A Comparative Epistemological study between Copenhagen and Many World's Quantum Mechanics Interpretations, with a Simple Educational Application

Rabah Ladj
Laboratory of N Corps and Matter structure, Laboratory of education of science and mathematics, School of Physics E.N.S, Kouba, Algiers Algeria.

E-mail: r.ladj@hotmail.com

(Received 6 June 2017, accepted 9 November 2017)

Abstract
Quantum mechanics describes the behavior of the quantum objects, but the description and the interpretation of its basic principles; (nature of the wave, interpretations of: superposition principle, complementary principle, and uncertainly principle), these latter have had many debates concerning their meanings and their concepts between the scientists. Since the beginning of its appearance until today, many different interpretations between many schools and scientists have emerged, about the descriptions of quantum mechanics. In our present contribution, a modest modeling for many worlds interpretation is presented, followed by solving and interpreting a famous example "particle in an infinite square potential well", using our modeling and then, a comparison between Copenhagen and many world interpretations.

Keywords: Quantum Mechanics; Copenhagen interpretation; Many Worlds interpretation; Modeling; Educational Application.

Resumen
La mecánica cuántica describe el comportamiento de los objetos cuánticos, pero la descripción y la interpretación de sus principios básicos; (naturaleza de la onda, interpretaciones de: principio de superposición, principio complementario y principio incertidumbre), estos últimos han tenido muchos debates sobre sus significados y sus conceptos entre los científicos. Desde el comienzo de su aparición hasta hoy, han surgido muchas interpretaciones diferentes entre muchas escuelas y científicos, sobre las descripciones de la mecánica cuántica. En nuestra presente contribución, se presenta un modesto modelado para la interpretación de muchos mundos, seguido por la resolución e interpretación de un famoso ejemplo "partícula en un potencial infinito cuadrado", utilizando nuestro modelado y luego, una comparación entre Copenhague y muchas interpretaciones del mundo.

Palabras clave: Mecánica cuántica; Interpretación de Copenhague; Interpretación de muchos mundos; Modelado; Aplicación educativa.

PACS: 01.40.Fk, 01.50, My. **ISSN 1870-9095**

I. INTRODUCTION

In order to understand quantum theory, you have to forget everything known about objects behavior in classical mechanics, especially its basic philosophical principles, such as the causes, the determinism, the effect, the reality, the certainty and much else besides. The objects in quantum mechanics behave in a way really different to that of classical one. It has its own ideas and rules. Concerning this subject, Richard Feynman said: "Quantum mechanics is the description of the behavior of matter and light in all its details and, in particular of the happenings on an atomic scale. Things on a very small scale behave like nothing that you have any direct experience about. They do not behave like waves, they do not behave like particles, they do not behave like clouds, or billiard balls, or weights on springs, or like anything that you have ever seen" [1]. As it is already known, an interpretation of quantum mechanics is a set of statements which attempt to explain how quantum mechanics informs our understanding of nature. Quantum mechanics has received many different interpretations through experimental testing. There is a number of contending schools of thoughts, having different opinions related to quantum mechanics, whether it can be understood to be deterministic; in which its elements can be considered "real" or something else? [2]. Copenhagen and Many-Worlds interpretations are the most widely schools, which are considered as objects' description of quantum mechanics. The following (Table) [3], shows their different point of view related to basic concepts of the field.

TABLE I. Differences between Copenhagen and Many-Worlds interpretation.

Copenhagen interpretation Author(s): Niels Bohr and Werner Heisenberg, 1927	Many-Worlds' Interpretation Hugh Everett 1957
Is not deterministic	Yes is deterministic
Wave function is not real	Wave function is real
Collapsing wave function	None Collapsing wave
None Hidden Variables	None Hidden Variables
Role of Observer is Causal	Role of Observer is not Causal
None Local	Yes is Local
Collapsing wave function	None Collapsing wave

