New quantum atomic spectrum of Schrödinger equation with pseudo harmonic potential in both noncommutative three dimensional spaces and phases

Abdelmadjid Maireche
Physics Department, Sciences Faculty, University of M'sila-Algeria.

E-mail: abmaireche@gmail.com

(Received 16 July 2014, accepted 23 January 2015)

Abstract

In present work, we obtain the modified bound-states solutions for three dimensional pseudo harmonic potential in both non-commutative three dimensional spaces and phases. We show the noncommutative new anisotropic Hamiltonian containing three terms, the first is the usual Hamiltonian in ordinary quantum mechanics; the first new term describes the spin-orbit interaction while the second new term describes the modified Zeeman effect. It has been observed that, the energy spectra in ordinary quantum mechanics was changed, and replaced by degenerate new states, depending on new discreet quantum numbers: *m*, *l*, *j* and *s*= ± 1/2, in addition to four infinitesimal parameters: α_2, $\bar{\varepsilon}_2$, Θ_1 and $\bar{\theta}_1$. The new symmetries were extended to include automatically new physics phenomena's.

Keywords: Star product, Boopp's shift method, three dimensional pseudo harmonic potential, Noncommutative space and Noncommutative phase.

Resumen

En este trabajo, obtuvimos las soluciones modificadas de los estados bound para tres potenciales dimensionales pseudoarmónicos tanto para tres espacios y fases dimensionales no conmutativos. Demostramos el nuevo hamiltoniano anisotrópico no conmutativo que contiene tres términos. El primero es el hamiltoniano usual en una mecánica cuántica ordinaria. El primer término nuevo describe la interacción spin-orbital; mientras que el segundo término nuevo describe el efecto modificado de Zeeman. Se ha observado que el espectro energético en una mecánica cuántica ordinaria fue cambiado y reemplazado por nuevos estados degenerados, dependiendo de los nuevos números cuánticos discretos: *m*, *l*, *j* y *s*= ± 1/2. Además de cuatro parámetros infinitesimales: α_2, $\bar{\varepsilon}_2$, Θ_1 y $\bar{\theta}_1$. Las nuevas simetrías fueron extendidas para incluir automáticamente nuevos fenómenos físicos.

Palabras clave: Producto Star, Metodo de derivada de Boopp, Potenciales dimensionales pseudoarmónicos, espacio no commutativo, fase no commutativa.

PACS: 11.10.Nx, 32.30-r, 03.65-w **ISSN 1870-9095**

I. INTRODUCTION

The central potentials have been studied in various fields of material sciences, nuclear physics, quantum molecules by using relativistic Klein-Gordon equation, Dirac equation and non-relativistic Schrödinger equation in commutative and noncommutative two and three-dimensional space and phase [1-44]. The Schrödinger equation rest major sources of principal information's and it consider as a big revolution like general relativity and special relativity for describing physics phenomena at microscopic (Planks scales) and macroscopic scales (the planets movements). In another hand, the notions of noncommutativity of space and phase, will be another big modern revolution to gives the profound interpretations in different fields. Based essentially on Seiberg-Witten map and Boopp's shift method and the star product, depended on the antisymmetric infinitesimal parameters ($\theta^{\mu\nu}$ and $\bar{\theta}^{\mu\nu}$) and equals $\frac{1}{2}\varepsilon^{ijk}\theta_k$ and $\frac{1}{2}\varepsilon^{ijk}\bar{\theta}_k$, respectively, in first order of two infinitesimal parameters θ_k and $\bar{\theta}_k$, the new star product defined as [23-43]:

$$f(x)*g(x)=f(x)*g(x)-\frac{i}{2}\theta^{\mu\nu}\left(\partial_\mu^x f(x)\right)\left(\partial_\nu^p g(x)\right)-\frac{i}{2}\bar{\theta}^{\mu\nu}\left(\partial_\mu^p f(x)\right)\left(\partial_\nu^p g(x)\right). \tag{1}$$

As a direct principal's result of the above equation, the two new non nulls commutators $\left[x_i, x_j\right]$ and $\left[\hat{p}_i, \hat{p}_j\right]_*$ in the notion of star product:

$$\left[x_i, x_j\right]_* = i\theta_{ij} \text{ and } \left[\hat{p}_i, \hat{p}_j\right]_* = i\bar{\theta}ij \,. \tag{2}$$

