Newton's law of cooling applied to microwave heating

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Abstract

We analyzed a way to save electricity when heating an object in a microwave oven. Taking water as an example, when heating it before boiling, is it more efficient to use the maximum power available in the microwave or not? The answer is yes and is given by the solution of the relevant differential equation, which includes Newton's law of cooling. Simple experiments qualitatively confirm this theoretical prediction. The results are relevant from an economic and environmental point of view and, therefore, encourage students to try to understand the physical and mathematical principles involved. It is explained that the extension of these results to a gas oven is not immediate, requiring caution.

Keywords: Energy use, cooling, cooking food, microwave radiation, electrical power, heat transfer, appliances. PACS numbers:44.25.+f, 44.27.+g, 78.70.Gq, 88.05.Rt

Resumen

Analizamos una forma de ahorrar electricidad al calentar un objeto en un horno microondas. Tomando el agua como ejemplo, al calentarla antes de hervir, ¿es más eficiente utilizar la potencia máxima disponible en el microondas o no? La respuesta es sí y viene dada por la solución de la ecuación diferencial relevante, que incluye la ley de enfriamiento de Newton. Experimentos simples confirman cualitativamente esta predicción teórica. Los resultados son relevantes desde un punto de vista económico y ambiental y, por lo tanto, alientan a los estudiantes a tratar de entender los principios físicos y matemáticos involucrados. Se explica que la extensión de estos resultados a un horno de gas no es inmediata, requiriendo precaución.

Palabras clave: Uso de energía, refrigeración, cocción de alimentos, radiación de microondas, energía eléctrica, transferencia de calor, electrodomésticos. PACS numbers:44.25.+f, 44.27.+g, 78.70.Gq, 88.05.Rt

I. INTRODUCTION

Students' interest in physics is often aroused when they try to solve a problem that directly affects their daily lives. One such situation, which also has a major impact on environmental issues, is that of energy conservation in machines and appliances used in everyday life. For example, LED (light-emitting diodes) lighting compared to fluorescent counterparts, better insulation for heating and cooling systems, high-efficiency washing machines and so on. Our work analyzes a particular case of this general theme.

We will study one aspect about cooking food in the kitchen. One might ask what the best way is to do it while trying to save gas, either in the oven or on the stove. This question, which involves the transformation of chemical energy in the gas into heat to be transferred to the food, although simple to formulate and important in practice, has a not so simple answer, involving various physical and chemical parameters [1-4]. For shure, there are a number of well-known rules that can be taken about saving gas: use a lid on pans, to avoid evaporation of water and consequent loss of heat to the environment; also lower the "fire" when the water reaches boiling, because when it reaches its boiling point, even receiving more heat, the water remains at this temperature until its complete evaporation. But a slightly more complex question can be asked: to save gas, should I use "high fire" or "low fire" on the stove, if I want to heat a certain amount of water, before it reaches boiling? Although it seems clear that the first option is the fastest in terms of time, would it also be the most economical? This

issue has been addressed in several articles [1-4], but in the case of gas stoves, there is no single answer.

For instance, the efficiency of a specific conventional natural gas stove [3] has a bell shape, with a maximum for a gas flow rate of *217 l/h*. On the other hand, for a different type of equipment (stove with a "perforated head"), although the overall efficiency is better, it decreases monotonically with increasing gas flow rate, starting from the minimum tested (*120 l/h*).

Another study [4] on heat transfer of a flame impinging on a flat surface, similar to a pan on a gas stove, shows that transfer efficiency depends on the distance and size of the flame, the Reynolds number involved (low, since the flow is laminar), and how close the gas/air ratio is too stoichiometric one (called the equivalence ratio).

In this paper, we will address a similar issue, but one that is more interesting from a teaching perspective, where the number of variables is reduced to arrive at a more definitive answer.

II. THEORETICAL FRAMEWORK

In the case of a gas stove or oven, one factor that complicates this analysis is that a large part of the heat generated by the gas oxidation reaction is lost directly to the environment [1-4] and not used to heat the system itself (for instance, an amount of water one wants to be heated). In this sense, one can distinguish the heat lost to the environment as direct (without passing through the water) or indirect (absorbed by the water initially but then transferred to the environment before the liquid reaches the desired final temperature).

