Thermal diffusivity measurement by means of the hot wire technique

J. Guarachi[1], U. Nogal[1], J. Hernández Wong[2], A. Calderón[1]*, E. Marín[1], J. B. Rojas Trigos[1], A. G. Juárez Gracia[1], R. Abdelarrague[3] and R. A. Muñoz Hernández[1]

[1]Instituto Politécnico Nacional, Centro de Investigación en Ciencia Aplicada y Tecnología Avanzada, Av. Legaria # 694, Col. Irrigación, C.P. 11500, Ciudad de México, México.

[2]CONACyT-Instituto Politécnico Nacional, Centro de Investigación en Ciencia Aplicada y Tecnología Avanzada, Unidad Legaria. Legaria No. 694, C.P. 11500 Ciudad de México, México.

[3]Universidad Autónoma del Estado de Mexico, Blev. Universitario s/n predio San Javier, Atizapan de Zaragoza, Estado de México.

E-mail: jcalderona@ipn.mx

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Abstract

Approximate models for the temperature difference ΔT as a function of the time t in the heating and cooling stages measured with the hot wire technique are reported. It is shown that considering up to the third term in the power series development of the exponential integral leads to a significantly greater approximation to the expression for ΔT. The utility of the model for the heating stage is demonstrated by the adjustment to experimental results of magnesium oxide powder. Likewise, minimum values to be measured of the thermal diffusivity α are reported for the cases of a single needle and the dual probe and it is shown that α is smaller by two orders of magnitude for the probe of a needle than for the case of a dual probe, which gives greater amplitude to the application of this technique. Finally, the application of the model to the cooling stage shows that the model does not reliably reproduce the experimental points due to the importance at this stage of the effects of edges not considered in the development of the model.

Keywords: Heat transfer, thermal properties, hot wire technique, thermal waves.

Resumen

Se reportan modelos aproximados para la diferencia de temperatura ΔT en función del tiempo t en las etapas de calentamiento y enfriamiento medidos con la técnica del alambre caliente (hot wire). Se demuestra que el considerar hasta el tercer término en el desarrollo en serie de potencias de la integral exponencial conduce a una aproximación significativamente mayor a la expresión para ΔT. Se demuestra la utilidad del modelo reportado para la etapa de calentamiento mediante el ajuste a resultados experimentales de óxido de magnesio en polvo. Asimismo, se reportan valores mínimos a medir de la difusividad térmica α para los casos de una sola aguja y la sonda dual y se demuestra que α es menor en dos órdenes de magnitud para la sonda de una aguja que para el caso de una sonda dual, lo cual da mayor amplitud a la aplicación de esta técnica. Finalmente, la aplicación del modelo a la etapa de enfriamiento muestra que el modelo no reproduce de forma confiable los puntos experimentales debido a la importancia en esta etapa de los efectos de bordes no considerados en el desarrollo del modelo.

Palabras clave: Transferencia de calor, propiedades térmicas, técnica del alambre caliente, ondas térmicas.

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I. INTRODUCTION

Heat transfer is the area which describes the energy transport between material bodies due to a difference in temperature, and its development and applications are of fundamental importance in many branches of engineering since provides economical and efficient solutions for critical problems encountered in many advanced equipment. Among the parameters that determine the thermal behaviour of a material, the thermal diffusivity (α), is especially important because it represents the rate of heat transfer into the media. Moreover, the thermal diffusivity is the ratio of the thermal conductivity k to the heat capacity ρc_p ($\alpha = k/\rho c_p$), hence, it measures the ability of a material to conduct thermal energy relative to its ability to store thermal energy. Materials of large α will respond quickly to changes in their thermal environment, whereas materials of small α will respond more sluggishly taking longer to reach a new equilibrium condition [1].

The hot wire (HW) technique is an absolute, non-steady state and direct method, which is considered an effective procedure to determining the thermal diffusivity of a variety of materials, including ceramics, fluids, food and polymers [2,]. The HW technique is based on the measurement of the temporal history of the temperature rise caused by a linear heat source (a hot wire) embedded in a test material. If the wire is heated by Joule´s effect passing a constant electrical current [3]. In the mathematical formulation, the hot wire is assumed an ideal, infinitely thin and long heat source, which is in an infinite surrounding material to be studied.

