Teaching equivalent statements of the second law of thermodynamics

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Abstract

It has been noticed that undergraduates are comfortable as far as their understanding of various equivalent statements of the second law of thermodynamics is concerned, but they are unable to work out by themselves the equivalence of one with the others. The aim of the present article is to help students following the textbook Fundamentals of Physics written by Halliday, Resnick, and Walker in this endeavour.

Keywords: Teaching, thermodynamics, second law statements, equivalence.

Resumen

Se ha observado que los estudiantes universitarios se sienten cómodos en lo que respecta a su comprensión de varios enunciados equivalentes de la segunda ley de la termodinámica, pero no son capaces de determinar por sí mismos la equivalencia de uno con los otros. El objetivo del presente artículo es ayudar a los estudiantes que siguen el libro de texto Fundamentos de Física escrito por Halliday, Resnick y Walker en este esfuerzo.

Palabras clave: Enseñanza, termodinámica, enunciados de la segunda ley, equivalencia.

I. INTRODUCTION

While teaching the second law of thermodynamics to undergraduates belonging to the discipline agriculture from the classic textbook Fundamentals of Physics by Halliday, Resnick, and Walker [1] (hereafter HRW), it was noticed that the students were comfortable as far as their understanding of various statements of the second law of thermodynamics and their detailed elaborations were concerned, but they found difficulty in working out by themselves the equivalence of one with the others. The aim of this article is to accomplish this task for the benefit of the students. The next section proceeds with background material borrowed from the above-mentioned book.

II. BACKGROUND MATERIAL

An engine operating between a higher temperature T_H and a lower temperature T_C, following the clockwise Carnot cycle (figure 1), has the maximum efficiency

$$\eta_{Car} = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}. \qquad (1)$$

Here Q_H is the heat extracted from the hot reservoir during isothermal expansion of the working substance at temperature T_H. An amount of work W is performed by the working substance, and heat Q_C is rejected to the cold reservoir during isothermal compression of the same substance at temperature T_C. As it is a cyclic process, the internal energy of the working substance remains unchanged after each cycle, that is, $\Delta E_{int} = 0 = Q_H - W - Q_C$. This gives us a relationship

$$W = Q_H - Q_C. \qquad (2)$$

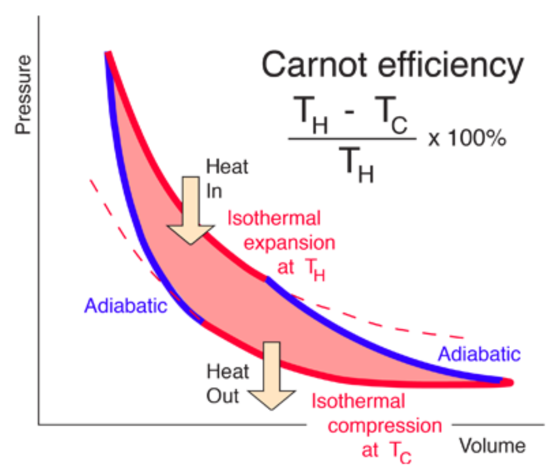

FIGURE 1. Pressure-Volume Diagram of a Clockwise Carnot Cycle for Engines from http://hyperphysics.phy-astr.gsu.edu/hbase/thermo/carnot.html. Thanks to Prof Carl Rodney

Nave.

Next, the counterpart of an engine, that is, a refrigerator operational between the same two temperatures, $\boldsymbol{T_H}$ **and** $\boldsymbol{T_C}$ performs on the reverse Carnot cycle (figure 2). It starts with the adiabatic expansion of the working substance from point **1** to point **2**, followed by the isothermal expansion of the same substance up to point **3** while absorbing heat $\boldsymbol{Q_C}$ from the object to be cooled. It is now compressed adiabatically up to the point **4** followed by isothermal compression arriving back to the starting point **1**. Once again, $\Delta \boldsymbol{E_{int}} = \boldsymbol{0} = -\boldsymbol{Q_H} + \boldsymbol{W} + \boldsymbol{Q_C}$. The working substance transfers heat $\boldsymbol{Q_H} = \boldsymbol{Q_C} + \boldsymbol{W}$ to the reservoir at a higher temperature $\boldsymbol{T_H}$; here $\boldsymbol{W}$ is the amount of work done on the working substance. The expression for the maximum coefficient of performance for the Carnot refrigerator is

