Kicking a cold soccer ball

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Abstract
A soccer ball is conceptually split into halves so that it can be modeled as two identical blocks connected by a spring. A constant kicking force applied to one block simultaneously compresses the spring and imparts velocity to the center of mass of the system. If the half in contact with the foot detaches from it when it reaches a specified speed, the final center-of-mass velocity depends on the spring constant. That could explain why it is hard to kick a soccer ball as far on a cold day (when the spring is stiffer) as on a warm day.

Keywords: Physics of sports, Newton's second law, Mass-spring oscillator.

Resumen
Una pelota de fútbol se divide conceptualmente en dos mitades para que se pueda modelar como dos bloques idénticos conectados por un resorte. Una fuerza de patada constante aplicada a un bloque comprime simultáneamente el resorte e imparte velocidad al centro de masa del sistema. Si la mitad en contacto con el pie se separa cuando alcanza una velocidad específica, la velocidad final del centro de masa depende de la constante del resorte. Eso podría explicar por qué es difícil patear un balón de fútbol tan lejos en un día frío (cuando la primavera está más rígida) que en un día cálido.

Palabras clave: Física de los deportes, Segunda ley de Newton, Oscilador masa-resorte.

I. INTRODUCTION

Football players report is it more difficult to kick a long field goal when the weather is cold. Data from the NFL corroborates this statement [1]. The longest kicks in history were either at Mile High Stadium in Denver, with an altitude of over 1.5 km above sea level, or else in warmer indoor or outdoor conditions. Mile High Stadium, where three of the five longest kicks took place, has reduced air density, thereby lowering the drag on the ball. But how can one understand the increased difficulty of kicking a ball in cold weather, when some players report it feels like kicking a rock? Sports balls have previously been analyzed as a single mass on a spring, with losses of mechanical energy represented by a coefficient of restitution [2] or by a viscous damper [3]. The present paper instead proposes a loss-free model of a kicked ball as two masses (representing the kicked hemisphere and the opposite hemisphere of the ball) connected by a spring (representing the elasticity of the ball). The ideas are thus within the grasp of an elementary physics student. The equations are solved theoretically and compare favorably with available soccer ball measurements. This approach may appeal to a physics of sports class, or as an example in a general introductory physics course when discussing the concept of mechanical oscillations.

II. THEORETICAL MODEL

Two blocks of mass m on a frictionless surface are connected by a massless spring of stiffness constant k. Initially the two blocks are at rest and the spring is relaxed. Starting at $t=0$ a constant force F is applied to the left block 1 in the direction of the right block 2 in Fig. 1 until some later time $t=t_f$ when the left block has attained speed υ. What is the speed of the center of mass (cm) of the system at that final instant in time?

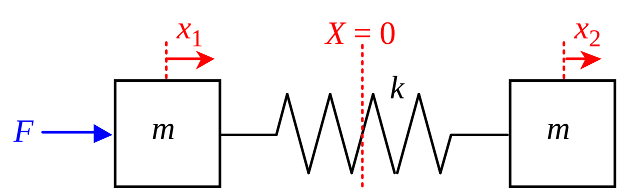

FIGURE 1. Two identical blocks of mass m are connected by a massless Hookean spring of stiffness constant k. A constant force F pushes the left block 1 toward the right block 2, but no other external horizontal forces act on the system. The displacements of the left block, right block, and center of mass of the system are respectively x_1, x_2, and X.

Choose a coordinate axis-pointing positive rightward with the origin at the initial position of the cm. At some arbitrary instant in time t such that $0<t<t_f$, block 1 has displaced

rightward by x_1 from its starting position and block 2 by x_2. Newton's second law (N2L) applied to block 1 implies

$$m\ddot{x}_1 = F - k(x_1 - x_2) . \quad (1)$$

Where overdots denote time derivatives. Likewise N2L for block 2 gives rise to

$$m\ddot{x}_2 = k(x_1 - x_2) . \quad (2)$$

Transform to center-of-mass coordinates such that the position of the cm is $X = (x_1 + x_2)/2$ and the compression of the spring is $x = x_1 - x_2$. Then the sum of Eqs. (1) and (2) becomes

