone of proximal development of the learner [17]. If complex situations are presented at the beginning of the study, they may cause a barrier for subsequent learning.

Regarding QM, we consider that many situations where the main concepts are involved, are necessary to lead to the understanding by the students. As already mentioned, our project included the double-slit and the Stern-Gerlach experiments, polarization states of light, quantum entanglement and quantum cryptography, as situations to help the students to understand concepts related to states and determination of states of quantum objects; and for instance the non-compatibility of certain observables and results of measurements. Also, situations presented in classical physics as superposition of waves and of geometrical vectors, were used to construct the concept of linear superposition to be used in QM. Such structure found in classical areas of physics was of help at the contextualization of important concepts in QM, but it was not adopted to explain quantum phenomena with classical concepts.

B. Spin and the Stern-Gerlach experiment

The choice of SG as one of the problem-situations to use in the course rests on various motivations as for instance:
the importance of the spin in physics and its technological consequences, and the richness of such experiment to provide the discussion of the main QM concepts and basic principles.

Spin or intrinsic angular momentum, is mentioned in SSC usually in chemistry disciplines, where it is presented in general as an intrinsic characteristic of elementary particles and of systems, like atoms and molecules. In physics, one is very interested in realizing how this property was (and is) observed, how it influences important properties of single and complex systems, and also how the knowledge about it was, and still may be used to generate advances in technology. To provide information about the spin, the theory of QM was essential (a need to obtain answers that physicists asked for). In fact, understanding the spin may be seen as an example of equilibrium between developments of experiment and theory in QM. Furthermore, important properties of physical systems observed in old times as the ones related to magnetism, were only understood after the introduction of spin characteristics of particles in QM, as this is a property not previously identified in classical physics. Up to then classical theory "explanations" on the subject were presented, but they consisted mainly in describing the effects (behavior) of the different known magnetic materials.

The Stern-Gerlach experiment [18] allied to the Uhlenbeck-Goudsmith hypothesis [19] showed the presence of this intrinsic angular momentum in nature, and opened a door to a whole field of consequences, still surprising us now-a-days. To inform to the community about the existence of such a characteristic of particles, is then to allow them to join to mass and charge.

For instance, a new notion to be classified non-relativistically as an intrinsic manifestation of nature.

In the context of teaching QM, SG is an excellent situation to be explored. Since the spin of the electron is $(½)\hbar$, it may be described in a two-dimensional complex space, the smaller non trivial space to explore effects of non-compatible observables, collapse of states due to measurements, probabilities of occurrence of results of measurements and others. Though the experiment itself is in general not available to SSC students, interesting simulations of it may be found in the literature, and this was in fact a choice for these classes. In this study, we used simulations freely available at the PhET website[2].

Basically the experiment performed by Stern and Gerlach consisted in passing a beam of silver atoms through an inhomogeneous magnetic field (in a certain direction), collecting the emergent beam in a screen. The result of the experiment showed two groups of atoms deviated from the central region of the screen.

An unexpected appearance (initially), if the angular momentum of these atoms were only of orbital nature (as the orbital angular momentum of the silver atoms – and of its last electron - is equal to zero in the fundamental state)–.

The fact that the projections of angular momentum are not compatible observables, the use of such problem-situation enriches the scenario.

III. RESEARCH METHODOLOGY

The course was developed in 24 class-hours, one of its goals being to analyze the potentiality of SG as a problem-situation to the learning of some of the first concepts and principles of QM. The contents programmed and discussed are the following ones.

- Introduction
- Summing waves, vectors and states
- Summing waves
- Summing vectors on the real plane
- Double-slit experiments – the first oddities
- Vector spaces
- Describing physical systems
- Binary systems
- Quantum bits
- Spin and the Stern-Gerlach experiment
- Non-binary systems
- Which are the main fundaments of the theory that explains quantum phenomena successfully?
- Is summing states a consequence of quantum mechanics?
- What do we understand as state of a physical system?
- Quantum objects
- Measurements in quantum mechanics
- What problems may we face when measuring physical quantities?
- Limiting the values of measurements
- Linear superposition of states and quantum entanglement
- Security at transmission of information: quantum criptography

[2] The simulations used in the course were available at http://phet.colorado.edu/pt_BR/simulations/category/physics/quantum-phenomena.