$$K_{Car} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C}. \quad (3)$$

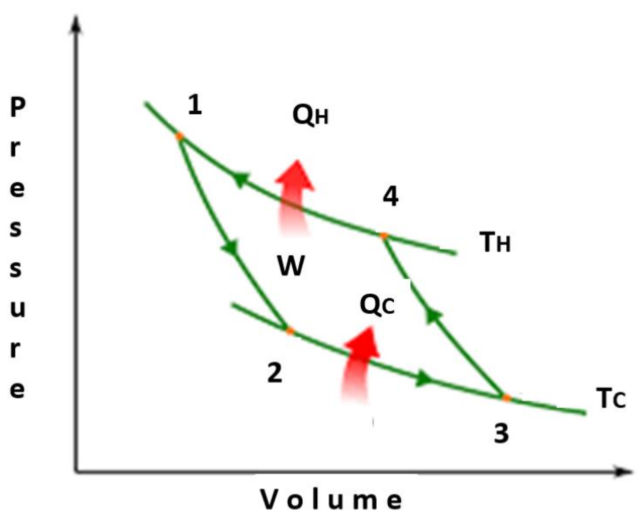

FIGURE 2. Pressure-Volume Diagram of the Reversed Carnot Cycle for Refrigeration.

The corresponding two devices based on these cycles can be visualized one after another. As far as the Carnot engine is concerned, heat Q_H is extracted from the hot reservoir at a certain temperature T_H, an amount of work W is produced, and heat Q_C is rejected to the sink at a lower temperature T_C; this device is depicted pictorially in figure 3. The next figure 4 shows the other device, a refrigerator, where Q_C is being borrowed from the object to be cooled, an amount of work W is performed, and the total heat $Q_H = Q_C + W$ is rejected at a higher temperature T_H.

The examination of the relation (1) and the pictorial view of the engine (figure 3) suggests that one can achieve hypothetical efficiency unity for an engine provided the heat Q_C rejected to the sink happens to be zero, that is, it does not reject any heat during the whole cycle and converts input heat Q_H wholly into work W; the corresponding diagram will appear as shown in figure 5; such an engine has been termed a *perfect engine* by HRW. Similarly, a *perfect refrigerator* is one that collects heat Q_C from the body to be cooled and directly transfers the same to the hot reservoir without the expenditure of any work; this can be understood from the relation (2) and the diagram shown in figure 4 if $W = 0$; the coefficient of performance of such a perfect refrigerator would be hypothetically infinity. A perfect refrigerator is depicted in Figure 6.

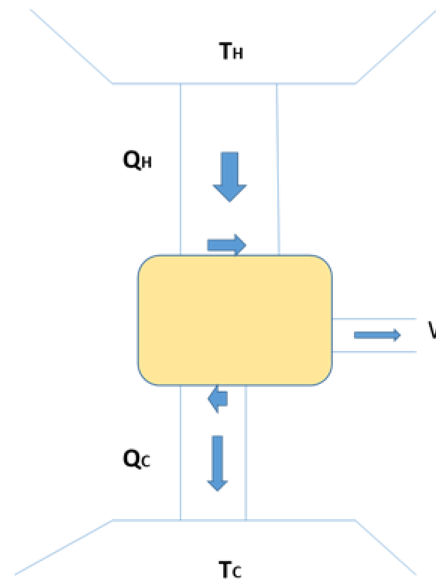

FIGURE 3. Diagram indicating an engine by encircling it with arrows pointing clockwise along the path of the working substance: the heat $\boldsymbol{Q_H}$, is extracted from the hot reservoir at $\boldsymbol{T_H}$, a part of it $\boldsymbol{W}$ is converted into work, and the rest $\boldsymbol{Q_C}$ is discharged to the sink at $\boldsymbol{T_C}$.