In this work, we want to study three dimensional pseudo harmonic potential in noncommutative 3D space and phase to discover the new symmetries by apply the Boopp's shift method, the Schrödinger equation will be treated by using directly new product procedure [23-44]:

$$\left[\hat{x}_i, \hat{x}_j\right] = i\theta_{ij} \text{ and } \left[\hat{p}_i, \hat{p}_j\right] = i\bar{\theta}_{ij} \,. \tag{3}$$

The study of three dimensional pseudo harmonic potential has relevance in the dynamical properties in solid-state physics and the history of molecular structures and interactions [6].

The rest of present search is organized as follows: in next section, we briefly review the Schrödinger equation with for pseudo harmonic potential in three dimensional spaces. In section 3, by applying Boopp's shift method to derive the deformed Hamiltonians of the Schrödinger equation with three dimensional pseudo harmonic potential in noncommutative three dimensional space-phases. The forth section reserved to present the formalism of Boopp's shift method and the construction of global noncommutative Hamiltonian for three dimensional pseudo harmonic potential.

In the fifth section we apply standard perturbation theory to find the exact quantum spectrum of the bound states in (NC-3D) space and phase for studied potential in first order of two infinitesimal parameters Θ and $\bar{\theta}$. Finally, the important found results and the conclusions are discussed in the last section.

II. REVIEW OF PSEDOHARMONIC POTENTIAL IN THREE DIMENSIONAL

In this section, we review the orthonormalization functions and energy eigenvalues of three-dimensional pseudo harmonic potential $V_{3dph}(r)$ [6]:

$$V_{3dph}(r) = D_e\left(\frac{r}{r_e} - \frac{r_e}{r}\right)^2 . \tag{4}$$

Where D_e, is the dissociation energy and r_e is the equilibrium intermolecular separation. The potential (4) can be rewritten in the form of an isotropic Harmonic oscillator plus inverse quadratic potential [6]:

$$V_{3dph}(r) = ar^2 + \frac{b}{r^2} + c \,. \tag{5}$$

Where $a = D_e r_e^{-2}$, $b = D_e r_e^{+2}$ and $c = -2D_e$, the radial function $\Psi(r)$ in 3-dimensional space for three-dimensional pseudo harmonic ($\hbar = c = 1$) is [6]:

$$\frac{d^2\Psi(r)}{dr^2} + \frac{2}{r}\frac{d\Psi(r)}{dr} + \left[E - V_{3dph}(r) - \frac{l(l+1)}{r^2}\right]\Psi(r) = 0. \tag{6}$$

Where l denote to the orbital angular momentum quantum numbers. Furthermore, to remove the first derivative from the above equation, we introduce a new radial function:

$$R(r) \; (\Psi(r) = \frac{R(r)}{r}),$$

then, the eq. (6) reduced to the form:

$$\frac{d^2R(r)}{dr^2} + \left[2\mu\left(E - V_{3dph}(r)\right) + \frac{l(l+1)}{r^2}\right]R(r) = 0 \,. \tag{7}$$

The complete orthonormalization functions and energy eigenvalues in three dimensional spaces for fondamental state (E_0^δ and $\Psi^0(r)$), first excited state (E_1^δ and $\Psi^1(r)$) and p and excited state (E_p^δ and $\Psi^p(r)$) are given by, respectively [6]:

$$E_{0\delta} = c + \sqrt{\frac{2a}{\mu}}\left(\delta + 2\right),$$
$$\Psi^0(r) = a_0 r^{\delta+\frac{1}{2}} \exp\left(\sqrt{2a\mu} r^2\right). \tag{8}$$

$$E_1^\delta = c + \sqrt{\frac{2a}{\mu}}\left(\delta + 4\right),$$
$$\Psi^0 1(r) = \left(a_0 + a_1 r^2\right) r^{\delta+\frac{1}{2}} \exp\left(\sqrt{2a\mu} r^2\right). \tag{9}$$

$$E_p^\delta = c + \sqrt{\frac{2a}{\mu}}\left(2p + \delta + 2\right),$$
$$\Psi^p(r) = \left(a_0 + a_1 r^2 + ...a_p r^{2p}\right) r^{\delta+\frac{1}{2}} \exp\left(\sqrt{2a\mu} r^2\right). \tag{10}$$