Here we focus on the concept of intrinsic angular momentum (spin) of the electron. The concepts of state of a physical system and of linear superposition of states, were discussed before the one of the spin and of the presentation of SG, and so, could be considered as a subsumer for the new topic. At the same time those concepts were reinforced with results of several possibilities of successive SGs.

Eight groups of students of two public secondary schools of Santa Catarina, Brazil, participated of the project.

Application of the instruction was performed in two stages:

The first one, to three groups of students (two of first-year and one of third-year, of one of the school) in a total of 79 scholars.

The second, one to five groups of students (two of first-year and three of third-year, of the other school) totalizing 147 scholars.

A written text including the contents to be discussed and related exercises, was prepared and distributed to the students.

The introduction of intrinsic angular momentum (spin) of the electron was reinforced with the use of a computer simulation (applet) of SG, freely available at the PhET website. Individual determinations of the emergent spin ½ particles of a beam for different orientations of the incident beam and successive passage of beams, through different inhomogeneous fields were exercised, and the results explored. Several concepts were discussed and exemplified along the instruction, as the ones of physical system and states of a physical system, linear superposition of states, eigenvalues as values of measurements, compatible and non-compatible variables, probabilities of obtainment of eigenvalues and collapse of states under measurement.

To estimate the understanding of the students about the fundamental concepts of QM considered, the two sets of students have been submitted to an evaluation, seven questions; at the end of the instruction one of them specifically about SG. In addition, students of the second phase have been previously submitted to a quiz about SG at the end of one of the classes. In total 143 students (70 first-year and 73 third-year) presented answers to the quiz. And 226 students (57 first-year and 22 third-year of the first-phase, and 70 first-year and 77 third-year of the second phase) responded to the evaluation questions.

Additional data not considered in this paper, concerning the learning and attitudes of the students, were taken from mental maps and from opinions registered in questionnaires answered at the end of the course. Answers presented to each question were evaluated according to Table I bellow:

TABLE I. Scores attributed to performance on questions of the quiz and of final evaluation.

Criteria of evaluation	*Scores*
Incorrect answer	0
Incorrect answer with some correct elements	1
Correct but laconic answer or with few incorrect elements	2
Correct answer	3

The analysis of the results of quizzes and final evaluation questions was performed according to variation coefficients (VC), with the exploratory analysis of Kendall [20] proposed by Barbancho [21], to verify the homogeneity of the group.

Student´s t-test [22] was used both on quizzes and on evaluation questions to verify the occurrence of statistically significant differences between groups of different secondary school years at the same phase and between groups of the same years at distinct phases.

IV. FINDINGS

A. Quiz results

Quizzes adopted were short tests applied to verify the immediate learning related to the contents of one class. At the end of one of the encounters, where SG was presented and used to help the understanding of some concepts of QM, the following quiz was proposed to students to get hints about the learning, in this first contact with the problem-situation.

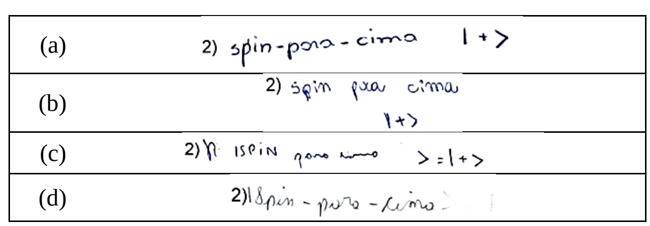
"Um elétron que foi sujeito ao experimento de Stern-Gerlach (campo na direção vertical) é registrado no grupo que está na parte de cima do anteparo. Em que estado podemos dizer que se encontra este elétron?[3]"

Of 147 students 57% presented correct answers, 19% answered partially correctly and 21% incorrectly. Only four students (less than 3%) did not answer the question.

