Spatial frequency response of a human eye apodized with the stiles crawford effect of the first kind in coherent illumination

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Abstract
A beam of light passing near the centre of the pupil of a human eye stimulates the retina more than an identical beam making an entry from the edges of the pupil. This is Stiles-Crawford effect of the first kind (SCE I). This diminution in the effective brightness of a beam is retinal in origin. But in the computation of how the human eye responds to different spatial frequencies in the object, often the SCE I is incorporated as a pupil apodization. We have shown here that in the absence of aberrations and defocus this response is predominantly governed by the coherence of the incident beam rather than the beam's pupil entry point, a characteristic of the traditional Stiles Crawford effect.

Keywords: Stiles-Crawford effect, Apodization, Coherent illumination, Frequency response.

Resumen
Un haz de luz que pasa cerca del centro de la pupila de un ojo humano estimula la retina más que un haz idéntico que haga un registro de los bordes de la pupila. Este es el efecto de Stiles-Crawford de la primera clase (SCE I). Esta disminución en el brillo eficaz de un haz es retinal en origen. Sin embargo, en el cálculo de cómo el ojo humano responde a diferentes frecuencias espaciales en el objeto, a menudo el SCE I es incorporado como una apodización pupilar. Hemos demostrado que en ausencia de aberraciones y desenfoques, esta respuesta se rige principalmente por la coherencia del haz incidente más que del punto de entrada de la pupila del haz; una característica del tradicional efecto Stiles Crawford.

Palabras clave: Efecto Stiles-Crawford, apodización, iluminación coherente, frecuencia de respuesta.

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I. INTRODUCTION

All the photoreceptors (both rods and cones) in the retina of a human eye share the same light coming from a single pupil. Still the light coming from different parts of the pupil stimulates the photoreceptors, especially the cones not in the same degree. Rather a beam passing near the edge of the pupil appears dimmer than a beam entering centrally. This is Stiles-Crawford effect of the first kind [1]. Observation of the effect in cones only in a retinal region populated with equal density of rods and cones clearly showed that it is retinal in origin and is primarily cone-specific. Thus, the extra retinal factors like the passage of light from the pupil to the retina had no role to play in the diminution of effective brightness [2].

The same kind of diminution in the effective brightness in an image is also observed when the light admitting pupil is covered with a filter which becomes more and more opaque from the centre toward the edge [3]. This is known as apodization.

Due to this outward resemblance between the retinal Stiles-Crawford effect and pupil apodization, the SCE I has been modeled as a pupil apodization in studying retinal light distributions in many imaging situations [4, 5, 6, 7, 8].

In all these diffraction computation of the light distribution Fourier theory is employed, that is, any complex object can be broken down into its simplest sine wave components which are imaged separately. This paper very methodically attempts a build up involving Stiles-Crawford effect of the first kind, apodization, spatial frequency approach, Fourier analysis and synthesis, coherent imagery, finally leading to computation of visual response of a human eye to a rectangular grating stimulus in coherent illumination in presence of Stiles-Crawford effect of the first kind.

II. STILES-CRAWFORD EFFECT OF THE FIRST KIND

The Stiles-Crawford effect of the first kind (SCE I) was first reported in 1933 [1]. It is an experimental observation that light passing near the edges of a pupil is less efficient in stimulating the retina than is light entering through the centre.

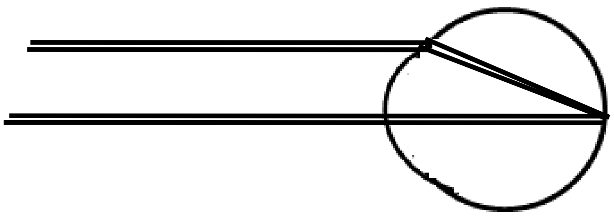

FIGURE 1. A wide pupil admitting both axial and peripheral rays.

The SCE I visibility curve is found out by using oblique rays and adjusting the light in them to match the brightness produced by a constant beam passing through the centre of the pupil. For an illustration, one can say that to match the apparent brightness of a beam entering the eye through a 10 mm^2 pupil, the light entering through a 30 mm^2 pupil is to be reduced by only a factor of two instead of the expected three [9]. This lack of reciprocity between pupil area and light needed for equal brightness was found to be present prominently in photopic vision [2]. A bundle of rays passing through the centre of the pupil strikes the retina head-on. But an identical bundle of rays entering near the edge of a fully dilated pupil of approximately 4 mm radius will make an angle of nearly 10^0 at the retina, which lies at a distance of 22.2 mm from the pupil [10]. Thus changing the pupil entry point is equivalent to varying the angle of incidence at which a ray strikes the retina. This is how the visibility curve of figure 2 was obtained [1, 11].