This work report the mathematical approximation of the temperature difference as a function of the time in the heating and cooling stages measured with the hot wire technique, showing the percentage error in the approximation and the limits of application in the determination of thermal diffusivity. The application of the theoretical models in samples of magnesium oxide in powder is shown.

II. HEATING STAGE

In the hot wire technique, the thermal properties of the study material are determined by adjusting the data ΔT vs t during the heating process, $0 < t < t_c$, According to [4]:

$$\Delta T = -\frac{q}{4\pi k} E_i\left(-\frac{r^2}{4\alpha t}\right) \tag{1}$$

where q is the linear ratio of heat dissipation by the source (W/m), t_c is the heating time and E_i is the integral exponential given by [5]:

$$E_i(x) = \int_{-\infty}^{x} \frac{e^u}{u} du \quad ; \quad x \neq 0 \tag{2}$$

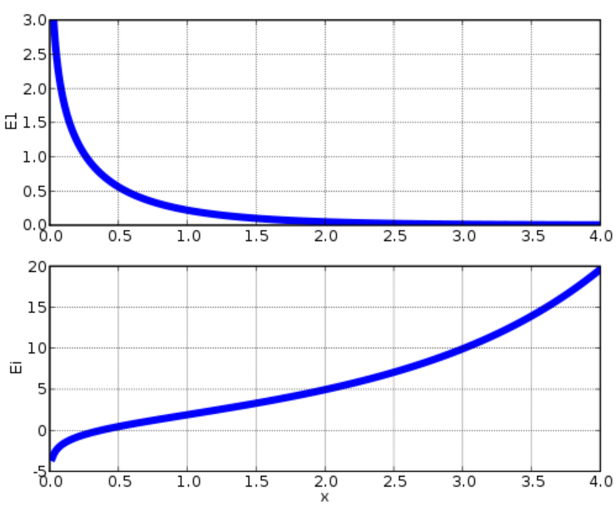

FIGURE 1. Graph of the function E_1 (top) and function E_i (bottom).

By integrating the Taylor series of e^u/u thge following serial representation of $E_i(x)$ is obtained:

$$E_i(x) = \gamma + ln|x| + \sum_{n=1}^{\infty} \frac{x^n}{nn!} \quad ; \quad x \neq 0 \tag{3}$$

where $\gamma = 0.57721566\ldots$ is the Euler constant.

From (2) y (3), it follows:

$$-E_i(-x) \approx -\gamma - ln|x| + x - \frac{1}{4}x^2 + \frac{1}{18}x^3 - \frac{1}{96}x^4 + \cdots \quad ; \quad x \neq 0 \tag{4}$$

Figure 2 shows the curve of:

$y(x) = -\gamma - ln|x|$,

$y_1(x) = -\gamma - ln|x| + x$,

$y_4(x) = -\gamma - ln|x| + x - (1/4)x^2 + (1/18)x^3 - (1/96)x^4$,

By comparison a significant approximation of y_1 to $-E_i(-x)$ is observed for small values of x.

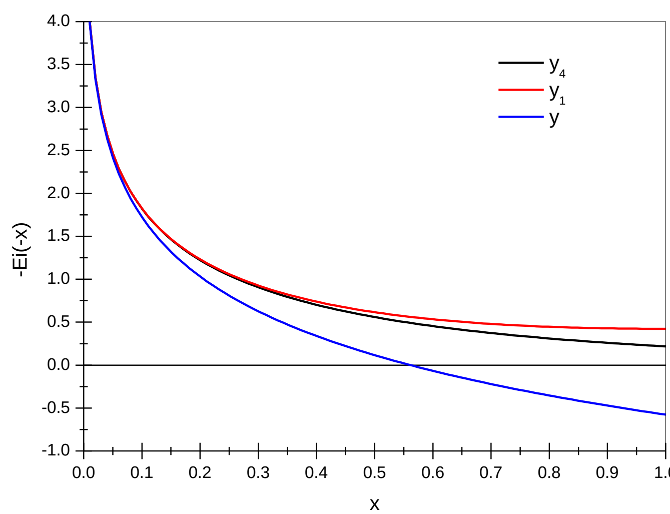

FIGURE 2. Graphs of $-E_i(-x)$ and their power series approximations.