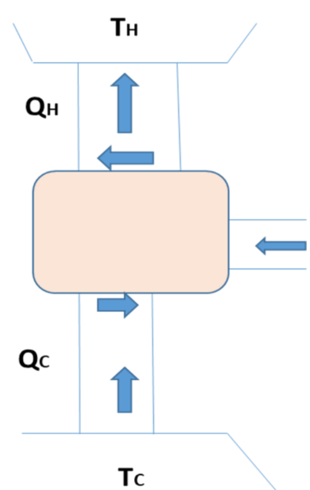

FIGURE 4. Diagram indicating a refrigerator encircled with arrows pointing in an anticlockwise direction for the working substance. The heat $\boldsymbol{Q_C}$ is extracted from the object to be cooled by the working substance, work $\boldsymbol{W}$ is performed on it, and lastly, the heat $\boldsymbol{Q_H} =$

$\boldsymbol{Q_C + W}$ is discharged at a higher temperature $\boldsymbol{T_H}$.

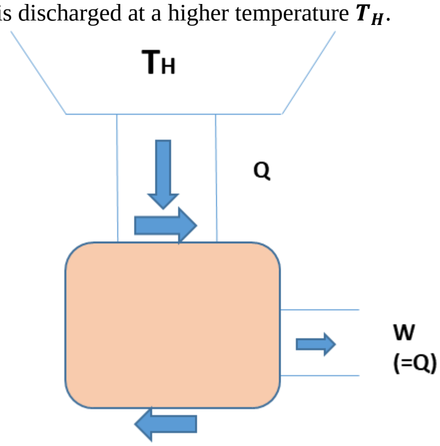

FIGURE 5. A pictorial representation of a perfect engine.

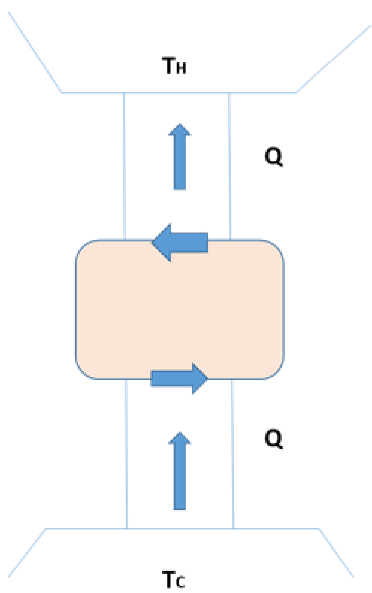

FIGURE 6. A pictorial view of a perfect refrigerator.

III. STATEMENTS AND THEIR EQUIVALENCE

Having summed up the necessary background material, let us look at various equivalent statements for the second law of thermodynamics provided by HRW; these are

1. ‘There are no perfect engines.’
2. ‘There are no perfect refrigerators.’
3. ‘No real engine operating between two specified temperatures can have a greater efficiency than that of a Carnot engine operating between those same two temperatures.’ Next, a corresponding fourth statement for refrigerators would be as mentioned below.
4. No real refrigerator operating between two specified temperatures can have a greater coefficient of performance than that of a Carnot refrigerator operating between those same two temperatures.

The above first three equivalent statements are mentioned by HRW, and the 4th statement for refrigeration results from the 3rd one. Next, we turn to the discussion of the equivalence of these four statements by considering them in all possible combinations. These are as follows:

- ***Equivalence of the 1st and 2nd statements***

Let us begin with the equivalence of the first two statements enunciated and described by HRW. They have debated it by demonstrating that if there is a perfect engine, a perfect refrigerator will also be available; to show this, they have drawn the diagram (Figure 7). Here we have a perfect engine that directly converts input heat into equivalent work without rejecting any heat whatsoever to the sink. This accessible work is now utilized for the operation of a real refrigerator; thereby, this unabridged device is equivalent to a perfect refrigerator, as shown; this perfect refrigerator transfers heat Q directly from the object to be cooled to the hot reservoir. Thus, the first two statements are equivalent in the sense that if we can have a perfect engine, then this amounts to having a perfect refrigerator also, since the former is not possible and therefore the latter is also impossible.