A. Copenhagen's interpretation

Up today Copenhagen's description is the common interpretation used in teaching. It is based on two major principles. The first is the Complementarity principle; which means that the object in the atomic scale behaves like wave or particle [4]. The second one is the Heisenberg's uncertainly principle. Adding to last two major principles, Max Born has given a statistical interpretation for the wave function density [5]. Unfortunately, those two basic principles violated the two pillars principles of classical mechanics' philosophy; (final causality and determinism principles). Although, even, the Copenhagen interpretation showed a successful description of the atomic world, it failed and showed some flaws in the interpretation of certain thought experiments, like Schrodinger Cat paradox, collapsing and no reversible of the wave function, etc…. For Niels Bohr in his complementarity, "an object is whatever it is measured is to be, a wave or a particle, when it looks like a wave, it is a wave, when it looks a particle, it is a particle"; in other words, the concept wave and particle of the object are not an intrinsic aspect, but are only an appearances [6]. More than that, for Bohr, it is meaningless to talk about any properties or even existence of any object which has not been measured. Bohr said: "nothing is real unless it is observed" adapted from [7]. The uncertainly principle means that: you cannot determine either the position or momentum of an object as accurately as you like, in addition, the act of doing that, makes your measurement of the other property much less [4]. For the Copenhagen school, Human being may, someday, build a device which is able to transport objects across the galaxy, but no one will ever be able to measure both the momentum and the position of an object at the same time [8].

B. Many- world's interpretation

Hugh Everett considered that, as soon as, a potential exists, for any object to be in any state, the universe of that object will be a series of parallel and reversible universes; "branches" equal to the number of possible states, in which that the object can exist in each universe. Furthermore every universe has a unique single possible state of that object [9]. According to Everett, there is only one universal wave function describes the entire world. In other words; the oneness of presence or existence and the other presences are just copies. Then any changes noticed in a single copy, will induce instantly all the observables in the other split worlds, without any information's exchange between them, and whatever the distance between, this doesn't mean that violate the classical speed of light, because there's no "movement" through space. According to Everett, Bohr's complementarity is just the particle's world split into two worlds in one the particle is a wave and in the other world is particle. We mention here that many theoretical scientists in the field were and are convinced with Everett's interpretation, among them; Stephen Hawking and Richard Feynman. We note that, the idea of a single universal wave function philosophy which represents the universal world, is nearly closed in the meaning to the *solutions' idea* of Islamism's sophism philosophy, also of old Indian's faith, both of them believe that there is only a single existence or there is just one reality, "Pantheism" named "ALLAH" (GOD), and all other existences are copies of HIM [10].

C. An important notice

The above sentence's meaning are: **the paragraph:** *"existence of potential"* represents the <u>cause</u>, and **the paragraph:** *"transmits to series of parallel universes"* represents the <u>final</u>. So many worlds interpretation is subject to final causality principle. And since; there is not any mutual influence between observer and the observable, the object's behavior is deterministic.

II. A MODEST MODELING OF HUGH EVERETT INTERPRETATION

A. Case of stationary states

As is discussed above, and according to Everett: as soon as, a potential exists for any object to be in any state, the universe of that object transmutes or splits into a series of parallel universes "branches" which are equal to the number of possible states where the object can exist, and each universe containing a unique single possible state of that object, Everett H. (1957). Because of parallel of states; we symbolize the universe by $\left|\psi\right\rangle$, where:

$$\begin{aligned} \left|\psi\right\rangle &= \left|\varphi\right\rangle_1 \otimes \left|\varphi\right\rangle_2 \ldots\ldots\ldots \otimes \left|\varphi\right\rangle_n \\ \left|\psi\right\rangle &= \prod_{j=1}^{n\geq 2} \left|\varphi\right\rangle_j \otimes \left|\varphi\right\rangle_{j+1}, \quad j(\leq n) \end{aligned} \tag{1}$$

$$j(\leq n) = \begin{cases} 1,3,\ldots(n-1); & if \quad 'n'\ Even \\ 1,3,\ldots..n; & if \quad 'n'\ Odd \\ 'n' \quad worlds' \quad number \\ \left|\varphi\right\rangle_{j_{Max}+1} = \left|\varphi\right\rangle_n \end{cases}$$

Note 1: Parallel universes; mean that there is no interference (superposition) between them. As a result of this, there is no information exchanged between the states. So the universal wave function which describes the universal world is a tonsorial product of states or worlds.