The constants δ, a_0 and $a_2$are given by [6]:

$$\delta = -1 + \sqrt{2\mu b + \eta^2},$$
$$a_0 = \sqrt{\frac{2}{(\delta+1)!}}\left(\sqrt{2\mu a}\right)^{\frac{\delta}{2}+1}, \tag{11}$$
$$a_1 = \frac{1}{\delta+2}\sqrt{\frac{2}{(\delta+1)!}}\left(\sqrt{2\mu a}\right)^{\frac{\delta}{2}+2}.$$

And

$$\eta = l + \frac{1}{2}.$$

III. NONCOMMUTATIVE THREE DIMENSIONAL SPACE-PHACE HAMILTONIAN FOR PSEUDO HARMONIC POTENTIAL

III.A Formalism of Boopp's shift method

The quantum noncommutative Schrödinger equation, can be determined by apply the following steps for pseudo harmonic potential [23, 24, 25, 26,27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44]:

$$\begin{gathered}\text{Ordinary Hamiltonian: } \hat{H}\left(p_i, x_i\right) \rightarrow \\ \text{NC-Hamiltonian: } \hat{H}\left(\hat{p}_i, \hat{x}_i\right) \\ \text{Ordinary-complex wave function:} \\ \Psi\left(\vec{r}\right) \rightarrow \text{NC-complex wave function: } \widehat{\Psi}\left(\vec{\vec{r}}\right) \\ \text{Ordinary-energy: E} \rightarrow \\ NC-Energy: E_{nc-ph} \\ \text{Ordinary-product} \rightarrow \\ \text{New star producr-acting on phace and space:} * \end{gathered} \tag{12}$$

Which allow us to writing the three dimensional space-phase quantum noncommutative Schrödinger equations as follows:

$$\hat{H}\left(\hat{p}_i, \hat{x}_i\right) * \widehat{\Psi}\left(\vec{\vec{r}}\right) = E_{nc-ph}\widehat{\Psi}\left(\vec{\vec{r}}\right). \tag{13}$$

The Boopp's shift method permutes to reduce the above equation to the form:

$$H\left(\hat{p}_i, \hat{x}_i\right)\Psi^p\left(r\right) = E_{nc-ph}\Psi^p\left(r\right). \tag{14}$$

Here the two $\hat{x}_i$ and $\hat{p}_i$ operators in (NC-3D) phase and space are given by [23-43]:

$$\hat{x}_i = x_i - \frac{\theta_{ij}}{2}p_j \text{ and } \hat{p}_i = p_i - \frac{\bar{\theta}_{ij}}{2}x_j. \tag{15}$$

On another hand, the two operators $\hat{r}^2$ and $\hat{p}^2$ in (NC-3D) spaces and phases are given by [37, 39]:

$$\begin{aligned}\hat{r}^2 &= r^2 - \vec{\mathbf{L}}\vec{\Theta}, \\ \hat{p}^2 &= p^2 + \vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}.\end{aligned} \tag{16}$$

Where $\mathbf{L}\Theta$ and $\vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}$ are given by, respectively:

$$\begin{aligned}\mathbf{L}\Theta &\equiv L_x\Theta_{12} + L_y\Theta_{23} + L_z\Theta_{13} \\ \vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}} &\equiv L_x\bar{\theta}_{12} + L_y\bar{\theta}_{23} + L_z\bar{\theta}_{13}.\end{aligned} \tag{17}$$

Where $\Theta \equiv \frac{\boldsymbol{\theta}}{\mathbf{2}}$, based, on the Eq. (17), in the first order of two infinitesimal parameters Θ and $\bar{\theta}$, the two important terms which will be used to determine the noncommutative pseudo harmonic potential can be written explicitly, in (NC-3D) spaces and phases as:

$$\begin{aligned}a\hat{r}^2 &= ar^2 - a\vec{\mathbf{L}}\vec{\Theta}, \\ \frac{b}{\hat{r}^2} &= \frac{b}{r^2} + \frac{b}{4r^4}\mathbf{L}\Theta.\end{aligned} \tag{18}$$

