Analysis of the Motion of Maxwell's wheel Using Tracker Video Analysis and Measurement of the Moment of Inertia

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Abstract

This study investigates the motion of Maxwell's wheel using Tracker Video Analysis software. Maxwell's wheel is composed of a wheel suspended by two strings rolling up around an axis through its center of mass. When released, the flywheel unwinds and converts gravitational potential energy into both translational and rotational kinetic energy. The analysis aims to provide students with hands-on experience in using Tracker for experimental analysis. By recording the motion with a smartphone and analyzing it using Tracker, we obtained quantitative data to confirm theoretical models and explore energy transformations within the system. The results show a gradual decrease in mechanical energy due to dissipative forces, with the flywheel not reaching its initial height after each cycle. The study also estimates the moment of inertia of the flywheel, accounting for energy dissipation, and highlights the educational benefits of integrating video analysis in physics teaching. This approach helps students gain a better understanding of theoretical concepts and develop skills in scientific data analysis.

Keywords: Maxwell's wheel, Tracker Video Analysis software Educational Physics Laboratory, Classical Mechanics teaching.

Resumen

Este estudio investiga el movimiento de la rueda de Maxwell utilizando el software de análisis de video Tracker. La rueda de Maxwell está compuesta por una rueda suspendida de dos cuerdas que giran alrededor de un eje que pasa por su centro de masas. Al soltarse, el volante se desenrolla y convierte la energía potencial gravitatoria en energía cinética, tanto traslacional como rotacional. El análisis busca brindar a los estudiantes experiencia práctica en el uso de Tracker para análisis experimental. Al registrar el movimiento con un teléfono inteligente y analizarlo con Tracker, obtuvimos datos cuantitativos para confirmar los modelos teóricos y explorar las transformaciones de energía dentro del sistema. Los resultados muestran una disminución gradual de la energía mecánica debido a las fuerzas disipativas, ya que el volante no alcanza su altura inicial después de cada ciclo. El estudio también estima el momento de inercia del volante, considerando la disipación de energía, y destaca los beneficios educativos de integrar el análisis de video en la enseñanza de la física. Este enfoque ayuda a los estudiantes a comprender mejor los conceptos teóricos y a desarrollar habilidades en el análisis de datos científicos. Palabras clave: Rueda de Maxwell, software de análisis de vídeo Tracker, Laboratorio de Física Educativa, enseñanza de la mecánica clásica.

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I. INTRODUCTION AND EXPERIMENTAL SETUP

In the context of teaching physics, the use of innovative educational tools and practices aimed at the experimental study of properties of real systems is an excellent method for actively engaging the students, as shown by some of our previous work [1-7]. When students become active participants by analyzing something new or experimentally verifying relationships that have, until then, been studied only theoretically, their engagement increases. Consequently, they become more inclined to delve deeper into the topics addressed, enhancing their knowledge and strengthening their understanding of the analyzed concepts. With this goal in mind, during an afternoon physics enrichment course, which is part of the Scientific Degree Plan of the Department of Physics and Astronomy at the University of Catania [8], we decided to experimentally analyze the motion of the Maxwell pendulum to understand in detail the dynamics of its movement.

The Maxwell pendulum (or wheel) consists of a flywheel suspended by two cords wound in the same direction around an axis that passes through its center of mass (see Fig. 1). When the flywheel is released, it begins to unwind, acquiring both translational and rotational kinetic energy. Upon reaching the lowest point, due to rotational inertia, the cords wind up again, bringing the flywheel back to the highest point, and so on, continuously transforming gravitational potential energy into rotational kinetic energy and vice versa. Essentially, this is the working principle of a yo-yo.