Figure 3 shows the percentage error between the graphs of y_4 and y, as well as those of y_4 and y_1. It is observed that the percentage error for $y(x) = -\gamma - ln|x$ is significantly higher than for $y_1(x) = -\gamma - ln|x| + x$, so that, in the first case $e_{\%} < 1\%$ for very small values of x (< 0.03), and in the second case $e_{\%} < 1\%$ for $x < 0.22$. This is a difference of one order of magnitude in x!

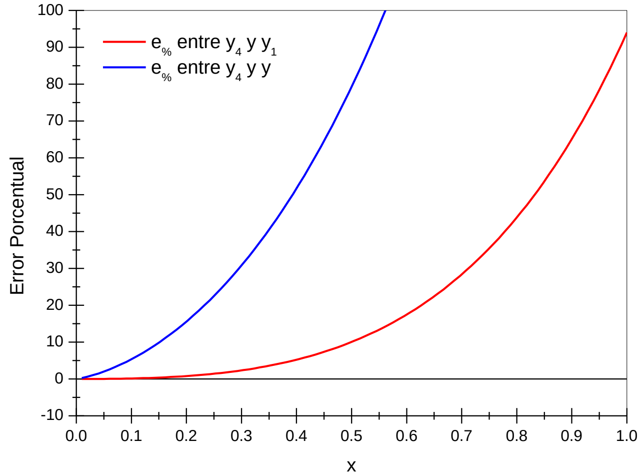

FIGURE 3. Graphs of percentage errors between -$E_i(-x)$ and their approximations in series of powers.

For $x < 0.22$:

$$x = \frac{r^2}{4\alpha t} < 0.22 \;\rightarrow\; \alpha > \frac{r^2}{0.88t}$$

Taking the value of $r = 6\ mm$ (for the case of a dual probe), it follows,

$$\alpha > \frac{(6mm)^2}{0.88t} = \frac{40.91mm^2}{t}$$

Thus, model y_1 can be used with a convenient approximation for materials with α values as small as 41 mm^2/s (0.41 cm^2/s) when fit from t = 1*s*, or 2.73 mm^2/s when fit from t = 15*s*.

If it is consider $r = 0.64mm$ (for tha case of a probe of a needle), it follows,

$$\alpha > \frac{(0.64mm)^2}{0.88t} = \frac{0.46mm^2}{t}$$

Thus, model y_1 s can be used with a convenient approximation for materials with α values as small as 0.46 mm^2/s (4.6 x10^{-3} cm^2/s) when fit from t = 1*s*, or 0.031 mm^2/s (0.31 x10^{-3} cm^2/s) when fit from t = 15*s*.

These results show that the minimum value of α decreases two orders of magnitude for the probe of a needle rather than the dual.

Fort the approximation $y_1(x)$ for -$E_i(-x)$, equation (1) takes the form:

$$\Delta T \approx \frac{q}{4\pi k}\left\{-\gamma - ln\left(\frac{r^2}{4\alpha t}\right) + \frac{r^2}{4\alpha t}\right\} \quad ; \quad 1 < t < t_c \tag{5}$$

But, taking into account that,

$$-\ln(u) = \ln(1/u) \text{ y } \gamma = \ln(e^{\gamma}).$$

The following final expression is obtained for ΔT in the heating stage:

$$\Delta T \approx \frac{q}{4\pi k}\left\{ln\left(\frac{4\alpha t}{r^2 e^{\gamma}}\right) + \frac{r^2}{4\alpha t}\right\} \quad ; \quad 1 < t < t_c \tag{6}$$

III. COOLING STAGE

This step could also be useful in determining the thermal properties of a given material. During the cooling stage ($t_c < t$) the behavior of ΔT vs t is given by [1]:

$$\Delta T = -\frac{q}{4\pi k}\left[-E_i\left(-\frac{r^2}{4\alpha t}\right) + E_i\left(-\frac{r^2}{4\alpha(t-t_1)}\right)\right] \tag{7}$$

where t_c is the heating time and E_i is the integral exponential given by Eq. (2).