The modified pseudo harmonic potential operators $V_{3\text{dph}}(\hat{r})$ in both (NC-3D) phase and space will be written as:

$$V_{3\text{dph}}\left(\hat{r}\right) = ar^2 + \frac{b}{r^2} + c + \left(-a + \frac{b}{4r^4}\right)\vec{\mathbf{L}}\vec{\Theta} + \frac{\vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}}{2\mu}. \tag{19}$$

The three first terms in above equation are given the ordinary pseudo harmonic potential in 3D spaces, while the rest terms are proportional's with two infinitesimals parameters (Θ and $\bar{\theta}$) and then gives the terms of perturbations for pseudo harmonic potential $H_{\text{ph-pert}}(r, \Theta, \bar{\theta})$ in (NC-3D) real space and phase as:

$$H_{\text{ph-pert}}\left(r, \Theta, \bar{\theta}\right) = \left(-a + \frac{b}{4r^4}\right)\vec{\mathbf{L}}\vec{\Theta} + \frac{\vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}}{2\mu}. \tag{20}$$

III.B The second part of noncommutative 3D: RSP

It is possible to replace $\vec{\mathbf{L}}\vec{\Theta}$ and $\vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}$ by $2\Theta\overleftrightarrow{S}\overleftrightarrow{L}$ and $2\bar{\theta}\overleftrightarrow{S}\overleftrightarrow{L}$, respectively, to obtain the new forms of $H_{\text{ph-pert}}(r, \Theta, \bar{\theta})$ for pseudo harmonic potential:

$$H_{\text{ph-pert}}\left(r, \Theta, \bar{\theta}\right) = \left(-a + \frac{b}{4r^4} + \frac{\bar{\theta}}{2\mu}\right)\vec{\mathbf{L}}\vec{S}. \tag{21}$$

Here $\overleftrightarrow{S} = \frac{\vec{1}}{2}$ denote the spin of electron, in quantum mechanics, it is possible also to replace $\left(\overleftrightarrow{S}\overleftrightarrow{L}\right)$ by $\frac{1}{2}\left(\overleftrightarrow{J}^2 - \overleftrightarrow{L}^2 - \overleftrightarrow{S}^2\right)$, which allow us to writing the perturbative terms for pseudo harmonic potential:

$$H_{\text{ph-pert}}\left(r,\Theta,\bar{\theta}\right)=\left(-a+\frac{b}{4r^4}+\frac{\bar{\theta}}{2\mu}\right)G^2 \qquad (22)$$

Where G^2 denote to the $\frac{1}{2}\left(\overleftrightarrow{J}^2-\overleftrightarrow{L}^2-\overleftrightarrow{S}^2\right)$. As it is known, this operator traduces the coupling between spin and orbital momentum. The ($\overleftrightarrow{J}^2$, $\overleftrightarrow{L}^2$, $\overleftrightarrow{S}^2$ and s_z) formed complete basis on quantum mechanics, then the operator $\left(\overleftrightarrow{J}^2-\overleftrightarrow{L}^2-\overleftrightarrow{S}^2\right)$ will be gives 2-eigenvalues $k_{\mp}\equiv\frac{1}{2}\left\{\left(l\pm\frac{1}{2}\right)(l+\frac{1}{2}\pm 1)+l(l+1)-\frac{3}{4}\right\}$, corresponding $j=l\pm\frac{1}{2}$ (spin up-spin down), respectively. Then, one can form a diagonal matrix $H_{nc-ph}\left(r,\vec{p},\Theta,\bar{\theta}\right)$ of order (3×3) with non null elements $\left(\left(H_{nc-ph}\left(r,\vec{p},\Theta,\bar{\theta}\right)\right)_{11}\right.$, $\left(H_{nc-ph}\left(r,\vec{p},\Theta,\bar{\theta}\right)\right)_{22}$ and $\left(H_{nc-ph}\left(r,\vec{p},\Theta,\bar{\theta}\right)\right)_{33}$, for pseudo harmonic potential in both (NC-3D) phase and space:

$$\left(H_{nc-ph}\right)_{11}=-\frac{\Delta}{2\mu}+ar^2+\frac{b}{r^2}+c+k_{+}\left(-a+\frac{b}{4r^4}+\frac{\bar{\theta}}{2\mu}\right)$$
$$\text{if } j=l+\tfrac{1}{2} \Rightarrow \text{spin up}$$
$$\left(H_{nc-ph}\right)_{22}=-\frac{\Delta}{2\mu}+ar^2+\frac{b}{r^2}+c+k_{-}\left(-a+\frac{b}{4r^4}+\frac{\bar{\theta}}{2\mu}\right)$$
$$\text{if } j=l-\tfrac{1}{2} \Rightarrow \text{spin down}$$
$$\left(H_{nc-ph}\right)_{33}=-\frac{\Delta}{2\mu}+ar^2+\frac{b}{r^2}+c\rightarrow \text{Non}-\text{polarised}-\text{electron}$$
(23)

III.C The third part of noncommutative 3D: RSP

In this sub section, we draw another interpretation for the production of modified three dimensional pseudo harmonic potential, we consider the two at infinitesimals parameters (Θ and $\bar{\theta}$) are the sum of two infinitesimals parameters to each one as [33-37]:

$$\Theta=\Theta_1+\Theta_2 \quad \text{and} \quad \bar{\theta}=\bar{\theta}_1+\bar{\theta}_2 \qquad (24)$$

The two parameters Θ_2 and $\bar{\theta}_2$ are arbitrary infinitesimals constants, then we can introduce an external magnetic field B as [33-41]:

$$\Theta_2=\alpha_2 B,\quad \bar{\theta}_2=\bar{\varepsilon}_2 B \quad \text{and} \quad \overleftrightarrow{B}=B\overleftrightarrow{k}. \qquad (25)$$

Where α_2 and $\bar{\varepsilon}_2$ are two new proportional constants, which allow us to obtain the following results:

$$\left(-a+\frac{b}{4r^4}+\frac{\bar{\theta}}{2\mu}\right)\vec{\mathbf{L}}\Theta_2+\vec{\mathbf{L}}\bar{\boldsymbol{\theta}}\rightarrow\left(\alpha_2\left(-a+\frac{b}{4r^4}+\frac{\bar{\theta}}{2\mu}\right)+\bar{\varepsilon}_2\right)BL_Z. \quad (26)$$

Thus, the second part of noncommutative Hamiltonians operator for three dimensional pseudo harmonic potential $H_{ph-mag}\left(r,\vec{p},\alpha_2,\bar{\varepsilon}_2\right)$ can be determined as follows:

$$H_{ph-mag}\left(r,\vec{p},\alpha_2,\bar{\varepsilon}_2\right)=\left(\alpha_2\left(-a+\frac{b}{4r^4}+\frac{\bar{\theta}}{2\mu}\right)+\bar{\varepsilon}_2\right)BL_Z \qquad (27)$$

III.D The global noncommutative anisotropic Hamiltonian operator

The previous obtained results permuted to deduce the global diagonal noncommutative Hamiltonian matrixes $H_{nc-ph}\left(r,\vec{p},\Theta,\bar{\theta}\right)$ of order (3×3), with non null elements:

$$\left(H_{nc-ph}\right)_{11},\ \left(H_{nc-ph}\right)_{22} \text{ and } \left(H_{nc-ph}\right)_{33}$$

for three dimensional pseudo harmonic potential in both (NC-3D) phase and space:

$$\left(H_{nc-ph}\right)_{11}=-\frac{\Delta}{2\mu}+ar^2+\frac{b}{r^2}+c+k_{+}\left(-a+\frac{b}{4r^4}+\frac{\bar{\theta}}{2\mu}\right)+$$
$$\left(\alpha_2\left(-a+\frac{b}{4r^4}+\frac{\bar{\theta}}{2\mu}\right)+\bar{\varepsilon}_2\right)\text{ if } j=l+\tfrac{1}{2} \Rightarrow \text{spin up}$$
$$\left(H_{nc-ph}\right)_{22}=-\frac{\Delta}{2\mu}+ar^2+\frac{b}{r^2}+c+k_{-}\left(-a+\frac{b}{4r^4}+\frac{\bar{\theta}}{2\mu}\right)+$$
$$\left(\alpha_2\left(-a+\frac{b}{4r^4}+\frac{\bar{\theta}}{2\mu}\right)+\bar{\varepsilon}_2\right)BL_Z\text{if } j=l-\tfrac{1}{2} \Rightarrow \text{spin down}$$
$$\left(H_{nc-ph}\right)_{33}=-\frac{\Delta}{2\mu}+ar^2+\frac{b}{r^2}+c\rightarrow \text{Non}-\text{polarised}-\text{electron}.$$
(28)