When a measurement is taken in the universe $|\psi\rangle = \prod_{j=1}^{n\geq 2} |\varphi\rangle_j \otimes |\varphi\rangle_{j+1}$ using a tool of observation "observer" represented by $\widehat{A}_i$ "operator", operate on $|\psi\rangle$ observes or measures, only its correspondent eigenvalue λ_i , and so on, for other observers $\widehat{A}_k$, $k = 1,2,.. \neq j$, each observer measures its correspondents eigenvalues $\lambda_k \neq \lambda_j$. We operate by just one observer let it be; $\widehat{A}_j$ on $|\psi\rangle$, as follows:

$$\begin{aligned}
&\widehat{A}_j \prod_{j=1}^{n} |\varphi\rangle_j \otimes |\varphi\rangle_{j+1} = \lambda_i \prod_{j=1}^{n} |\varphi\rangle_j \otimes |\varphi\rangle_{j+1} \\
&\Rightarrow \prod_{j=1}^{n} {}_{j+1}\langle\varphi| \otimes {}_j\langle\varphi|\widehat{A}_j|\varphi\rangle_j \otimes |\varphi\rangle_{j+1} = \lambda_j \\
&\textit{So the expectation value is } \langle \widehat{A}_j \rangle_{|\varphi\rangle_j \otimes |\varphi\rangle_{j+1}} \\
&= \lambda_j. \;\; \textit{the normalisation of } |\psi\rangle \textit{ gives}: \\
&\langle\psi|\psi\rangle = 1 \Rightarrow \langle\varphi|\varphi\rangle_{ij} = \delta_{i,j}
\end{aligned} \tag{2}$$

When a measurement is taken by all observers: $\sum_{j=1}^{n} \widehat{A}_j$ in which every observer $\widehat{A}_j$ measure only its correspondent Eigenvalue λ_j in his world $|\varphi_j\rangle$ " because of worlds' paralleled; by other word there is no superposition of states". The general form of the universo $|\psi\rangle$ in function of all universes takes the form:

$$|\psi\rangle = \prod_{j=1}^{n\geq 2} \sum_{i=1}^{n\succ 2} (\pm)^{i-1} \alpha_i \alpha_j |\varphi\rangle_i \otimes |\varphi\rangle_j \quad . \tag{3}$$

The factor $(\pm 1)^{i-1}$ is due to mirroring (reflection) property (copy).

Note 2: $\alpha_i \alpha_j |\varphi\rangle_i \otimes |\varphi\rangle_j \neq \alpha_i |\varphi\rangle_i \otimes \alpha_j |\varphi\rangle_j$, the first part $\alpha_i \alpha_j |\varphi\rangle_i \otimes |\varphi\rangle_j$ indicates to no separate states "entanglement", while the second one $\alpha_i |\varphi\rangle_i \otimes \alpha_j |\varphi\rangle_j$ refers to separated states. The factor (the coefficient) $\alpha_{j,j+1}$ represents the entanglement "entanglement factor".

Now and because the universo which is represented by the wave function $|\psi\rangle$ is normalized to the unit, so we write:

$$\begin{aligned}
\langle\psi|\psi\rangle &= \int_{-\infty}^{+\infty} \psi^* \psi \, dv = 1 \\
&= \prod_{j\neq i}^{n\geq 2} \sum_{i=1}^{n\geq 2} (-)^{i-1} |\alpha_i|^2 |\alpha_j|^2 = 1 \\
& \quad i = 0, 1, ...n
\end{aligned} \tag{4}$$

Where the entangled factor is egual to $\alpha_i \alpha_{j,} = \sqrt{\frac{1}{n}}$.

Note 3: Here all the wave functions are real and reversible and there is no collapsing of waves, and no degeneration of eigenvalues. Because of the entanglement of states, once any observer in its states, measures its Eigen value, all other eigenvalues of the other observers will be instantly known, even there's no exchange of information between them.