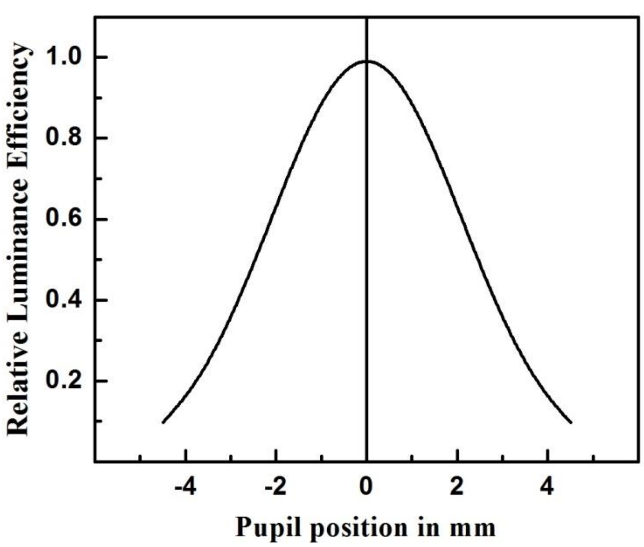

FIGURE 2. Relative luminous efficiency for light rays as a function of pupil location of the incident beam.

The equation characterizing SCE I visibility is thus given as

$$\ln\frac{\eta_o}{\eta_r} = \rho r^2. \quad (1)$$

Where η_o and η_r are the luminous efficiencies of light entering centrally and at r distance from the peak of visibility ($r = 0$). ρ is the shape parameter which is equal to 0.115 [11, 12]. The significance of SCE I lies in the supposed difference between using coherent or incoherent light as incident illumination, for the simple reason that this difference can be used as a tool for controlling visual response of the retina.

How to incorporate the directional sensitivity of the retina, now that it is known to be present in a human eye as revealed through the SCE I? One way is to treat the photoreceptors as optical waveguides as the directional sensitivity is a known feature of waveguides [13, 14, 15].

The main feature of a waveguide model is that light is accepted only at a single aperture and is guided from there into the interior space of the receptor (Fig. 3). The waveguide model demonstrates directional sensitivity in addition to the existence of modes [16].

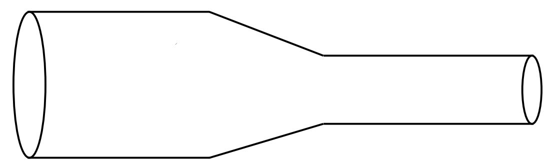

FIGURE 3. Waveguide model of a cone receptor.

But the other way is to treat the SCE I as a pupil apodization.

And the remarkable thing is that in the absence of aberrations, both the approaches (retinal waveguiding and pupil apodization) give identical results [13]. We will consider the second approach of pupil apodization in computing visual response.

III. APODIZATION

For a round pupil like the eye and in the absence of aberrations, the pattern in the plane of focus is the familiar Airy disc (figure 4), whose diameter is related directly to the wavelength and inversely to the pupil aperture [17]. In order to remove the undesired second hump of the Airy disc a pupil that becomes gradually less transparent (figure 4) from the centre toward the edge is employed [3]. And this is mainly used to resolve binary stars (where a weak companion star is present next to a brighter star) [18].

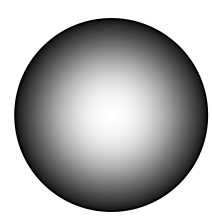

FIGURE 4. Visual sensitivity against pupil entry point of light; apodizing filter; a radially symmetric neutral density wedge in the pupil plane.