FIGURE 1. Maxwell's wheel or wheel

In textbooks, the analysis of the Maxwell pendulum's motion is often presented as a way of exploring fundamental concepts such as the transformation of mechanical energy into its different forms, including rotational kinetic energy, the concept of rotational inertia and the effects of friction. A very detailed and thorough analysis of the dynamics of the Maxwell pendulum was carried out several years ago by Pecori and Torzo [9]. In that work, unlike ours, a commercial MBL system was used to track the flywheel, specifically with an ultrasonic sensor connected to a computer. In contrast, we employed a different approach, based on video analysis using open-source software, and we estimated the moment of inertia of the flywheel while accounting for the dissipated energy, aiming to provide a quantitative estimate of the source of this mechanical energy loss. Besides the work by Pecori and Torzo, there are other studies on the Maxwell pendulum or similar systems that analyze different aspects [10-12]. Given that the goal of the enrichment course was to learn the basics of using Tracker Video Analysis software, we decided to perform an experimental analysis of the pendulum's motion using this software. Tracker is an open-source program designed for video analysis of physical phenomena, allowing the recording and analysis of object movement in videos, providing quantitative data with good accuracy that can be used to both confirm theoretical models and conduct empirical analyses of mechanical systems [13]. The use of Tracker Video Analysis in studying the motion of the Maxwell pendulum offers numerous educational advantages. Primarily it allows students to directly compare experimental data with theoretical predictions, fostering a deeper understanding of the studied concepts. Additionally, it develops the students' practical skills in using technological tools for scientific analysis, better preparing them for future academic and professional activities. The use of Tracker also helpsreducing manual measurement errors and obtain more precise and reproducible data. The results produced by the students during the course, which included not only the study of the Maxwell pendulum but also the analysis of other mechanical systems, was all presented at the XIX PLS Meeting held on 29/05/2024 at the Aula Magna of the Department of Physics and Astronomy in Catania [14].

II. KINEMATIC ANALYSIS

The video that captured the motion of Maxwell pendulum was recorded with a smartphone set to 60 FPS. The tracking was done automatically using Tracker, setting the point of interest to be the location where the disk intersects the horizontal axis. Figure 2 shows the initial kinematic analysis of the motion. In particular, on the left-hand side, the video frame of the Maxwell pendulum is displayed, showing the measurement rod (in blue) used to calibrate the system, the Cartesian coordinate system (in magenta) with the origin placed at the lowest point of the pendulum's motion reversal, and the tracked points (in red). On the right-hand side the kinematic diagrams provided by Tracker are presented: from top to bottom, these include the vertical position diagram of the flywheel, the vertical velocity diagram, and the acceleration diagram. It is important to note that the position diagram is derived from the direct tracking of the flywheel's position, while the velocity and acceleration diagrams are constructed by the system using the recorded position values. As shown in Figure 2, during the descent and ascent phases, the time diagram follows a parabolic arc with downward concavity, indicating uniformly accelerated motion with acceleration directed downward, as the coordinate system's vertical axis points upward. The velocity diagram supports this observation, as the velocity remains linear with a negative slope during the descent and ascent phases. The acceleration diagram shows a near-zero average acceleration during both descent and ascent phases, with noticeable positive peaks corresponding to sudden changes in velocity, i.e. when the flywheel reaches the end of its motion and bounces upward. Nonetheless, when scaling is adjusted, it is observed that the acceleration during both descent and ascent phases remains nearly constant throughout, with a very low negative value.

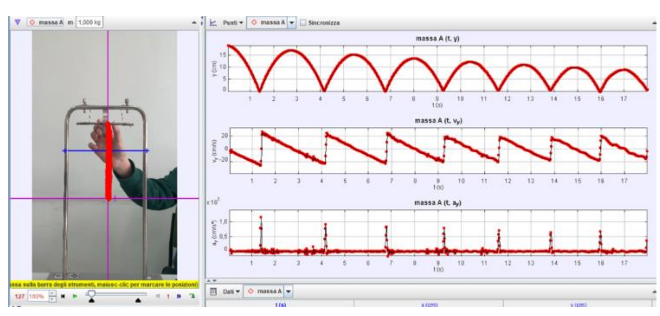

FIGURE 2. On the left, the image of the Maxwell pendulum flywheel frame, with the calibration rod (blue), the reference system (magenta), and the tracked points (red). On the right, the kinematic diagrams of the flywheel's vertical translational motion produced by Tracker. From top to bottom, the diagrams show: positions (in cm),

velocities (in cm/s), and accelerations (in cm/s²) as functions of time (in s).

Performing a more quantitative analysis reveals that both the position and velocity diagrams indicate that the acceleration value remains consistent during the different descents and ascents of the flywheel. Indeed, by applying a parabolic best-fit to the position diagram for each descent and ascent of the flywheel, including the initial descent, we found that the acceleration, barring experimental errors, remains constant. This is illustrated in Figure 3, which shows the accelerations deduced from the best-fit for each ascent and descent of the flywheel.