Note 4: Here the Hilbert space is constructed of tonsorial victors' product.

Now we would like to clarify the idea, by the following case: we consider an universe $|\psi\rangle$ "equation (3)" has the potential to split into four universes, in this case, this $|\psi\rangle$, in function of the different four universes related to the four observes $\widehat{A}_i$ in which everyone observes or measures only its correspondent eigenvalue λ_i, $i = 1,2,3,4$, takes the form:

$$|\psi\rangle \Leftrightarrow \begin{cases} \rightarrow \left(|\varphi\rangle_1 \otimes |\varphi\rangle_2 \otimes |\varphi\rangle_3 \otimes |\varphi\rangle_4\right)_{w_1} \\ \rightarrow \left(|\varphi\rangle_2 \otimes |\varphi\rangle_3 \otimes |\varphi\rangle_4 \otimes |\varphi\rangle_1\right)_{w_2} \\ \rightarrow \left(|\varphi\rangle_3 \otimes |\varphi\rangle_4 \otimes |\varphi\rangle_1 \otimes |\varphi\rangle_2\right)_3 \\ \rightarrow \left(|\varphi\rangle_4 \otimes |\varphi\rangle_1 \otimes |\varphi\rangle_2 \otimes |\varphi\rangle_3\right)_4 \end{cases} \tag{5}$$

Note 5: The above universes' ranking; (tonsorial product), mathematically is not required, but here it is important, to refer to every operator's universe.

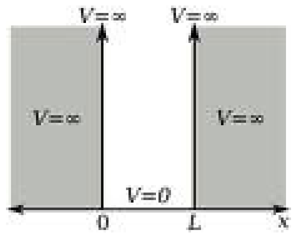
III. APPLICATION

Our educational application aims to compare Copenhagen's and many worlds' interpretations.

A. Copenhagen's interpretation

We take the famous following example; an infinite one dimension square well potential; Particle in a box. (Case: particle moves in only one direction).

$$V(x) = \begin{cases} 0 & 0\langle x\langle L \\ \infty & elsewhere \end{cases}.$$



The aim is to calculate the energies occupied by the particle in this kind of potential, so the tool of observation is a Hamiltonian operator $\widehat{H}$. From boundary conditions which are $\Psi(x)$ vanish at x=0 and x=L, the particle is free in moving within the well and has no possibility of moving into the region outside of the well. Therefore, because $\widehat{H}$ is independent of time, the stationary Schrodinger equation is needed in order to solve the problem. As it is well known the solution gives the following results:

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \quad and$$
$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi}{L}x) \ , \ n = 1,2,...$$

The general wave function $\Psi(x)$ describes the particle in the box is: $\Psi(x) = \sum_1^n c_i\varphi_i$.

Where $\sum_1^n |c_i|^2$ =1, and $|c_i|^2$ is the probability of finding the particle in state $\psi_i(x)$ with the energy: $E_i = \frac{i^2\pi^2\hbar^2}{2mL^2}$. If we consider the particle is in two first states only, then:

$$\varphi_1(x) = \sqrt{\frac{2}{L}}\sin(\frac{\pi}{L}x) \ , \ E_1 = \frac{\pi^2\hbar^2}{2mL^2}.$$
$$\varphi_2(x) = \sqrt{\frac{2}{L}}\sin(\frac{2\pi}{L}x) \ , \ E_2 = \frac{4\pi^2\hbar^2}{2mL^2}.$$

According to Copenhagen interpretation the above results are interpreted as follows:

-Before measurement, the particle is in the superposition of two states $\varphi_1(x)$ and $\varphi_2(x)$ in the same time. So, is in $\psi(x) = \frac{1}{\sqrt{2}}\varphi_1(x) + \frac{1}{\sqrt{2}}\varphi_2(x)$ with energies E_1 and E_2.

- Once a measurement is done, the two states will collapse into only a single one, (single wave function, either φ_1 or φ_2), with the correspondent energy E_1 or E_2. And because of the irreversible of the wave function! When another measurement is done, the second energy will be measured.