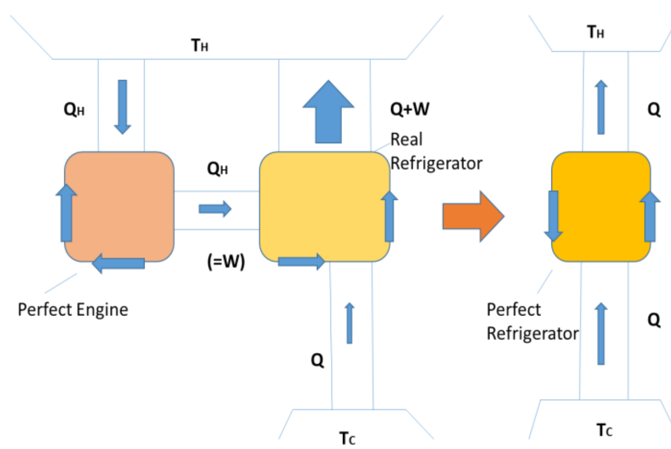

FIGURE 7. A pictorial view of the existence of a perfect refrigerator based on the hypothesis that we do have a perfect engine. A perfect engine is coupled with a real refrigerator and provides the work part for its operation in cooling; the whole device appears to be a perfect refrigerator.

- ***Equivalence between the 2nd and 1st statements***

Now it is worth asking to prove it the other way, that is, to show that if there is a perfect refrigerator, it is equivalent to having a perfect engine. This has been left as an exercise for students by HRW; students indeed approached us for the solution. This solution is shown in the following figure 8. Here we have a device consisting of a perfect refrigerator coupled with a real engine. The perfect refrigerator transfers heat Q_C from the cold reservoir to the hot one without performing any work, while the real engine performs as usual. The combined device appears to be a perfect engine.

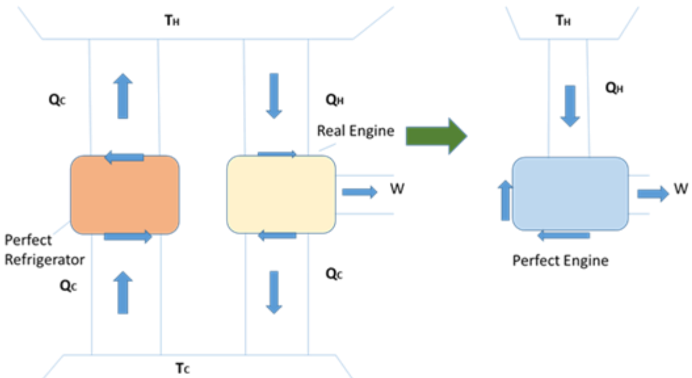

FIGURE 8. A pictorial view of the existence of a perfect engine based on the hypothesis that we do have a perfect refrigerator. A device consisting of a perfect refrigerator coupled with a real engine; the perfect refrigerator cools the sink by transferring its heat Q_C directly to the hot reservoir while the real engine performs as usual; the whole device appears as a perfect engine.

- ***Equivalence of the 3rd statement with the 1st***

This says that if an engine is more efficient than the Carnot one, then it is equivalent to having a perfect engine (Figure 5). This equivalence has been demonstrated by HRW through figure 9; here an engine X, more efficient than its counterpart Carnot engine, is coupled with a Carnot engine, supposed to be operating backward as a Carnot refrigerator in the present case. The refrigerator is adjusted so that the work required per cycle is provided by the engine X. As the efficiency of the engine X is higher than the corresponding Carnot one, we have

$$\frac{W}{Q'_H} > \frac{W}{Q_H}. \tag{4}$$

Here, the right side of the inequality is the efficiency of the Carnot refrigerator when it operates as an engine. This inequality gives

$$Q_H > Q'_H. \tag{5}$$

For every cycle, the work produced by the engine X is equal to the work required for the operation of the Carnot refrigerator. We have from (2)