After a straightforward calculation, one can show that, the radial function $R(r)$ satisfied the following the equation, in (NC-3D: RSP) for three dimensional pseudo harmonic potential:

$$\frac{d^2R(r)}{dr^2}+\left[2\mu\begin{pmatrix}E-V_{3dph}(r)-\left(-a+\frac{b}{4r^4}+\frac{\bar{\theta}}{2\mu}\right)G^2-\\ \left(\alpha_2\left(-a+\frac{b}{4r^4}+\frac{\bar{\theta}}{2\mu}\right)+\bar{\varepsilon}_2\right)BL_Z\end{pmatrix}+\frac{l(l+1)}{r^2}\right]R(r)=0.$$
(29)

V. NONCOMMUTATIVE SPECTRUM

Know, we want to obtain the energies: ($E_{\text{nc0-up}}$, $E_{\text{0nc-d}}$) and

($E_{\text{nc1-up}}$, $E_{\text{1nc-d}}$) of a particle fermionic with spin up-spin down at first order of two infinitesimals parameters (Θ and $\bar{\theta}$) corresponding ($\left(H_{nc-ph}\left(r,\vec{p},\Theta,\bar{\theta}\right)\right)_{11}$ and $\left(H_{nc-ph}\left(r,\vec{p},\Theta,\bar{\theta}\right)\right)_{22}$ for fundamental state and first excited state corresponding three dimensional pseudo harmonic potential in both (NC-3D) phase and space:

$$\begin{cases} E_{\text{nc0-up}} = c + \sqrt{\dfrac{2a}{\mu}}\left(\delta+2\right) + E_{ph0-up}\left(\Theta_1,\bar{\theta}_1\right) \\ +E_{ph0-mag}\left(\Theta_2,\bar{\theta}_2\right) \\ E_{\text{nc0-d}} = c + \sqrt{\dfrac{2a}{\mu}}\left(\delta+2\right) + E_{ph0-d}\left(\Theta_1,\bar{\theta}_1\right) \\ +E_{ph0-mag}\left(\Theta_2,\bar{\theta}_2\right). \end{cases} \tag{30}$$

And

$$\begin{cases} E_{\text{nc1-up}} = c + \sqrt{\dfrac{2a}{\mu}}\left(\delta+4\right) + E_{ph1-up}\left(\Theta_1,\bar{\theta}_1\right) \\ +E_{ph1-mag}\left(\Theta_2,\bar{\theta}_2\right) \\ E_{\text{nc1-d}} = c + \sqrt{\dfrac{2a}{\mu}}\left(\delta+4\right) + E_{ph1-d}\left(\Theta_1,\bar{\theta}_1\right) \\ +E_{ph1-mag}\left(\Theta_2,\bar{\theta}_2\right). \end{cases} \tag{31}$$

Where:

($E_{ph0-up}\left(\Theta_1,\bar{\theta}_1\right)$, $E_{ph0-d}\left(\Theta_1,\bar{\theta}_1\right)$,

$E_{ph0-mag}\left(\Theta_2,\bar{\theta}_2\right)$) and ($E_{ph1-up}\left(\Theta_1,\bar{\theta}_1\right)$,

$E_{ph1-d}\left(\Theta_1,\bar{\theta}_1\right)$, $E_{ph1-mag}\left(\Theta_2,\bar{\theta}_2\right)$),

are the exact modifications of spin-orbital (up-down) and magnetic for the three dimensional pseudo harmonic potential corresponding fundamental state and first excited state, respectively.