When considering the approximations of the previous section, it follows ($t_c < t$):

$$\Delta T \approx -\frac{q}{4\pi k}\left[-\gamma - ln\left(\frac{r^2}{4\alpha t}\right) + \frac{r^2}{4\alpha t} + \gamma + ln\left(\frac{r^2}{4\alpha(t-t_c)}\right) - \frac{r^2}{4\alpha(t-t_c)}\right] \tag{8}$$

Which, when ordering terms acquires the reduced form:

$$\Delta T \approx \frac{q}{4\pi k}\left[-ln\left(\frac{t}{t-t_c}\right) + \frac{r^2}{4\alpha t}\left(\frac{t_c}{t-t_c}\right)\right] \quad ; \quad t_c < t \tag{9}$$

For the temperature difference during the cooling stage.

IV. EXPERIMENTAL RESULTS

Figure 4 shows the experimental data of ΔT vs t for the measurement of a powder sample of MgO with the probe of a needle. The continuous curve represents the best fit of the Eq. (6) to experimental data while maintaining k and α as fitting parameters. The result k = 0.2 W/mK and α = 0.135 mm^2/s corresponds to those reported in the literature.

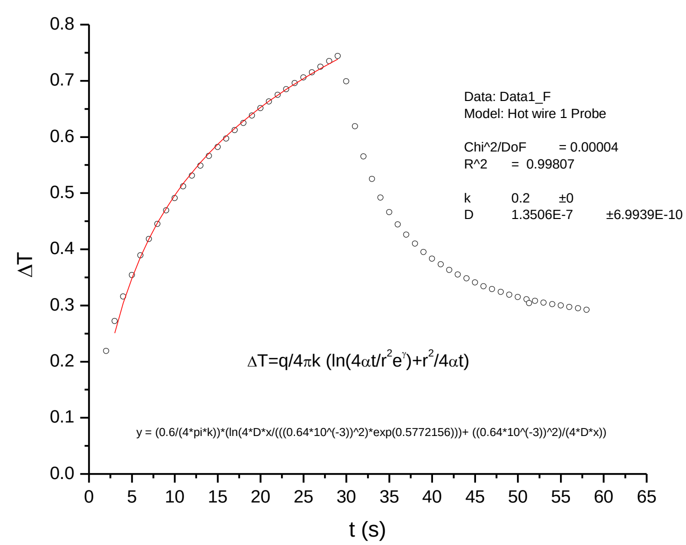

FIGURE 4. Experimental results of ΔT vs t for a simple of MgO powder by the hot wire technique of a needle. The continuous curve represents the best fit of Eq. (6) to the experimental data.

Figure 5 shows the graph of Eq. (6) in red and its comparison with the same but without the term $x = r^2/4\alpha t$. It is evident that not considering this term leads to significant deviations, especially to lower values of t, which can cause notable deviations when fitting to the experimental data.

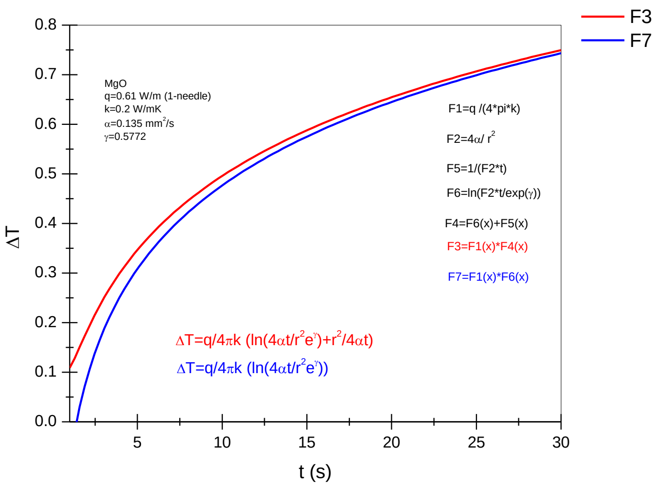

FIGURE 5. Graphs of ΔT vs t for the complete Eq. (6), in red, and without the term $x = r^2/4\alpha t$, blue curve.