As the visibility curve of the SCE I is obtained against variable pupil entry point, naturally it has resemblance with apodization in spite of the fact that it is being retinal in origin. So it can be modelled as pupil apodization while calculating the visual response of a humane eye to a grating stimulus. So the methods to obtain the visual response of a humane eye can be outlined as: a) to choose the type of illumination from among completely coherent, incoherent, partially coherent b) to choose a stimulus or a test target from a sinusoidal, square, rectangular, triangular, saw-tooth grating c) to model SCE I as pupil apodization. The approach is frequency response techniques and the tool is Fourier theory.

IV. SPATIAL FREQUENCY APPROACH

The image quality of a human eye is ascertained by determining its response to spatial frequency supplied in the form of a grating stimulus. The properties of a grating are specified by its waveform, contrast, spatial frequency, orientation, and phase. The waveform of a grating refers to the grating's intensity distribution. When the intensity alternates abruptly between high and low it is a square -wave grating. But when the grating's intensity alternates more gradually between high and low it is a sine-wave grating.

FIGURE 5. Sine wave and square wave grating.

The spatial frequency of a grating is specified either in number of cycles (one white bar plus one black bar) per unit distance across the grating or cycles per degree of visual angle. The visual angle of an object is the angle between two lines that extend from the observer's eye, one to one end of the object and the other to the other end of the object [19-20].

How the spatial frequency of a grating stimulus is determined by a human eye? Evidence suggests that there are spatial frequency channels, each of which is sensitive to a narrow range of frequencies [21]. Thus, we are able to detect spatial frequency present in a grating stimulus. But, how to get from a collection of different spatial frequencies the perception of the original stimulus? Here comes the mathematical tool of Fourier analysis and Fourier Synthesis.

V. FOURIER ANALYSIS AND SYNTHESIS

According to this analysis, any intensity pattern can be broken into a number of sine-wave components. For example, the square-wave grating of the figure can be broken down into a sine-wave with a frequency equal to that of the square-wave plus sine waves with amplitudes equal to one third and one fifth the amplitude of the square wave, and with frequencies three and five times the frequency of the square wave, respectively. Evidence supports the idea that actually the visual system breaks a stimulus (let a square-wave grating) down into spatial frequency components [22, 23].

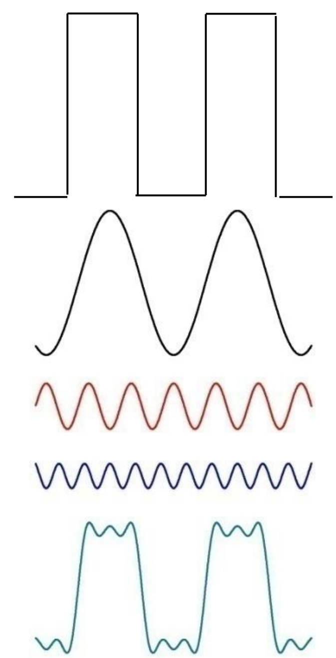

FIGURE 6. The sine-wave components for the square-wave grating at the top of the figure, determined by Fourier analysis. The intensity of the square wave grating is broken down to sine wave components.

The visual system first carries out a Fourier analysis by breaking a scene (here a square wave grating) into a number of sine-wave components. Next, this information is contained in the firing of spatial frequency detectors (neurons). Finally, the information contained in the neurons is combined through the Fourier synthesis (sine components combined to obtain square- wave).

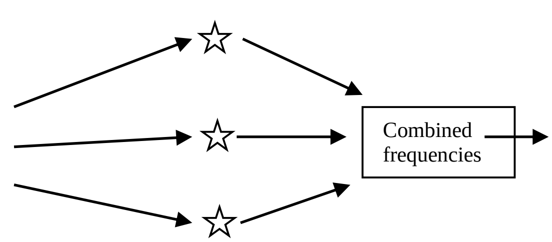

FIGURE 7. Diagram showing how a scene is broken down into spatial frequency components by Fourier analysis and then

reconstotuted RECONSTITUTED into the scene by Fourier synthesis

VI. COHERENT IMAGERY

When the periodic grating (be it the sine-wave or the square-wave) is illuminated by a perfectly coherent light the image formation on the retina takes the following route. Since the grating is illuminated by a plane wavefront, the image formed in the plane of the eye's pupil will be the Fraunhofer diffraction pattern as shown in Fig. 8 [24]. And the amplitude distribution in this diffraction image is the Fourier transform of the grating's amplitude function.