The VC analysis and Student´s t-test showed two interesting and different sides about the knowledge of the students on the topics questioned at the end of the classes.

The VC values of the tests for the first-year and third-year students, were respectively 0.38 and 0.50. So, according to the VC of each year in this question, there was not homogeneity of the groups in the understanding of the concepts involved. Comparison between the answers of the groups of the two years, in a significance level of 95%, shows the value 1.56 for Student´s t-test, showing no statistically significant difference between the two years, in spite of their non homogeneity. Also, only two students of first-year and two students of third-year did not answer this question; is an indication of involvement of the learners with the subject.

As an illustration some answers of students are shown in Figure 1.



[3] Translation – An electron that was subjected to the Stern-Gerlach experiment (field at the vertical direction) is registered at a group on the upper part of the screen. In which state may we say that this electron is?

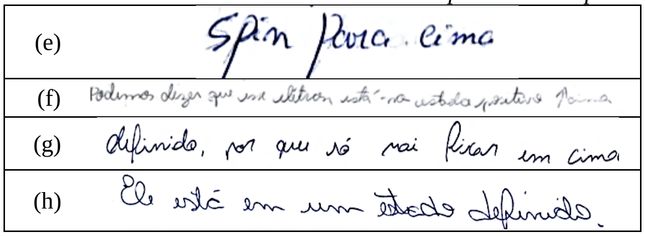


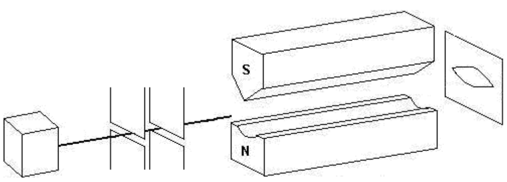
FIGURE 1. Answers[4] to quiz of students (a) 20102; (b) 20111; (c) 20211; (d) 20214; (e) 20329; (f) 20336; (g) 20413; (h) 20507.

The numeration system refers to secondary school year, class and student in each class, avoiding public identification.

B. Results on the Stern-Gerlach question of evaluation

Students were submitted to the final evaluation on the area after the presentation of contents of QM and some problem-situations. The question about SG (one in seven) reads:

Qual dos estados a seguir pode ser atribuído a elétrons que, submetidos ao experimento de Stern-Gerlach, apresentam resultados como os indicados na figura[5] abaixo? Justifique sua escolha.[6]



a) $\left|\Psi\right\rangle = \left|-\right\rangle$ c) $\left|\Psi\right\rangle = \left|+\right\rangle + \left|+\right\rangle$

b) $\left|\Psi\right\rangle = \left|+\right\rangle$ d) $\left|\Psi\right\rangle = \left|+\right\rangle + \left|-\right\rangle$

Table II presents the number of scores attributed in each year to the SG question of the final evaluation at the two phases of the investigation, according to the criterion of valorization presented before.

TABLE II. Quantity of scores for the SG question at the evaluation.

Phase	*Year*	*0*	*1*	*2*	*3*	*Blank*
1	1st	14	2	19	10	12
	3rd	4	0	10	5	3
2	1st	5	0	33	32	0
	3rd	8	0	32	37	0
TOTAL		31	2	94	84	15

According to this table one obtains as a result of the responses of the 226 students submitted to the evaluation that:

- 39.8% of the answers presented were correct;
- 44.5% of the answers presented were almost correct;
- 15.6% of the answers presented were incorrect (scores 0 and 1);
- 6.6% of the students did not answer the question, all of them pertaining to the groups of the first phase of the investigation.

We can see that 84.4% of the students attained a high score of correct, or almost correct answers (scores 2 and 3).

This majority of good answers can be considered as an indication of some understanding about the subject (although it is not possible to state that this was in fact meaningful learning).