To simplify the analysis and still give an approximate answer to the original question, let's replace the gas oven (or stove) with a microwave oven. In this case, most of the electrical energy does not directly heat the environment, because microwaves do not essentially heat the surrounding air, the container and other parts of the environment. The internal metallic walls also heat up a little, especially when the oven is empty [5]. Therefore, in essence, heat losses to the environment are always indirect: the system (water) gives off heat as its temperature rises and becomes higher than the temperature of the environment. Eliminating direct energy losses to the environment simplifies the analysis of the problem. The equivalent question to "which is more economical: high fire or low fire?" would be "which is more economical: high power or low power (in the microwave oven)?". Again, it's clear that the first option is the fastest time-wise, but is it also the most economical option?

To try to answer this question, we must first understand how a hot body loses heat to a colder environment. A basic description of this is Newton's law of cooling [6,9], which can be written as:

$$\frac{dq}{dt} = -KA(T - T_{env}) \tag{1}$$

In the above equation q is the heat given off by the body to the environment, t is the time, T is the temperature of the body, T_{env} is the ambient (constant) temperature, A is the surface area of the body and K is a positive constant. It shows that, under certain conditions, the loss of heat from a heated body to the environment is proportional to the difference in temperature between the body and the environment.

This law was empirically explored by Newton, although at that time the concept of heat was not yet developed and an expression closer to Newton's original results would be:

$$\frac{d(T - T_{env})}{dt} = -k(T - T_{env}) \tag{2}$$

where k is a new positive constant.

To simplify the notation, let's rename the temperature difference $(T\text{-}T_{env})$ as y:

$$\frac{dy}{dt} = -ky \tag{3}$$

This law is approximate but all the truer the more certain assumptions are verified, such as those listed below:

1) Heat is lost by convection and/or conduction, not by irradiation or evaporation.
2) Convection is forced and not natural.
3) The temperature variation is not too high [10].
4) The body temperature is uniform. That is, although it varies in time, it does not vary in space.
5) The heat capacity of the body has a constant value with temperature.
6) There is no loss of mass from the body, such as evaporation.

In our case, the body will be simultaneously heated by the microwaves and cooled according to Newton's law. In our experiment, described below, some of the above conditions are not strictly met. For example, there is only natural air convection inside the microwave oven and some minor water evaporation.

Considering at first only the heating process and if the microwave provides constant power, we will have:

$$\frac{dQ}{dt} = mc\frac{dy}{dt} = a \tag{4}$$

Q (not to be confused with q, in equation 1) is the heat absorbed by the body during the heating process and a is a positive constant, measured in the International System (SI) in watts, representing the useful power of the microwave and considering that all of it has been converted into heat in the test body. We assume that the mass m has not changed (there has been no evaporation) and no variation in specific heat c with temperature.

If there is simultaneous heating and (Newton) cooling, we will have, by equations 3 and 4:

$$\frac{dy}{dt} = -ky + \alpha \tag{5}$$

with $\alpha = a/mc$, being a new positive constant, measured in SI in kelvin/second.

The general solution of this linear first-order separable ordinary differential equation is [11,12]:

$$y = -\frac{e^{-kt-c_1}}{k} + \frac{\alpha}{k} \tag{6}$$

Since we have an initial condition such that $y = 0$ when $t = 0$, we can determine the integral constant, which will be $c_1 = -\ln(\alpha)$. So, our solution is:

$$y = -\frac{e^{-kt+\ln(\alpha)}}{k} + \frac{\alpha}{k} = \frac{\alpha}{k}(1 - e^{-kt}) \tag{7}$$

If we isolate the time, we get:

$$t = \frac{\ln\left(\frac{\alpha}{\alpha - ky}\right)}{k} \tag{8}$$

We can see that, as the logarithm of negative numbers is not real defined, we must have $\alpha > ky$. Furthermore, as ky is positive, the above equation is restricted to non-negative times, $t \geq 0$. This must occur throughout the process, remembering that the ky term increases progressively with heating. Therefore, to keep the solution mathematically acceptable, it is necessary that $\alpha > ky_f$, where y_f is the final temperature difference of the body. Physically, this means that the microwave oven must be powerful enough to raise the temperature of that mass of the sample (water) to the desired final temperature difference. Otherwise, this will never happen, no matter how long the microwave is left on.