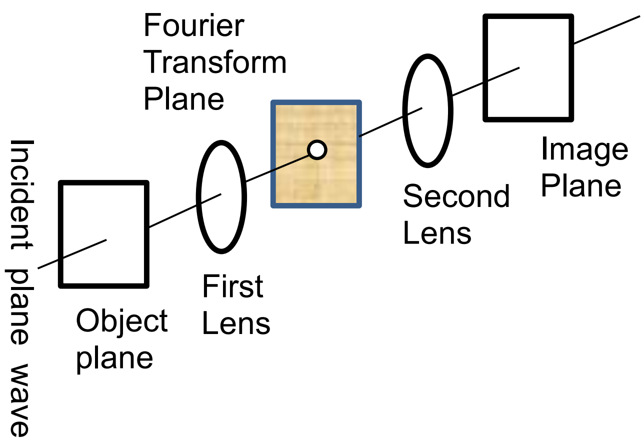

FIGURE 8. The spatial frequency components of the object placed in the object plane are displayed in the Fourier transform plane. The small hole filters out the high frequency components.

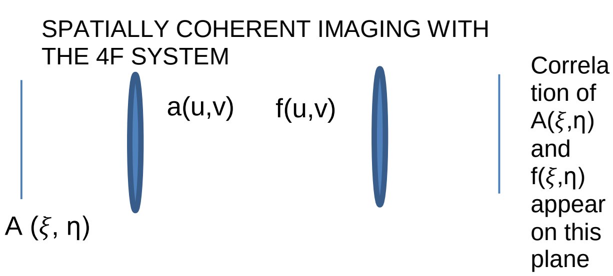

FIGURE 9. The Fourier transform of A(ξ,η) is formed as a(u,v) on the back focal plane of the lens. A transmission mask containing the Fourier transform of the seconf function f(ξ,η) is placed on this plane as f(u,v).Thus, the product a(u,v)f(u,v) lies on the front focal plane of the second lens. So the Fourier transform of a(u,v)f(u,v) equivalent to the convolution of A(ξ,η) and f(ξ,η) is obtained on the back focal plane.

The Fraunhofer diffraction pattern of a point object can also be expressed in terms of the spatial frequency domain. A grating with sinusoidal intensity pattern with a spatial period p (in mm) may also be said to have a spatial frequency of 1/p cycles/mm. In the spatial frequency domain, the way of describing the performance limitation of an eye is to state the demodulation experienced by a sinusoidal grating target in the process of being imaged. When this quantity is plotted against spatial frequency we obtain the modulation transfer function. The mathematics of coherent image formation can be outlined as follows

A plane wavefront illuminates a sinusoidal grating. Hence in the plane of the eye's pupil is formed Fraunhofer diffraction pattern. The amplitude distribution in this diffraction image is the Fourier transform of the target amplitude function. If $A(\xi,\eta)$ is the distribution of amplitude in the object, the distribution of amplitude in the disturbance in the plane of the pupil is given by

$$a(u,v) = \text{FT of } A(\xi,\eta). \tag{2}$$

Naturally, only that portion of a (u,v) that is transmitted by the pupillary apertura OPENING can be effective in the formation of the image amplitude distribution $A'(\xi,\eta)$, which is again the FT of the pupil amplitude function.

Thus $A'(\xi,\eta)$ = IFT of modified amplitude function

$$[\ = a(u,v)f(u,v)]. \tag{3}$$

Finally, the image intensity distribution $B'(\xi,\eta)$ is the product of the image amplitude distribution with its complex conjugate.

When it is coherent, the amplitude of light emerging from the plane of the periodic grating is being spatially Fourier analyzed (displayed as a diffraction image in the plane of the pupil), filtered, and then resynthesized as a distribution of amplitude of light in the retinal plane. Ultimately the amplitude is squared at each point in the image and this constitutes the image intensity pattern It is seen that filtering the spatial frequency in the domain of amplitude of light will lead to results which, when looked at in the realm of the intensity pattern, may be non-linear. As an example, consider the case of an amplitude distribution that is sinusoidal. When squared this becomes an intensity distribution that is also sinusoidal but has twice the frequency.