$$2m\ddot{X} = F , \quad (3)$$

whose solution for the given initial conditions is

$$X = \frac{F}{4m} t^2 . \quad (4)$$

On the other hand, the difference of Eqs. (1) and (2) leads to

$$m\ddot{x} = F - 2kx \quad (5)$$

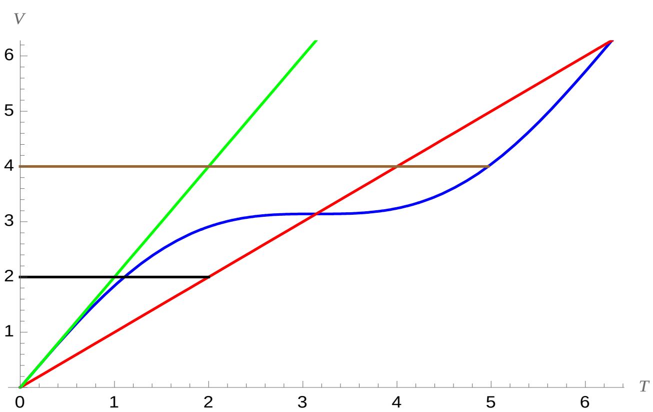

FIGURE 2. Dimensionless speed *V* of the left mass 1 as a function of dimensionless time *T* starting from the instant of application of the external force *F*. The blue curve plots the general solution from Eq. (10), the red curve plots the limiting case of $k \to \infty$ from Eq. (11) which is also equal to the center-of-mass speed in general from Eq. (16), and the green curve plots the opposite limiting case of $k \to 0$ from Eq. (13). As examples discussed in the text, the horizontal black and brown lines intersect the colored curves at times *T* corresponding to a speed *V* of 2 and 4, respectively.

With solution

$$x = \frac{F}{2k}(1 - \cos\omega t) \quad \text{where} \quad \omega = \sqrt{\frac{2k}{m}} . \quad (6)$$

Inverting Eqs. (4) and (6) gives

$$x_1 = \frac{x}{2} + X = \frac{F}{4k}(1 - \cos\omega t) + \frac{F}{4m} t^2 , \quad (7)$$

so that

$$\dot{x}_1 = \frac{F}{4k}\omega \sin\omega t + \frac{F}{2m} t . \quad (8)$$

Recast Eq. (8) into dimensionless form by defining

$$V = \frac{\sqrt{8mk}}{F}\dot{x}_1 \quad \text{and} \quad T = t\sqrt{\frac{2k}{m}} , \quad (9)$$

to get,

$$V = \sin T + T , \quad (10)$$

as plotted in blue in Fig. 2.

Connecting the two masses rigidly together corresponds to an infinitely stiff spring $k \to \infty$ so that $\omega / k \to 0$ while $\sin\omega t$ remains bound within the range –1 to 1. Thus Eq. (8) becomes

$$V_{\text{rigid}} = T \quad (11)$$

in dimensionless form, as plotted in red in Fig 2. At the opposite limit, disconnecting the two masses is equivalent to a maximally floppy spring $k \to 0$ so that

$$\frac{F}{4k}\omega \sin\omega t \to \frac{F}{4k}\omega^2 t = \frac{F}{2m} t , \quad (12)$$

and hence Eq. (8) gives rise to

$$V_{\text{floppy}} = 2T \quad (13)$$

as plotted in green in Fig. 2.

The left mass has a dimensionless final speed of

$$V_{\text{f}} = \frac{\sqrt{8mk}}{F}\upsilon , \quad (14)$$

at a dimensionless time of T_{f} found by substituting Eq. (14) into the appropriate choice of Eqs. (10), (11), or (13) and then inverting it, numerically in the case of Eq. (10). The speed of the center of mass is found by taking the time derivative of Eq. (4) to get

$$\dot{X} = \frac{F}{2m} t . \quad (15)$$

It can be rewritten in dimensionless form as

$$V_{\text{cm}} = \frac{\sqrt{8mk}}{F}\dot{X} = T , \quad (16)$$

using Eq. (9), which is again plotted in red in Fig. 2.

For example, suppose the dimensionless final speed of block 1 is $V_f = 2$ as indicated by the black horizontal line in Fig. 2. Then the dimensionless final time T_f and hence, the dimensionless final center-of-mass speed is 1 for the floppy case, approximately 1.1061 for the spring-coupled system, and 2 for the rigid connection. For that value of V_f, rigidly connecting the two masses together thus gives rise to the largest final cm speed of the system.

On the other hand, suppose the dimensionless final speed of block 1 is instead $V_f = 4$ as indicated by the brown horizontal line in Fig. 2. Then the dimensionless final time and cm speed is 2 for the floppy case, approximately 4.9676 for the spring coupling, and 4 for the rigid connection. So now connecting the two masses with the spring gives rise to the largest final center-of-mass speed of the system. If this situation can be taken as a model for an inflated ball, then an underinflated floppy ball or a stiff cold ball both result in lower final cm speeds than does a ball which is properly inflated and warm, assuming the kicker applies to all balls the same constant force F until the side of the ball in contact with the foot reaches the specified detachment speed υ. The system will lie in a range of parameter values where this behavior can occur only if

$$\frac{\sqrt{8mk}}{F}\upsilon > \pi \ . \qquad (17)$$

Provided the left-hand side of this inequality is also smaller than 2π. (More generally, the quantity on the left-hand side must lie between an odd-integer multiple of π and the next larger even-integer multiple of π).