Let Q_f be the amount of energy delivery from the microwave to the body to reach a certain final temperature difference y_f. Since the rate at which energy is supplied by the microwave (power) is constant and equal to a, we will have:

$$\frac{dQ}{dt} = a \rightarrow dQ = adt \rightarrow \int_0^{Q_f} dQ = a\int_0^{t_f} dt \tag{9}$$

So, using the equation *(8)* for time and remembering that $\alpha = a/mc$, we can write:

$$Q_f = at_f = -\frac{a}{k}\ln\left(1 - \frac{mcky_f}{a}\right) \tag{10}$$

The standard way to find the minimum of Q_f (to be energy efficient) would be to find the roots of the derivative of the above equation, regarding the microwave power a. Better than that, let's find the minimum of Q_f graphically. To analyze the general behavior of the function, we can simplify some terms, without altering the result of the analysis. We'll set the parameters k and mc numerically equal to *unity* and the final temperature difference, y_f, will be arbitrarily taken as having the value *10*. The equation becomes:

$$Q_f = -a\ln\left(1 - \frac{10}{a}\right) \tag{11}$$

The graph of this function (see figure *1*) shows a behavior with two asymptotes. One of the asymptotic limits of this function occurs when $a \rightarrow \infty$, which results in an indeterminacy of the "*zero times infinity*" type in the Q_f function. By rearranging the terms, one can apply L'Hopital's rule [13] to lift the indeterminacy and obtain $Q_f \rightarrow 10$ when $a \rightarrow \infty$. This limit corresponds, on the graph, to the horizontal asymptote, parallel to the x-axis. The other limit occurs when $a = 10$ and $Q_f \rightarrow +\infty$. Graphically, it corresponds to the vertical asymptote, parallel to the y-axis.

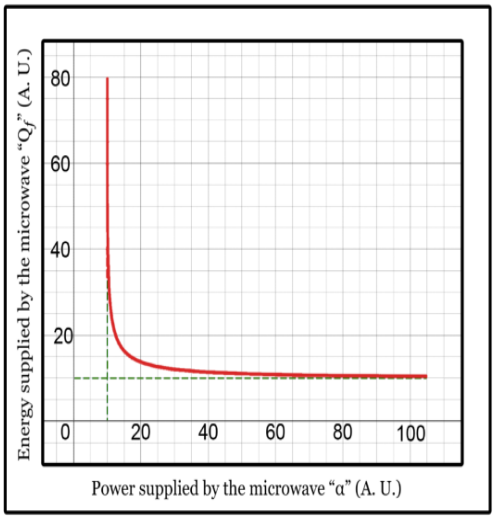

FIGURE 1. Graph of theoretical equation number *11*, from the text. In arbitrary units, it represents the amount of energy delivery from a microwave oven to heat a body to a given final temperature as a function of the power of the appliance. As can be seen from the horizontal asymptote, this energy is minimal when the power tends to infinity.

Physically, the vertical asymptotic behavior with $a = 10$ corresponds to the minimum power that the microwave must supply to the water for it to reach a given final temperature difference, y_f . The time required for this also tends to *infinity*, resulting in the total energy, Q_f , also tending to *infinity*. Below this value of $a = 10$, the mass of water will never reach the desired final temperature difference with this specific microwave, it doesn't matter how long it stays on. In contrast, the other asymptote, vertical with $Q_f = 10$, shows that this is the minimum value of total energy, Q_f, to reach y_f. However, a microwave power α tending to *infinity* is required. Therefore, in practice, the graph shows that to reduce the energy costs of heating a body with a microwave, one should operate at the highest power technically available in the device. As we will see below, this theoretical result is consistent in quality with the experimental data shown in this work.

III. EXPERIMENTAL RESULTS AND DISCUSSION

To heat a mass of water, we use a *1.2kW* nominal manufacturer's rated power *2.45GHz*-microwave oven model Consul CMP 25 ABHNA. The water was initially at room temperature [14], in our case, *(30.6 ± 1.7)ºC*. The four final temperatures were: *(59.9 ± 0.4)ºC, (73.1 ± 0.6)ºC, (84.4 ± 0.8)ºC* and *(91.0 ± 1.1)ºC*.