Data obtained led to Table III, where the average score and the standard deviation are registered for each group of students separately.

Scores produce at phase 1 values 0.75 and 0.58 for the VC of first and third years respectively, and at phase 2 values 0.35 and 0.40, for first and third years respectively. These numbers indicate that, the average scores of the four groups considered do not reflect their conceptual reality in this evaluation, showing that we are dealing with heterogeneous groups. Nevertheless, when one compares only the phase 2, groups one sees more homogeneous results more homogeneous, and also an improvement on the homogeneity relative to the quiz responses, homogeneity relative to the quiz responses, signaling to a better understanding of the matter under consideration.

TABLE III. Average score and standard deviation of each group of students for the SG question of the evaluation.

Phase	*Year*	*Average score*	*Standard deviation*
1	1st	1.56	1.16
	3rd	1.84	1.07
2	1st	2.31	0.81
	3rd	2.27	0.91

In a confidence level of 95% the Student´s t-test result presents the value t=3.832 in the comparison between the groups of first-year of both phases, and t=1.618 in the third-year case. This shows that there was statistically significant difference only between the average scores of the first-year groups. When looking at groups of a same phase t=0.956 and t=0.293 (first and second phases respectively), indicating that there was not a statistically significant difference between the years of a same phase, in spite of the lack of homogeneity.

Figure 2 shows the answers of seven students distinguished with the use of the same labels as before, but concerning different identifications now, as compared to

[4] Translation – Students 20102, 20111, 20211, 20214 and 20329: “spin up”. Student 20336: “We may say that this electron is in the positive “up” state. Student 20413: “defined, because it will only stay on top”. Student 20507: “It is in a defined state”.

[5] Adapted from [23].

[6] Translation – Which of the following states may be attributed to electrons that, submitted to the S-G experiment present, results as those indicated at the figure bellow? Justify your choice.

Figure 1. Notice that in this case students of both first and third years are quoted.

Considering the answers of the students, is possible to find signs of learning and to suppose the adequacy of the problem-situation used.

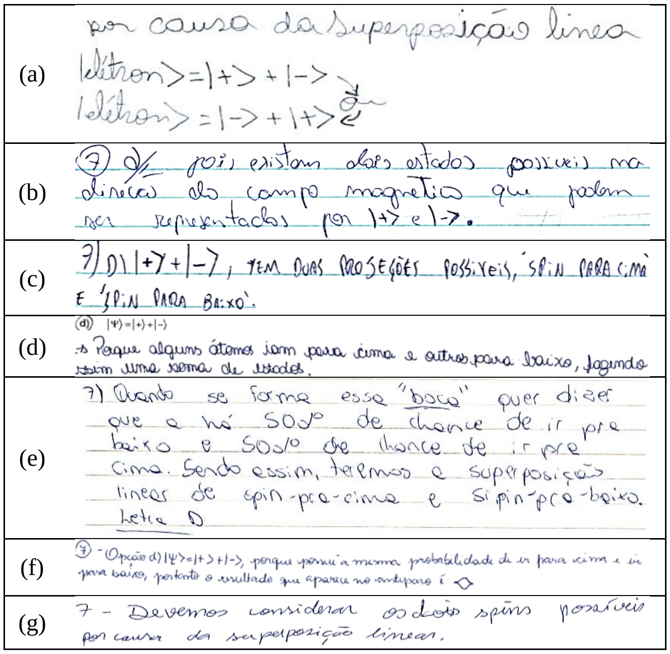


FIGURE 2. Answers[7] to the question about SG at the evaluation of students: (a) 10119, (b) 10303, (c) 10312, (d) 20115, (e) 20125, (f) 20231, (g) 20515.

The considerations presented focused on the understanding of QM concepts with the use of SG as a problem-situation.

Answers mentioned in figure 2 above, show the presence of elements suggesting an understanding about linear superposition of sates in relation to experimental results, particularly the ones of students 10119, 20115, 20125 and 20515. Several students marked correctly the alternative corresponding to the state of the electron involved at the experiment without justifying their answers, so they were evaluated with score 2.