III. COMPARISON WITH SPORTS DATA

Consider a soccer ball with a mass of 430 g so that $m = 0.215$ kg . For a long-range kick when the final cm speed is approximately 30 m/s, a peak force of about 3 kN must be applied to the ball [4]. In the present simplified model, it is assumed that the force is instead constant over the duration of the kick. So to give the same impulse, F must be about half of the peak force, or 1.5 kN. The compression of the ball during the kick is plotted in Fig. 3 using these values and a spring constant of $k = 25$ kN/m . The peak value of x is about 6 cm and the compression lasts about 12 ms. This graph is similar to measurements for a kicked soccer ball plotted by the dashed curve in Fig. 2 of either Ref. [4] or [5]. If the kicked side of the ball detaches from the foot when the latter is moving at $\upsilon = 24$ m/s , then the left-hand side of Eq. (17) is about 5% larger than π. There may be a good reason that a soccer ball is designed so that, when properly inflated and warm, a strong kick places its behavior near the crossing point between the red and blue curves in Fig. 2. The blue curve is quite flat in that crossing region, so that a skillful player can obtain a large increase in final cm speed of the ball for a small increase in foot speed □.

For these parameters, the ball is found to lose contact with the foot after about 9 ms and to end up with a final cm speed of 30 m/s. That speed is 26% larger than what would be obtained for a rigid ball, such as might be more likely to occur on a cold day when the skin of the soccer ball is less flexible.

IV. CONCLUSION

Mathematically it has been shown that under the right conditions, a change in the spring constant (due to a change in the elasticity of the ball with temperature) can result in a different final speed of the center of mass of a soccer ball for fixed values of the mass $2m$ of the ball, of the kicking force F, and of the maximum speed υ of the player's foot. It is instructive to end by stepping back and reviewing the explanation for this result at a conceptual level.

In the limiting case of a rigid ball when $k \to \infty$, every part of the ball is always moving at the same speed as the foot with which it is in contact. Thus, as the ball detaches from the foot when it is on the verge of outpacing the maximum speed υ of the foot, its center of mass must be traveling with that same speed υ. The only way to increase the final center-of-mass speed is to keep the ball in contact with the foot for a longer time, given that the kicking force F is constant in the simple model adopted here. That is done by connecting mass 1 to a spring (of finite nonzero stiffness) so that the mass will then oscillate about the end of the spring to which it is connected. If it so happens that the mass is oscillating with a *backward* velocity (relative to the forward motion of the center of mass) as it approaches the detachment speed, then it will remain in contact longer with the forward-moving foot. This reasoning explains why the blue curve in Fig. 2 must lie in a range that is vertically *below* the red curve.

It follows that the maximum enhancement in the final center-of-mass speed of the ball (relative to what would be achieved if the ball were perfectly rigid) is obtained when the vertical difference between the red and blue curves is maximized. According to Eqs. (10) and (11) that happens at a dimensionless time of $T_f = 3\pi/2$, three-quarters of the way through a cycle of oscillation of the mass on the spring, as one might have guessed. In turn, that implies the final value of the dimensionless speed of mass 1 will be

$$V_f = \frac{3\pi}{2} - 1 \, . \qquad (18)$$

According to Eq. (10). Reverting to dimensional quantities using Eq. (9), this value corresponds to a spring of stiffness constant

$$k = \left(\frac{3\pi}{2} - 1\right)^2 \frac{F^2}{8m\upsilon^2} \qquad (19)$$

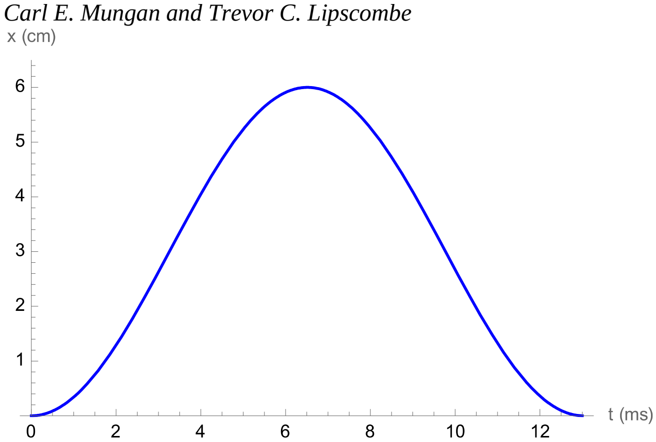

FIGURE 3. Graph of the compression *x* as a function of time *t* after the kicking force *F* is applied to the two-block system with parameters chosen to model a soccer ball.

Given that the final value of $\dot{x}_1$ is υ. For the parameters in Sec. III of $2m = 0.43$ kg, $F = 1.5$ kN, and $\upsilon = 24$ m/s, the optimum stiffness to demonstrate the enhancement experimentally would therefore be $k = 31$ kN/m.

On the other hand, a perfectly floppy ball will always perform worse than one that is either elastic or rigid. The green curve lies above the blue and red curves in Fig. 3 at all nonzero times. The reason for that is the kick will initially propel forward only the left mass in Fig. 1, while the right mass remains at rest. The center of mass will then have a speed of only *half* of the speed of the left mass. Assuming there is enough initial separation between them that the left mass gets up to speed υ and detaches from the foot before it hits the right mass, the final center of mass speed will thus be $\upsilon / 2$, only half of what it would be for a perfectly rigid body as explained above. That is true regardless of whether the subsequent collision between the two halves of the ball is elastic or inelastic. Intuitively no one would want to play soccer with a ball that is deflated due to a tear in its skin!

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