B. Many word interpretation

Our aim in this application is to calculate the different energies occupied by the particle in a box; we need to use as tool of observation the Hamiltonian operator, $\widehat{H}$ (independent of time). So, we solve the stationary Schrodinger equation, just for one branch.

Using equation (1) and the following Schrodinger stationary equation;

$$: \ \hat{H}|\psi\rangle = E_i|\psi\rangle,$$

where; $\widehat{H}_i = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2}$ operates in the $|\varphi\rangle_i$ only, measures the observable E_i only, so we can write the previous equation in the following form:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2}\left\{|\varphi_i\rangle \otimes \left[\prod_{j=1}^n |\varphi\rangle_j \otimes |\varphi\rangle_{j+1}\right]\right\}.$$

$$= E_i|\varphi_i\rangle \otimes \left[\prod_{j=1}^n |\varphi\rangle_j \otimes |\varphi\rangle_{j+1}\right] \quad i \neq j$$

Because there is no interference between states, we solve only the following equation which is related to the observable measured by the correspondent observer $\widehat{H}_i$: So, $-\left(\frac{\hbar^2}{2m}\frac{\partial^2}{\partial {x_i}^2}|\varphi\rangle_i\right) = E_i|\varphi\rangle_i$. Where obviously its solution and after normalization, takes the form: $\varphi_i(x) = \sqrt{\frac{2}{L}}\sin(\frac{i\pi}{L}x)$, and the energy is: $E_i = \frac{i^2\pi^2\hbar^2}{2mL^2}$; $i = 1,2,.....n$. It is clear, that among all the entangled states, we took only the state $|\varphi_i\rangle$. We ignore the other remained tonsorial states' product, because the observer $\widehat{H}_i$ observes only its eigenvalue in the state $|\varphi_i\rangle$; in other words, that doesn't mean the state $|\varphi_i\rangle$ in the branch or universo $|\psi\rangle$ is separated to the other different $|\varphi_j\rangle$, $j = 1,2,... \neq i$, but all are entangled. In the case of:

$i=1$ the solution of the previous differential equation is $\varphi_1(x)=\sqrt{\frac{2}{L}}\sin(\frac{\pi}{L}x)$ and the corresponding Eigenvalue is: $E_1=\frac{\pi^2\hbar^2}{2mL^2}$

Let's suppose the universo $|\psi\rangle$, in function of all universes, "using relation (3)", we will get its general form as:

$$|\psi\rangle=\prod_{j\neq i}^{n=2}\left(\sum_{1=1}^{n=2}(\pm 1)^{i-1}\alpha_i\alpha_j\sin\frac{i\pi}{L}x\otimes\sin\frac{j\pi}{L}x\right) \quad .$$

As soon as, any Eigen value E_i is known, then any other Eigen value different to E_i is instantly measured. If we take only two universes (branches), the particle and its copy, each occupied just one universe. The observer in the entangled universe $|\varphi\rangle_1\otimes|\varphi\rangle_2$, observes or measures $E_1=\frac{\pi^2\hbar^2}{2mL^2}$, while the other observer in the entangled universe $|\varphi\rangle_2\otimes|\varphi\rangle_1$, measures or observes $E_2=\frac{4\pi^2\hbar^2}{2mL^2}$. (there is no a special observer for a special world). Using the previous results we can write:

$$\begin{aligned}|\psi\rangle &=\alpha_1\alpha_2|\varphi\rangle_1\otimes|\varphi\rangle_2-\alpha_2\alpha_1|\varphi\rangle_2\otimes|\varphi\rangle_1\\ &=\alpha_1\alpha_2\sin\frac{\pi}{L}x\otimes\sin\frac{2\pi}{L}x\otimes\\ &\quad-\alpha_2\alpha_1\sin\frac{2\pi}{L}x\otimes\sin\frac{\pi}{L}x\end{aligned}$$