Figure 6 shows the best fit of Eq. (6), without the term $x = r^2/4\alpha t$, for the same experimental data of the Fig. 4, while maintaining k and α as fitting parameters. The result $k = 0.22$ W/mK and $\alpha = 0.187$ mm^2/s present percentage errors of 10% y 38.5%, respectively, when compared to those obtained with complete Eq. (6). This shows a poor fit.

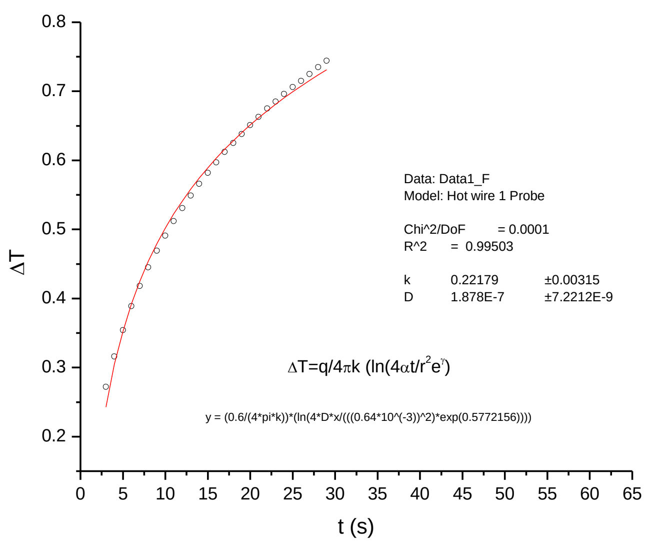

FIGURE 6. Fit of Eq (6) to the experimental data obtained for the sample of MgO powder, without the term $x = r^2/4\alpha t$.

On the other hand, in the case of the cooling stage, figure (7) shows the attempt to fit the Eq. (9) to the experimental data of Fig. 4. There is a poor fit of the model of Eq. (9), suggesting a cooling faster than the experimental data show.

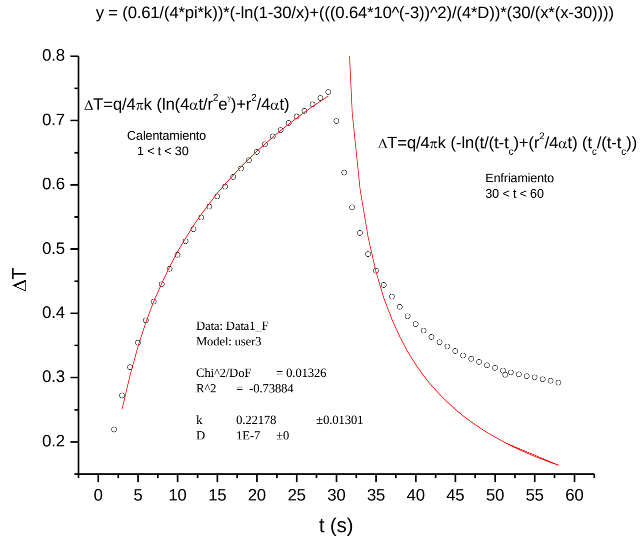

FIGURE 7. Fit of Eq. (9) to the experimental data of MgO powder during the cooling stage.

During the heating process, the model reproduces the behavior of the experimental data because in the first seconds the heat propagates from the source radially outwards. The interface, between the sample and the external medium, does not present any alteration to the propagation of heat. As the heat reaches the interface it no longer propagates with the same speed and there is an energy accumulation effect that is reproduced in the experimental data in a higher value of ΔT in the cooling process (even from the end of the process of heating) which is increased with t. It is worth mentioning that, the model developed suppose an infinite medium in which the heat should be propagated without obstacle.

VI. CONCLUSIONS

The mathematical approximation of the temperature difference as a function of the time in the heating and cooling stages measured with the hot wire technique it was presented. It showed the percentage error in the approximation and the limits of application in the determination of thermal diffusivity. In addition, the application of the theoretical models in samples of magnesium oxide in powder is demonstrated.

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