In this question 37.2% of the students presented answers considered correct (score 3), and 78.8% showed answers considered either correct or partially correct (scores 2 and 3).

This high number of answers with good scores seems to indicate that SG is a problem-situation with easy assimilation of QM concepts, and may be used with a relative success when introducing QM at the SSC level[8].

V. CONCLUSIONS

The results of the evaluation about the learning concerning the Stern-Gerlach experiment, presented during an introductory course on QM, show indications of initial mastering of the conceptual field of QM by the students. A deep mastering of a conceptual field demands in general a long time of study (besides maturity and experience), but one can say that this first contact with QM accomplished the role of a first correct scientific alphabetization of some of the basic concepts and principles of QM frequently mentioned at many popular information channels, often without a scientific rigor, or even incorrectly, and that may lead to wrong interpretations by the students.

In a more general aspect the chosen situations were important for the motivation and understanding about several concepts and principles discussed. SG in particular (together with the double slit experiment) showed to be of easy assimilation by the students, and may be considered adequate to introduce QM concepts at SSC (even at the first-year level). A comparison with questions related to the hydrogen atom (not focused in this paper) indicates that SG is more easily mastered both, because more students answered to SG question, as because the correct answers were more numerous. In fact against the 14 blanks presented at phase 1, first-year, for SG, the hydrogen atom answers for two questions showed an average of 20 blanks. Also the 3 blanks of SG at phase 1 third-year, are confronted with an average for hydrogen atom of 10 blanks.

As to phase 2 thought results for blanks are low in the situations mentioning the hydrogen atom, they are compared to zero blanks for SG (total of 36 blanks for hydrogen atom average, versus 15 blanks for SG). Also, a total of 178 scores 2+3 for SG answers, is compared with 79.5 for scores for the average of two hydrogen questions.

Considering that this was a first course of short duration on concepts and principles of QM for the majority of students involved and that it had a, the indication of some learning seems to be positive. Also it may be considered that SG is a good choice of a situation–problem, not only for an undergraduate level, but also for the SSC one (even at the first-year). The perception of the main differences of QM concepts as compared to classical ones will probably remain with these students even if they forget how they acquired them. We then suggest that SG be used at the SSC level (joining other problem-situations as the double-slit experiment, polarization states of photons, quantum entanglement and quantum cryptography) both to include in the curricula some MCP, as to motivate the students for the learning of a physics that is nearer to the scientific and

[7] Translation - (a) due to the linear superposition [and two examples of the electron state]. (b) Choice d, because exist two possible states in the direction of the magnetic field that can be represented by |+> and |->. (c) Choice d, |+> + |->, there are two possible projections, "spin up" and "spin down". (d) Choice d. Because some atoms went up and others down, making so a sum of states; (e) When this "mouth" is formed, it means that there is a 50% chance of going down and a 50% chance of going up. So that, we will have the linear superposition of spin-up and spin-down. Choice D. (f) Option d, because it has the same probability to go up and down, so the result shown in the screen is [image]. (g) We must consider the two possible spins because of the linear superposition.

[8] Preliminary results on this matter were presented at ICPE-EPEC 2013, Prague, Czech Republic

technological actual evolution. And to avoid big misconceptions about QM at an important moment of their studies, when they have the rare opportunity to be confronted with really new knowledge in several areas.

After SSC the majority of students will either prioritize to abandon formal studies and get some job or to follow studies in a particular area at universities. Very few of them will meet again QM in formal studies. Students with a basic correct feeling about QM will also to propagate misconceptions about the field, contributing to break a chain of wrong considerations about this theory frequently spread.

REFERENCES

[1] Terrazan, E. A., *A inserção da física moderna e contemporânea no ensino de física na escola de 2º grau*, Cad. Catarin. Ensino Fís. **9**, 209-214 (1992).

