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Investigation of plane waves on an arbitrary manifold and generalized uncertainty

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Abstract

In this paper we extend concept of wave on the Riemannian manifold (such as sphere) and by combination these waves, similar to flat space, we obtain uncertainty relations. Our result shows that the generalized uncertainty relation does not depend on the choice of the geodesic and the shape of the manifold, and only depends on the dimensions of the manifold.

Key words: Plane waves, Manifold, Uncertainty.

Resumen

En este trabajo extendemos el concepto de onda sobre la variedad Riemanniana (como la esfera) y mediante la combinación de estas ondas, similares al espacio plano, obtenemos relaciones de incertidumbre. Nuestro resultado muestra que la relación de incertidumbre generalizada no depende de la elección de la geodésica y la forma de la variedad, sino que solo depende de las dimensiones de la variedad.

Palabras clave: Ondas planas, Variedad, Incertidumbre.

I. INTRODUCTION

A wave packet is a concentrated train of quantum waves of various wave lengths or momenta with the property that the packet is confined within a small region of space. Such a packet can be constructed by adding a very large number of waves so chosen that their sum interferes destructively everywhere except in a small region. This situation is permitted by the principle of superposition. One of the leader of quantum mechanics, P.A.M. Dirac hold that the principle of superposition is one of two most important concepts in quantum mechanics; the other one is Schrodinger equation [1].

Usually, wave packets in quantum mechanics are associated with the concept of spreading and broadening [2]. However, there is a report about non-spreading wave packets in quantum mechanics [3]. It seems that the subject of wave packets on curved spaces has recently received more attention, for example from a classical view, it can be seen that in [4], and in another report, both experimentally and theoretically, accelerating wave packets have been investigated in curved spaces [5].

Recently, there has been a report about the translation of wave function in curved space [6].

II. METHOD

In the orthogonal coordinate plane, the plane wave equation can be written as $\frac{d^2\Psi}{d\xi^2}+\Psi=0$ where Ψ and ξ are two orthogonal coordinate axes. Obviously, a geodesic is a generalization of a straight line in any space, so in an arbitrary manifold, by choosing a geodesic as a straight line and another geodesic perpendicular to it, a normal coordinate system can be obtained, write the above differential equation for it, and then with the mapping that the manifold It takes the target as $\mathbb{R}^n$ and by putting the solutions of the above differential equation in it, it can be obtained the obtained wave equation on the manifold $\mathbb{R}^n$. Now, the equation is written in $\mathbb{R}^n$, we can find its Fourier transform for each component. Now by summing the distribution width $x(\Delta x)$ of $\mathbb{R}^n$ components and summing the distribution width $k(\Delta p)$ of $\mathbb{R}^n$ components and multiplying them together and placing the De Brogli-Einstein relation in it, the uncertainty in the desired space can be found.

For example, on the sphere, by choosing θ and φ as the geodesics of the sphere, a normal coordinate system can be made and in the equation $\frac{d^2\theta}{d\varphi^2}+k\theta=0$, where k is a constant value gave The answers to this equation are

$$\theta = A\sin k\varphi + B\cos k\varphi, \theta = f(\varphi). \tag{1}$$

As it is known, it is possible to continue the work by applying the condition $\frac{\pi}{2}=f(0)$ without reducing the generality of the problem.

$$\theta = \frac{\pi}{2}\cos k\varphi \,. \tag{2}$$

As we know, the mapping of sphere S^2 to $\mathbb{R}^3$ is as follows

$$\begin{aligned} x &= r\sin\theta\cos\varphi \\ y &= r\sin\theta\sin\varphi \,. \\ z &= r\cos\theta \end{aligned} \tag{3}$$

Now, by placing equation (2) in (3) and putting $r = 1$, we get the wave equation on the sphere in $\mathbb{R}^3$ so we can write

$$\begin{aligned} x(\varphi) &= \sin(\frac{\pi}{2}\cos k\varphi)\cos\varphi \\ y(\varphi) &= \sin(\frac{\pi}{2}\cos k\varphi)\sin\varphi \,. \\ z(\varphi) &= \cos(\frac{\pi}{2}\cos k\varphi) \end{aligned} \tag{4}$$

Now, by summing up a number of the above waves, i.e. $x(\varphi)$ and $y(\varphi)$ and $z(\varphi)$ (Figure 1) and calculating the distribution width of k and φ and knowing that in one dimension $\Delta x\Delta p \geq \frac{1}{4\pi}$, we calculate the uncertainty relation in the following formula

$$\Delta\varphi\Delta k = (\Delta\varphi_x + \Delta\varphi_y + \Delta\varphi_z)\left(\Delta k_x + \Delta k_y + \Delta k_z\right), \tag{5}$$

where $\Delta\varphi_i\Delta k_j \geq \frac{1}{4\pi}\delta_{ij}$ since $\Delta\varphi_x\Delta k_y \geq 0$. Therefore we obtain the equation as following

$$\Delta\varphi\Delta k \geq \frac{3}{4\pi} \,. \tag{6}$$

In the equation (6) we have

$$\Delta\varphi = (\Delta\varphi_x + \Delta\varphi_y + \Delta\varphi_z), \Delta k = \left(\Delta k_x + \Delta k_y + \Delta k_z\right).$$

As a result, the uncertainty relation is obtained by putting the De Brogli-Einstein relation in the following form

$$\Delta\varphi\Delta p \geq \frac{3\hbar}{2} \,. \tag{7}$$

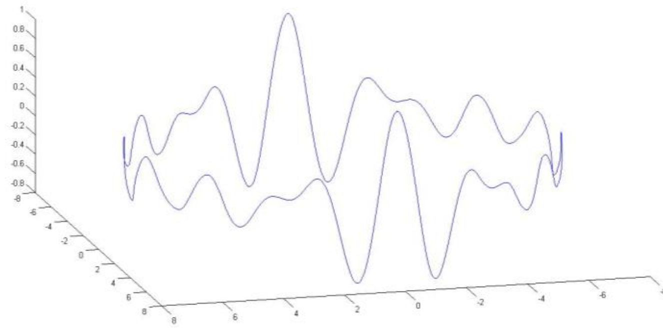

FIGURE 1. Superposition of some waves on the S^2 sphere.

III. CONCLUSION

As can be seen, the uncertainty obtained by the above method does not depend on the choice of the geodesic and the shape of the manifold, and only depends on the dimensions of the manifold. Figure (1) indicates the superposition of waves on the sphere correctly.

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