$$Q_H - Q_C = Q'_H - Q'_C. \tag{6}$$

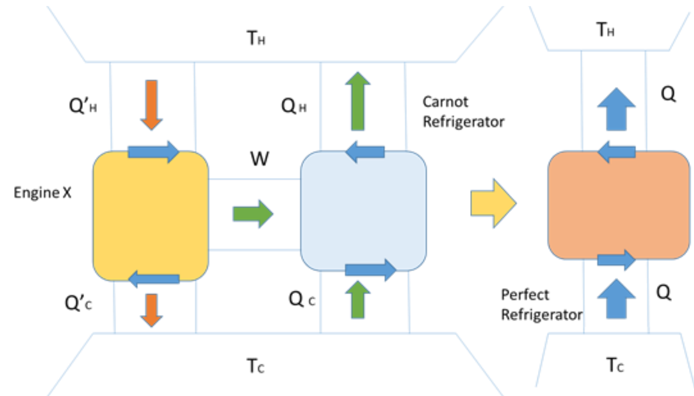

FIGURE 9. The engine X, which is more efficient than a Carnot engine, is coupled with a Carnot refrigerator; the whole system is equivalent to a perfect refrigerator.

This can be rewritten as

$$Q_H - Q'_H = Q_C - Q'_C \equiv Q. \tag{7}$$

Because of the inequality (5), the above Q must be positive. Thus, this coupled device is equivalent to a perfect refrigerator, as shown in Figure 9. Since we have already proved that a perfect refrigerator is equivalent to having a perfect engine, the equivalence of the third with the first one is justified.

- ***Equivalence of the fourth with the first one***

Next, we turn up to the equivalence of the 4th statement with the 1st one, for which we take the help of figure 10. Here, a refrigerator X supposed to be more efficient than a Carnot refrigerator, is coupled with a Carnot engine. The Carnot engine is adjusted to provide the necessary work W required for the operation of the refrigerator X. As the coefficient of performance of the refrigerator X is greater than that of the coupled engine if it works backward as a refrigerator, we have

$$\frac{Q'_C}{W} > \frac{Q_C}{W}. \tag{8}$$

Thus, we get the inequality,

$$Q'_C > Q_C.$$

Since the expressions W for the refrigerator X and the Carnot engine are both equal, we have

$$W = Q'_H - Q'_C = Q_H - Q_C.$$

This can be rewritten as

$$Q'_H - Q_H = Q'_C - Q_C \equiv Q.$$

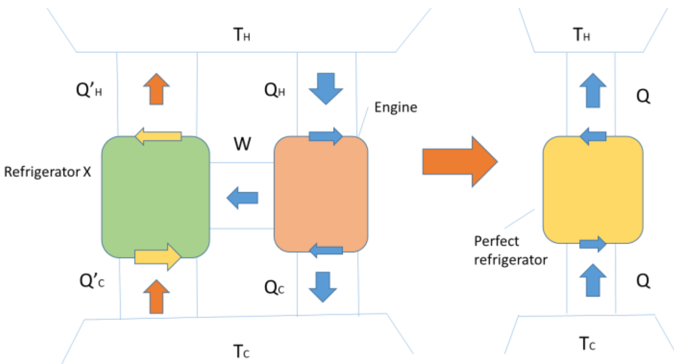

FIGURE 10. A refrigerator X that is more efficient than a Carnot refrigerator is coupled with a Carnot engine, which provides the necessary work to device X for its operation; the whole device is equivalent to a perfect refrigerator.

This algebra leads to the right-hand side of the above diagram, which is figure 10. This figure represents a perfect refrigerator. It is concluded that a refrigerator with a higher coefficient of performance than a Carnot refrigerator is equivalent to a perfect refrigerator and thereby to a perfect engine.

The equivalence between the 3rd and 4th statements has been left as an exercise for the students.

VI. SCOPE OF THE ARTICLE

It is our observation that if we study the same topic through a couple of resources, our understanding of the subject does improve. Of course, it goes without saying that the textbook Fundamentals of Physics[1] written by Halliday, Resnick, and Walker is among the best resource materials for the subject of physics, yet the present article will definitely be fruitful in developing confidence amongst students in their understanding of the second law of thermodynamics, so much so that they will attempt to prove equivalence between the 3rd and 4th statements and vice versa with a positive outcome.

REFERENCES

[1] Halliday, D., Resnick, R., and Walker, J., *Fundamentals of Physics* (John Wiley, New York, 1993), pp. 607-617.