[2] Oliveira, F. F., Vianna, D. M. and Gerbassi, R. S., *Física moderna no ensino médio: o que dizem os professores*, Rev. Bras. Ens. Fís. **29**, 447-454 (2007).

[3] Rezende Jr., M. F. and Cruz, F. F. de S., *Física moderna e contemporânea na formação de licenciandos em física: necessidades, conflitos e perspectivas*, Ciênc. Educ. **15**, 305-321 (2009).

[4] Fanaro, M. A., Otero, M. R. and Arlego, M., *Teaching the foundations of quantum mechanics in secondary school: a proposed conceptual structure*, Inv. Ens. Ciênc. **14**, 37-64 (2009).

[5] Ostermann, F. and Moreira, M. A., *Física contemporánea en la escuela secundaria: una experiencia en el aula involucrando formación de profesores*, Ens. Cienc. **18**, 391-404 (2000).

[6] Rocha, C. R., Moreira M. A., Herscovitz, V. E., *Introdução à Mecânica Quântica: uma proposta de minicurso para o ensino de conceitos e postulados fundamentais*, Rev. Bras. Ensino Ciênc. e Tecnologia **3**, 10-15 (2010).

[7] Ostermann, F. and Cavalcanti, C. J. H., *Um pôster para ensinar física de partículas*, Física na escola **2**, 13-18 (2001).

[8] Ausubel, D. P., *Educational psychology: a cognitive view*, (Holt, Rinehart and Winston, New York, 1968).

[9] Ausubel, D. P., *Acquisition and retention of knowledge: a cognitive view*, (Kluwer Academic Publishers, Dordrecht: NED, 2000).

[10] Vergnaud, G., *A comprehensive theory of representation for mathematics education*, J. Mat. Behav. **17**, 167-181 (1998).

[11] Vergnaud, G., *¿En qué sentido la teoría de los campos conceptuales puede ayudarnos para facilitar aprendizaje significativo?*, Investig. Ensino Ciênc. **12**, 285-302 (2007).

[12] Vergnaud, G., *La théorie des champs conceptuels*, Rech. Didact. Math., **10**, 133-170 (1990).

[13] Moreira, M. A., *A teoria dos campos conceituais de Vergnaud, o ensino de ciências e a pesquisa nesta área*, Investig. Ensino Ciênc. **7**, 7-29 (2002).

[14] Barais, A. W. and Vergnaud, G., *Students' conceptions in physics and mathematics: biases and helps*, In: Caverni, J. P.; Fabre, J. M.; Gonzalez, M. (Eds.), Cognitive Biases, (Elsevier Science Publishers, North Holland: NED, 1990).

[15] Vergnaud, G., *Teoria dos campos conceituais*, In: Nasser, L. (Ed.), Anais do 1º Seminário Internacional de Educação Matemática do Rio de Janeiro (1993).

[16] Vergnaud, G., *A classification of cognitive tasks and operations of thought involved in addition and subtraction problems*, In: Carpenter, T., Moser, J. and Romberg, T. Addition and subtraction. A cognitive perspective, (Lawrence Erlbaum, Hillsdale: USA, 1982).

[17] Vygotsky, L. S., *A formação social da mente*, (Martins Fontes, São Paulo, 1991).

[18] Gerlach, W. and Stern, O., *Der experimentelle nachweis des magnestischen moments des silberatoms*. Z. Phys. A**8**, 110-111 (1922).

[19] Uhlenbeck, G. E. and Goudsmit, S., *Spinning electrons and the structure of the spectra*, Nature **117**, 264-265 (1926).

[20] Kendall, M. G., *The advanced theory of statistics*, (Griffin, London, 1977).

[21] Barbancho, A. G., *Estadística elemental moderna*, (Ariel, Barcelona, 1992).

[22] "Student" Gosset, W. S., *The probable error of a mean*, Biometrika **6**, 1-25 (1908).

[23] Sakurai, J. J., Napolitano, J., *Modern quantum mechanics*, (Addison-Wesley, San Francisco, 2011).