VII. RESULTS AND DISCUSSION

For different duty cycle (α=0.1, 0.3, 0.4, 0.7) of the periodic grating target with a rectangular profile we have calculated the intensity distribution in the image by incorporating the Stiles Crawford effect of the first kind (SCE I), as a pupil apodization with the beam hitting the pupil having spatial coherence. And the results are illustrated in Fig.10 and 11. From the figure it is evident that when σ=3.086, that is the SCE I apodized eye's, intensity distribution is almost identical without SCE I apodization (σ=0) for both low (ω=0.5) and high (ω=1.0) normalized spatial frequency. In an earlier work we have studied the coherent response of a SCE I apodized human eye in coherent illumination by taking a sinusoidal grating [25].

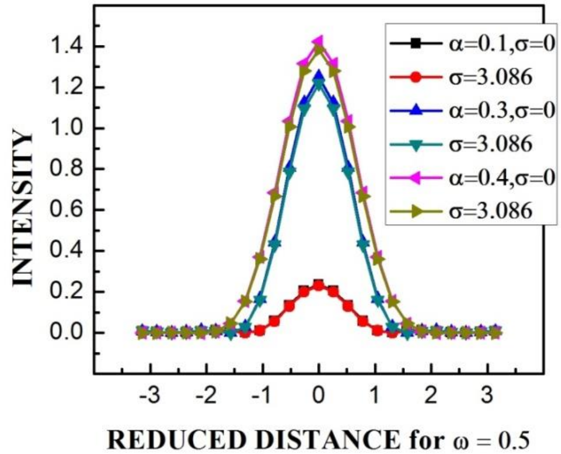

FIGURE 10. Variation of intensity with reduced distance for low normalized spatial frequency.

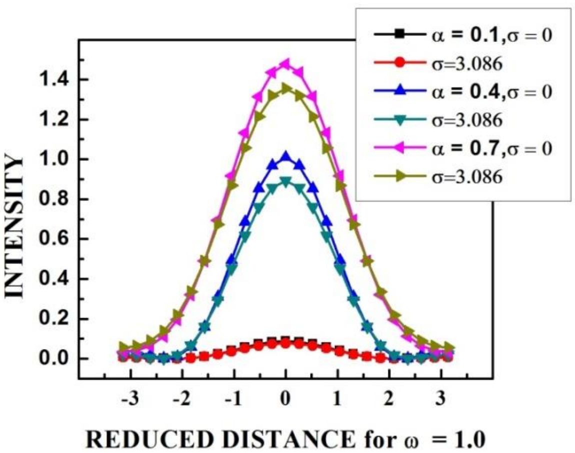

FIGURE 11. Variation of intensity with reduced distance for high normalized spatial frequency.

Similarly the modulation's response to spatial frequency is studied for a SCE I apodized human eye side by side with an eye without SCE I apodization.

Again for coherent illumination we do not find any modification in the modulation even for different duty cycle of a rectangular periodic object as revealed in Fig. 12.

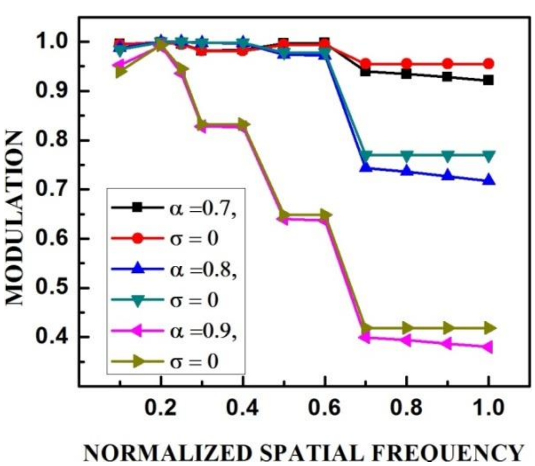

FIGURE 12. Frequency response of a SCE I apodized human eye in coherent illumination as shown from the variation of modulation with normalized spatial frequency for different duty cycles.

VIII. CONCLUSION

So, we see that for coherent illumination the treatment of Stiles Crawford effect of the first kind as a pupil apodization did not lead to any modification either in the modulation or in the intensity distribution in the image of an object of rectangular periodicity indicating the dominance of the coherence of the incident beam over the pupil entry point of the beam, a characteristic of traditional Stiles Crawford effect.

REFERENCES

[1] Stiles, W. S. & Crawford, B. H., *The luminous efficiency of rays entering the eye pupil at different points*, Proc. R. Soc. B **112**, 428-450 (1933).