TABLE 1. Heating process of *250ml* of water from room temperature *(30.6 ± 1.7)ºC* to *4* different final temperatures (represented in *4* different colors). The electrical energy consumed is the product of time interval for heating by the microwave power, in arbitrary units.

Final temperature (ºC)	*Time interval (min.)*	*Microwave power a (A.U.)*	*Energy consumed (A.U.)*
92.6	2:30s	100%	15,000
91.6	2:15s	100%	13,500
92.2	2:15s	100%	13,500
91.7	2:15s	100%	13,500
90.3	9:30s	30%	17,100
91.2	10:00s	30%	18,000
90.2	10:00s	30%	18,000
89.2	10:15s	30%	18,450
89.2	10:30s	30%	18,900
91.4	10:45s	30%	19,350
83.6	1:44s	100%	10,400
84.7	1:46s	100%	10,600
83.3	1:48s	100%	10,800
83.3	1:49s	100%	10,900
84.6	1:52s	100%	11,200
84.9	7:50s	30%	14,100
86.1	7:50s	30%	14,100
85.1	7:50s	30%	14,100
84.3	7:50s	30%	14,100
84.4	7:50s	30%	14,100
84.3	7:50s	30%	14,100
73.5	1:25s	100%	8,500
72.3	1:25s	100%	8,500
73.6	1:25s	100%	8,500
72.6	5:45s	30%	10,350
72.8	5:45s	30%	10,350
73.9	5:45s	30%	10,350
59.8	0:57s	100%	5,700
59.9	0:57s	100%	5,700
60.2	0:57s	100%	5,700
59.8	0:57s	100%	5,700
59.1	3:30s	30%	6,300
59.4	3:30s	30%	6,300
60.5	3:30s	30%	6,300
60.3	3:30s	30%	6,300

When heating between these temperatures, two different powers were selected: *100%* ("high heat") and *30%* ("low heat") of the maximum provided by our microwave. Each specific heating cycle was repeated at least *3* times, to have an average and a standard deviation. A volume of *250ml* of water was put in an *8cm* in diameter and *11.5cm* high glass beaker that was placed *7.5cm* from the center of a *27cm*-diameter turntable. In this way, the sample was always moving and avoided remaining fix at the nodes or antinodes of any standing microwave wave inside the oven [5]. The measurements are listed in table 1.

A measure of the efficiency with which a microwave oven heats a given mass of water would be the ratio between the heat received by the water and the electrical energy used by the oven. This ratio can be called efficiency. Note that this word should not be confused with the efficiency of a thermal machine (which is not our case, as there is no mechanical work done), which has a different definition [15]. However, to measure that efficiency, one must know the electric energy converts to heating water. If we take the microwave manufacturer's rated power (*=1.2kW*), this is not the power supplied to heat the water, as part of this electrical energy is spent in other ways.

As an example, taking a typical experiment from our work, *250ml* of water is heated for *90s* and the temperature variation observed corresponds to *13.5kcal*. As the microwave was at full power (*100%*), the nominal electrical energy would be *1.2kWx90s = 26kcal*. This back-of-the-envelope calculation tells us that the difference (*12.5kcal*) was consumed in ways other than heating the water. However, it is not easy to say how much of the latter was used to indirectly heat the environment (heat transferred by water during the 90 seconds it takes to heat it up) and how much was used to directly heat the oven metallic walls [5] or spending on internal parts of the appliance.

As the microwave operates at *127V* and has a rated current of up to *15A*, the (so called) apparent electrical power would be the product of these parameters [16], resulting in *1.9kW*. But as the voltage is alternating, any capacitances and inductances inside the appliance mean that it's not enough just to product the root mean square (r.m.s.) values, as above. There's the power factor [16] which, if it isn't *one*, would reduce the value to less than *1.9kW*. This would explain, at least in part, that the nominal power is just *1.2kW*. Furthermore, this power factor depends on the load put inside the microwave oven [17].

There's a further complicating factor in calculating the absolute efficiency: our microwave oven isn't an inverter type, like some new models [18]. This means that when we select *50%* power (for example), instead of reducing the power to *50%*, it operates intermittently. The power is maintained at *100%* but in a rapid on-off cycle, with *50%* of the time off. Observe that, despite this, we are approximating our calculations and assuming, as in the equation *4*, that the power of our microwave oven is always supplied continuously. This is a good approximation for the temporal average of the power.