V.A Noncommutative spin-orbital spectrum

To obtain the exact noncommutative spin-orbital modifications of energy $E_{ph0-up}\left(\Theta_1,\bar{\theta}_1\right)$, $E_{ph0-d}\left(\Theta_1,\bar{\theta}_1\right)$ for three dimensional pseudo harmonic potential corresponding fundamental states, we apply the standard perturbations theory:

$$\begin{aligned} \frac{E_{ph0-up}\left(\Theta_1,\bar{\theta}_1\right)}{\left|a_0\right|^2 k_+} &= \int_0^{+\infty} r^{2\delta+1}\exp\left(2\sqrt{2a\mu}r^2\right)\left(-a\Theta+\frac{b}{4r^4}\Theta+\frac{\bar{\theta}}{2\mu}\right)r^2dr \\ \frac{E_{ph0-d}\left(\Theta_1,\bar{\theta}_1\right)}{\left|a_0\right|^2 k_-} &= \int_0^{+\infty} r^{2\delta+1}\exp\left(2\sqrt{2a\mu}r^2\right)\left(-a\Theta+\frac{b}{4r^4}\Theta+\frac{\bar{\theta}}{2\mu}\right)r^2dr. \end{aligned} \tag{32}$$

It's convenient to rewrite the two above equations as follows:

$$\begin{aligned} E_{ph0-up}\left(\Theta_1,\bar{\theta}_1\right) &= \left|a_0\right|^2 k_+\left\{\left(-\Theta a+\frac{\bar{\theta}}{2\mu}\right)L_{so1}-\Theta\frac{b}{4}L_{so2}\right\} \\ E_{ph0-d}\left(\Theta_1,\bar{\theta}_1\right) &= \left|a_0\right|^2 k_-\left\{\left(-\Theta a+\frac{\bar{\theta}}{2\mu}\right)L_{so1}-\Theta\frac{b}{4}L_{so2})\right\}. \end{aligned} \tag{33}$$

Where:

$$\begin{aligned} L_{so1} &= \int_0^{+\infty} r^{m_1}\exp\left(-\beta r^2\right)dr \\ L_{so2} &= \int_0^{+\infty} r^{m_2}\exp\left(-\beta r^2\right)dr. \end{aligned} \tag{34}$$

With $m_1 = 2\delta+3$, $m_2 = 2\delta-1$ and $\beta = -2\sqrt{2a\mu}$. Know we apply the special integral [38]:

$$\int_0^{+\infty} x^m \exp\left(-\beta x^n\right)dx = \frac{\Gamma\left(\dfrac{m+1}{n},\beta x^n\right)}{n\beta^{\frac{m+1}{n}}}. \tag{35}$$

Where $\Gamma\left(\dfrac{m+1}{n},\beta x^n\right)$ denote to the incomplete Gamma function. After straightforward calculations, we can obtain the results:

$$\begin{aligned} L_{so1} &= \frac{\Gamma\left(\delta+1,-2\sqrt{2a\mu}\right)}{2\left(-2\sqrt{2a\mu}\right)^{\delta+2}} \\ &\text{and} \\ L_{so2} &= \frac{\Gamma\left(\delta,-2\sqrt{2a\mu}\right)}{2\left(-2\sqrt{2a\mu}\right)^{\delta}}. \end{aligned} \tag{36}$$

For the first excited states, the exact noncommutative spin-orbital modifications of energy $E_{ph1-up}\left(\Theta_1,\bar{\theta}_1\right)$, $E_{ph1-d}\left(\Theta_1,\bar{\theta}_1\right)$ of three dimensional pseudo harmonic potential:

$$\begin{aligned} E_{ph1-up}\left(\Theta_1,\bar{\theta}_1\right) &= k_+\int_0^{+\infty}\left(a_0+a_1r^2\right)^2 r^{2\delta+1}\exp\left(2\sqrt{2a\mu}r^2\right) \\ &\left(-a\Theta+\frac{b}{4r^4}\Theta+\frac{\bar{\theta}}{2\mu}\right)r^2dr \\ E_{ph1-d}\left(\Theta_1,\bar{\theta}_1\right) &= k_-\int_0^{+\infty}\left(a_0+a_1r^2\right)^2 r^{2\delta+1}\exp\left(2\sqrt{2a\mu}r^2\right) \\ &\left(-a\Theta+\frac{b}{4r^4}\Theta+\frac{\bar{\theta}}{2\mu}\right)r^2dr \end{aligned} \tag{37}$$