Because $|\psi\rangle$; is normalized to unit, we will finish to the final form:

$$\begin{aligned}|\psi\rangle &=\sqrt{\frac{1}{2}}\left(\sin\frac{\pi}{L}x\otimes\sin\frac{2\pi}{L}x\right)_{w_1}\\ &\quad-\sqrt{\frac{1}{2}}\left(\sin\frac{2\pi}{L}x\otimes\sin\frac{\pi}{L}x\right)_{w_2}\end{aligned}$$

$$\textit{where: } w_1 \textit{ and } w_2 \textit{ are the entangled worlds } 1 \textit{ and } 2.$$

The interpretation of the results mentioned above is the following: the particle in the box has two possibilities, or has the potential or the opportunity to be in two universes; then its universal world split into two branches. The observers in different universes measure two different Eigen value at the same time, without any information exchanged between them. This is why Hugh Everett interpretation, could interpret the Schrödinger Cat paradox, and gave a convincing interpretation for complementary principle. But as it is said, many worlds failed in the interpretation of electron interference; this is due to parallel worlds. In our humble opinion, the interference shown in the electrons' experiment is due to the entanglement factor, which is related to superposition of the different universes, "equation (3)". Concerning the information's exchange, between observers, I think, when a single universal world will have the opportunity to split into many universes, its initial information, will not be divided into different information, because information is undividable, but the issue is only that each observer estimates it according to the level or conditions of his state.

IV. CONCLISION

Our contribution to the quantum theory is to improve the students' knowledge and skills in order to they show more importance and interest to the subject. In our experience, in teaching the Quantum field; we noticed that many students are unrelieved to Copenhagen's point of view concerning objects behavior, especially with its non-causal and non-deterministic interpretation. Concepts in quantum mechanics are not familiar for the students, and this is due to the influences of classical mechanics' principles. Because of the previous difficulties, we always introduce the many worlds interpretation and also we discuss shortly some other important interpretation's schools during our class of teaching quantum field, in addition to Copenhagen's one. We believe because of that we noticed more animations and contributions of students during the class. We believe that teaching physics as general case and quantum theory as special one, is not just deriving a logical equations and relations related to the subject, but we believe that each step of deriving those equations, or relations should be followed by deep discussions and interpretations of concepts and their meanings.

Finally, I suggest adding an extra course, apart from all other physics courses, to the curriculum, which should be related to the epistemological aspects of quantum theory.

ACKNOWLEDGEMENT

We would like to express our sincere gratitude Professor Benchikou, A. for his valued guidance, really thanks to him; I learnt the basic concepts of the physics' epistemology.

REFERENCES

[1] Aspect, A., Balibar, F., *Le monde quantique*, (Edition du Seuil Science et Avenir, France, 1984), pp. 7-8.
[2] Goldstein, S. and Lebowitz, J. L. "Does the Wave Function Provide a Complete Description of Physical Reality"?, in *The Physical Review- The First Hundred Years: A Selection of Seminal Papers and Commentaries*, Henry Stroke, H. ed., (AIP Press, USA, 1995).
[3] Bub, J. and Clifton, R., *Studies in History and Philosophy of science Part B*, Studies in History and Philosophy of Modern Physics **27**, 181-219 (1996).
[4] Camilleri, K., *Bohr, Heisenberg and the Divergent Views of Complementarity*, Studies in History and Philosophy of Modern Physics **38**, 514-528 (2007).
[5] Stephen, G., Quantum Physics, 3rd ed. (John Wiley and Sons, USA, 2003), pp. 28-36.
[6] Esfeld, M., *Causal realism. Profanities, Laws and Structures*, (Springer, Dordrecht, 2012), pp. 157–168.
[7] (Think Quest) What is Quantum Physics library think quest, viewed 24th September (2012), http://www.library.thinkquest.org/3487/qp.html^.1
[8] Ladj, R., *About the wave's concepts in quantum mechanics and how do university students understand its aspects, Algerian students' case*, Educational Journal of the University of Patras Unesco Chair **3**, 37-46 (2016).
[9]. Arini, H., Diwan, Al Hallaj, translated from Arabic into English languish, first published 1931, (2001).
[10]. Everett, H. *Relative State Formulation of Quantum Mechanics*, Review of Modern Physics **29**, 454–46 (1957).