In short, an absolute calculation of efficiency would be unreliable without more data available. So what we will do is compare the relative efficiency for heating a given mass of water depending on whether the microwave is operating at full power (*100%*) or at a specific reduced power (less than *100%*). The first case would correspond to "high heat" and the second to "low heat". In our experiment, we choose *30%* for “low heat”. Then we can get a relative comparison of efficiency for these two different situations.

In this sense, we will define the so-called “percentage of unnecessary electrical energy” (“*pued*”) as the ratio between the electrical energy consumed in the “low heat” situation

(E_{lh}) and that consumed in the "high heat" situation (E_{hh}) being greater than one. Remembering that electrical energy is electrical power (P) multiplied by the time (t) that the microwave is turned on, we have:

$$pued = \frac{E_{lh}}{E_{hh}} - 1 = \frac{P_{lh}t_{lh}}{P_{hh}t_{hh}} - 1 \qquad (12)$$

As we said earlier, we do not have the absolute values of P_{lh} and P_{hh}, but we do have the P_{lh}/P_{hh} ratio. This ratio is provided by the microwave control panel itself. Therefore, it is possible to calculate this "*pued* " variable.

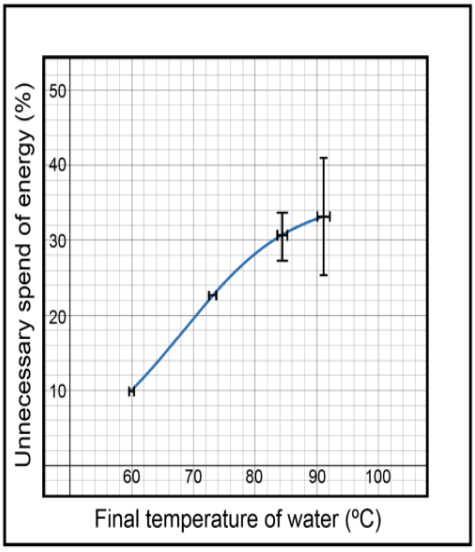

FIGURE 2. "Percentage of unnecessary electrical energy" ("pued") when operating the microwave on "low heat" (*30%* of total power) instead of "high heat" (*100%* of total power), to reach a final temperature of a mass of water. The curve connecting experimental points is only a visual guide.

As shown in the graph in fig. 2, the energy consumed at low heat is always greater than at high heat. Also, the higher the final temperature reached by the water sample, the greater the percentage difference between these two situations. For example, at the highest final temperature we tested *(91.0 ± 1.1)ºC*, the unnecessary expenditure of electricity when operating on "low heat" instead of "high heat" reaches (*33 ± 8)%* .

This experimental graph in figure 2 shows the qualitative behavior expected by the theoretical graph in Fig. *1*. Thus, the greater the power applied by the microwave to the water sample, the greater the electricity savings to raise the water temperature by a certain temperature range. The curve in figure 2 is only a guide to see the qualitative behavior of the unnecessary spend of electrical energy (operating on "low heat") as a function of final temperature of water. As we explained, it is not possible to make a comparison in absolute terms, but only in comparative terms, as we do not have the data necessary to calculate the actual power of the microwave oven.

IV. CONCLUSIONS

It can be concluded that when we want to heat a certain mass of a body without changing its phase, we will achieve greater energy savings by operating at the maximum power available in the microwave oven. In addition, there is also greater time savings. This is predicted by the relevant differential equation, analyzed in the theoretical framework, within the approximations of Newton's cooling law. Experiments in heating water below its boiling temperature confirm this theoretically expected behavior. These results have great practical application, due to the widespread use of microwave ovens in kitchens and the need for energy efficiency, both from an economic and environmental point of view. Let us remember that around *50%* of global energy used in buildings (in *2022*) was for heating water and spaces [19]. The conclusions of this work were taken to public schools in the state of Paraná, Brazil, addressing mainly the more qualitative aspects of the process. They aroused enormous interest among the students where they were presented [20]. The extension of these results to gas ovens is not obvious, for reasons listed in the text, although the appropriate response to be applied to the latter devices is also of enormous general interest.

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