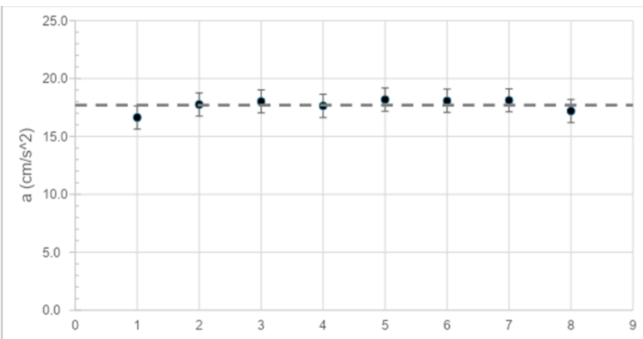

FIGURE 3. Acceleration during the different phases of descent and ascent of the flywheel, estimated through the parabolic fit of the time diagram obtained from tracking with Tracker. It is observed that the acceleration remains approximately constant.

Calculating an average, the translational acceleration of the flywheel is estimated to be (17,7 ± 0,8) cm/s^2. Further analysis of the position diagram reveals that the maximum height reached by the flywheel decreases over time, indicating the presence of dissipative forces. Specifically, plotting the maximum height reached by the flywheel during the different descent-ascent phases shows a linear trend with a decrease of approximately $\delta = 1.52$ cm per complete cycle of descent and ascent, as illustrated in Figure 4.

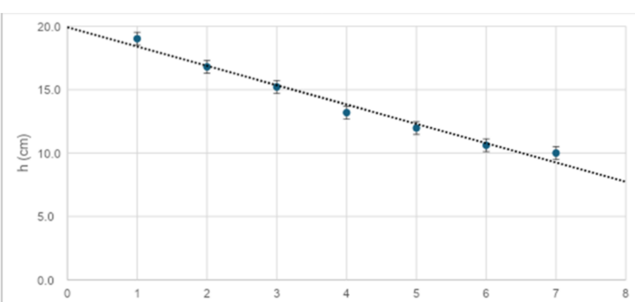

FIGURE 4. Graph of the maximum height reached by the flywheel in various cycles. The trend line shows a decreasing rate of approximately 1.52 cm per complete cycle of descent and ascent, as clearly indicated by the equation.

III. DYNAMIC ANALYSIS AND ENERGY CONSIDERATIONS

Considering that in each cycle of descent and ascent, an energy equivalent to the decrease in maximum height is dissipated, we can reasonably hypothesize that this dissipation is half of the total one during the single descent. Therefore, during a single descent, the energy lost is approximately equal to $\varepsilon = mg\frac{\delta}{2}$. Therefore, let ε represent the amount of energy dissipated during a single descent due to frictional forces:

$$E_{initial} = E_{final} + \varepsilon, \quad (1)$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mg\frac{\delta}{2}. \quad (2)$$

In which the following variables are defined: m is the mass of the flywheel, h is the maximum height, v and ω are respectively the linear velocity of the center of mass and the angular velocity relative to the axis of rotation of the flywheel at the lowest point, and I is the moment of inertia of the flywheel. Considering that

$$\omega = \frac{v}{r} \quad (3),$$

where r is the radius of the spindle section around which the cord is wound. With some straightforward mathematical manipulations, we can derive from the relation (2) that:

$$I = \frac{2mgh - mv^2 - mg\delta}{v^2}r^2 = mr^2\left(\frac{2gh}{v^2} - 1 - \frac{\delta g}{v^2}\right). \quad (4)$$

By measuring the following: the mass of the flywheel with a digital scale, $m = (360.2 \pm 0.1)g$; the diameter of the spindle around which the cord is wound with a digital caliper, $d = 2r = (6.05 \pm 0.01)mm$; and evaluating the drop height and the velocity reached at the end of the descent just before the rebound using Tracker, we estimated a moment of inertia of:

$$I_{Flywheel-Friction} = (1.8 \pm 0.1) \cdot 10^{-4} kgm^2. \quad (5)$$

If we neglect friction in the conservation of mechanical energy, we obtain a moment of inertia that can be compared with that expressed by equation (5). Specifically, we find:

$$I_{Flywheel-NoFriction} = (2.0 \pm 0.1) \cdot 10^{-4} kgm^2. \quad (6)$$

Therefore, applying energy conservation to determine the moment of inertia does not introduce a significant error. For comparison, one could calculate the moment of inertia of the flywheel its formal definition, but this is quite challenging due to its complex shape and non-uniform mass distribution. However, to provide a rough comparison with experimental estimates, approximating the flywheel as a uniform disk with a diameter $D = 2R = (69.60 \pm 0.01)\,mm$ (the actual value measured with a digital caliper, we obtain a moment of inertia of:

$$I_{Disk} = (2.181 \pm 0.001) \cdot 10^{-4} kgm^2. \quad (7)$$

This is essentially comparable to the dynamic estimates if we limit ourselves to one significant figure. We have determined, through measurements with Tracker, that energy is not conserved, indicating the presence of dissipative forces. We then investigated when and how much energy is dissipated during the motion. As an initial analysis, we estimated the mechanical energy dissipation during the rebound phases, evaluating the magnitude of the translational velocities of the flywheel at the lowest point and when it starts moving upward again. This was done by measuring the slope of the position diagram near the point of reversal. Performing this analysis for each rebound, we estimated a translational kinetic energy dissipation of $(1.1 \pm 0.5) \cdot 10^{-3} J$ per rebound. Given that the radius of the spindle around which the cord is wound is relatively small, we can neglect the corresponding rotational energy dissipation. Additionally, considering that there are 7 rebounds throughout the motion, we can estimate the total energy loss at the points of motion reversal to be:

$$\delta E_{rebounds} = (7.7 \pm 3.5) \cdot 10^{-3} J. \quad (8)$$

Regarding the dissipation of energy during the ascent and descent phases, we analyzed the behavior of the total mechanical energy as a function of time, which is the sum of gravitational potential energy and both translational and rotational kinetic energies. For this analysis, we defined a new quantity in Tracker:

$$E_{tot} = mgy + \frac{1}{2} m v_y^2 + \frac{1}{2} I \left(\frac{v_v}{r}\right)^2. \quad (10)$$

In this analysis, we used relation (3) to express the angular velocity ω in terms of the measured quantities. Tracker, among its many applications, allows for the creation of new quantities based on existing default ones and the construction of the respective graphs. Figure 5 shows the trend of total mechanical energy as a function of time, using the measured values of m, $I_{Flywheel-Friction}$, r and with $g = 9.81 m/s^2$. The graph clearly illustrates that the energy decreases over time. The graph features sudden drops in total mechanical energy corresponding to the rebounds. These drops can be attributed to two main factors:

- Theoretical Limitations. According to relation (8) used for calculating total mechanical energy, when the translational velocity becomes zero at the reversal point, the angular velocity is also expected to become zero. However, this is not the case, as the flywheel continues to rotate as it ascends.
- Tension Forces. During the reversal of motion, the tension forces in the cord lead to a temporary storage of elastic energy in the cord, which contributes to the observed drops in mechanical energy.

Both effects contribute to the apparent disappearance of mechanical energy during the motion reversal. [9]

The black line represents the linear best-fit line estimated with Tracker, which predicts a rate of energy decrease of $(21.2 \pm 0.3) \cdot 10^{-3} J/s$. Therefore, the total energy loss is:

$$\delta E_{tot} = (381 \pm 5) \cdot 10^{-3} J. \quad (11)$$

From this analysis, we can conclude that approximately 98% of the energy loss is attributable to viscous air friction.

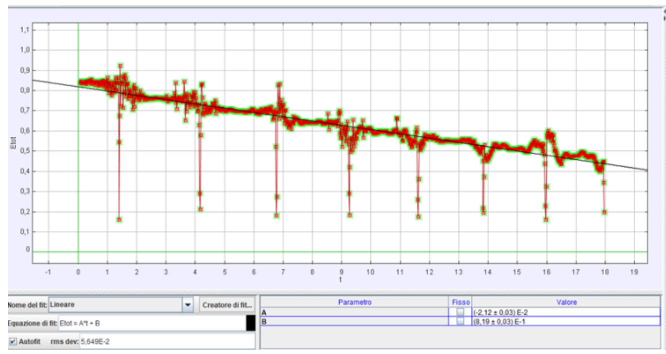

FIGURE 5. Total mechanical energy of the Maxwell wheel, calculated as the sum of gravitational potential energy, translational kinetic energy, and rotational kinetic energy (see text).

IV. CONCLUSIONS

The study of the Maxwell's wheel motion using Tracker Video Analysis demonstrated the effectiveness of innovative educational tools for teaching physics. The video analysis allowed us to obtain precise quantitative data and compare it with theoretical models, confirming the importance of experimental analysis in the learning experience. The results revealed a gradual decrease in the system's mechanical energy due to dissipative forces, as evidenced by the flywheel's failure to reach its initial height after each cycle. The estimation of the flywheel's moment of inertia provided values consistent with theoretical predictions, showing that energy conservation principles can be applied with good approximation despite friction losses. This work enabled students to deepen their understanding of concepts such as kinetic and potential energy, rotational inertia, and friction, enhancing their skills in using technological tools for scientific analysis. In conclusion, integrating Tracker Video Analysis into the physics curriculum not only facilitates the understanding of theoretical concepts but also prepares students for future academic and professional activities, developing essential practical skills for analyzing and solving complex problems in physics.

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