[2] Flamant, F. & Stiles, W. S., *The directional and spectral sensitivities of retinal rods to adapting fields of different wavelengths*, J. Physiol. **107**, 187-202 (1948).

[3] Jacquinot, P., Bougron, P., and Dossier, B., *Calcul et realisation des distributions d'amplitude pupillaire, permettant a suppression des franges laterale dans les figures de diffraction*, In: La Theorie des Images Optiques, P. Fleury, A. Marechal, and C. Anglade, eds. (Editions de la Revue d'Optique, 1949), pp. 183-193.

[4] Westheimer, G., *Retinal light distributions, the Stiles-Crawford effect and apodization*, J. Opt. Soc. Am. A **30**, 1417-1421 (2013).

[5] Westheimer, G., *Specifying and controlling the optical image on the retina*, Prog. Retinal Eye Res. **25**, 19-42 (2006).

[6] Metcalf, H., *Stiles-Crawford apodization*, J. Opt. Soc. Am. **55**, 72-74, 1411-1416 (1965).

[7] Carroll, J. P., *Apodization model of the Stiles-Crawford effect*, J. Opt. Soc. Am. A **70**, 1155-1156 (1980).

[8] Atchison, D. A., Joblin, A. and Smith, G. *Influence of Stiles-Crawford effect apodization on spatial visual performance*, J. Opt. Soc. Am. A **19**, 1073-1083 (2002).

[9] Troland, L. T., *On the measurement of visual stimulation intensities*, J. Exp. Psychol. **2**, 1-34 (1917).

[10] Emsley, H. H., *Visual Optics* Vol. 1. , 5th ed. (Hatton, UK, 1955).

[11] Applegate, R. A. & Lakshminarayanan, V., *Parametric representation of Stiles-Crawford functions: normal variation of peak location and directionality*, J. Opt. Soc. Am. A **10**, 1611-1623 (1993).

[12] Artal, P., *Incorporation of directional effects of the retina into computations of optical transfer functions of human eyes*, J. Opt. Soc. Am. A **6**, 1941-1944 (1989).

[13] Vohnsen, B., *Photoreceptor waveguides and effective retinal image quality*, J. Opt. Soc. Am. A. **24**, 597-607 (2007).

[14] Snyder, A. W., and Pask, C., The Stiles-Crawford effect-explanation and consequences, Vision Res. 13, 1115-1137 (1973).

[15] Vohnsen, B., Iglesias, I., and Artal, P., Guided light and diffraction model of human eye photoreceptors, 22, J. Opt. Soc. Am. A **22**, 2318-2329 (2005).

[16] Stiles, W. S., *The directional sensitivity of the retina,*

Ann. R. Coll. Surg. Engl. **30**, 73-101 (1962).

[17] Born, M., & Wolf, E., *Principles of Optics*, 7th (expanded edition) (Reissued by Cambridge University Press, 1997).

[18] Pattnaik S., Kamilla, S. K., Roy, G. S., Nayak, M. K., Das, R., and Sharma, N. K., *Binary star system-a spectral analysis*, Lat. Am. J. Phys. Educ. **5**, 438- 442 (2011).

[19] Westheimer, G., *Retinal light distribution for circular apertures in Maxwellian view*, J. Opt. Soc. Am. **49**, 41-44 (1959).

[20] Hopkins. H., *21st Thomas Young Oration (The application of frequency response techniques in optics)*, Proc. Phys. Soc. **79**, 889-919 (1962).

[21] Campbell, F. W., & Robson, J. G., *Application of Fourier analysis to the visibility of gratings*, J. Physiol., **197**, 551-566 (1968).

[22] Goodman, J. W., Introduction to Fourier optics, 3rd ed., (Viva books, USA, 2007)

[23] Goldstein, E. Bruce, *Sensation and Perception,* 4th ed., (Brooks/Cole Publishing Company, USA, 1996).

[24] Ghatak, A., *Optics*, 5th ed., Tata McGraw-Hill (2009).

[25] Sharma, N. K., Roy, G. S., Kamilla, S. K., Mishra, K., and Pattnaik, S., *Coherent sine wave response of a human eye in the presence of the Stiles-Crawford effect of the first kind*, J. Pure & Appl. Phys. **23**, 51-58 (2011).