A direct simplification gives:

$$E_{ph1-up}\left(\Theta_1,\bar{\theta}_1\right)=k_+\int_0^{+\infty}\frac{\left(a_0{}^2r^2+a_1{}^2r^6+2a_0a_1r^4\right)}{r^{2\delta+1}\exp\left(2\sqrt{2a\mu}r^2\right)}\left(\left(\frac{\bar{\theta}}{2\mu}-a\Theta\right)+\frac{b}{4r^4}\Theta\right)dr$$

$$E_{ph1-d}\left(\Theta_1,\bar{\theta}_1\right)=k_-\int_0^{+\infty}\frac{\left(a_0{}^2r^2+a_1{}^2r^6+2a_0a_1r^4\right)}{r^{2\delta+1}\exp\left(2\sqrt{2a\mu}r^2\right)}\left(\left(\frac{\bar{\theta}}{2\mu}-a\Theta\right)+\frac{b}{4r^4}\Theta\right)dr. \quad (38)$$

It's convenient to rewrite the two above equations as follows:

$$E_{ph1-up}\left(\Theta_1,\bar{\theta}_1\right)=k_+\left\{\begin{array}{l}\left(-\Theta a+\dfrac{\bar{\theta}}{2\mu}\right)\left(L_{so3}+L_{so5}+L_{so7}\right)\\-\Theta\dfrac{b}{4}\left(L_{so4}+L_{so6}+L_{so8}\right)\end{array}\right\}$$

$$E_{ph01-d}\left(\Theta_1,\bar{\theta}_1\right)=k_-\left\{\begin{array}{l}\left(-\Theta a+\dfrac{\bar{\theta}}{2\mu}\right)\left(L_{so3}+L_{so5}+L_{so7}\right)\\-\Theta\dfrac{b}{4}\left(L_{so4}+L_{so6}+L_{so8}\right)\end{array}\right\}. \quad (39)$$

Where:

$$L_{so3}=a_0{}^2\int_0^{+\infty}r^{m_3}\exp\left(-\beta r^2\right)dr,$$
$$L_{so4}=\frac{ba_0{}^2}{4}\int_0^{+\infty}r^{m_4}\exp\left(-\beta r^2\right)dr$$
$$L_{so5}=a_1{}^2\int_0^{+\infty}r^{m_5}\exp\left(-\beta r^2\right)dr,$$
$$L_{so6}=\frac{ba_1{}^2}{4}\int_0^{+\infty}r^{m_6}\exp\left(-\beta r^2\right)dr \quad (40)$$
$$L_{so7}=2a_0a_1\int_0^{+\infty}r^{m_7}\exp\left(-\beta r^2\right)dr$$
$$\text{and}\quad L_{so8}=\frac{ba_0a_1}{2}\int_0^{+\infty}r^{m_8}\exp\left(-\beta r^2\right)dr.$$

With $m_3=2\delta+3$, $m_4=2\delta-1$, $m_5=2\delta+7$, $m_6=2\delta+1$, $m_7=2\delta+5$, $m_8=2\delta+1$ and $\beta=-2\sqrt{2a\mu}$. Know we apply the special integral (35), we obtains:

$$L_{so3}=\frac{a_0{}^2}{2}\frac{\Gamma\left(\delta+2,\beta r^2\right)}{\left(-2\sqrt{2a\mu}\right)^{\delta+2}},L_{so4}=\frac{ba_0{}^2}{8}\frac{\Gamma\left(\delta,\beta r^2\right)}{\left(-2\sqrt{2a\mu}\right)^{\delta}}$$
$$L_{so5}=\frac{a_1{}^2}{2}\frac{\Gamma\left(\delta+2,\beta r^2\right)}{\left(-2\sqrt{2a\mu}\right)^{\delta+2}},L_{so6}=\frac{ba_1{}^2}{8}\frac{\Gamma\left(\delta+1,\beta r^2\right)}{\left(-2\sqrt{2a\mu}\right)^{\delta+1}} \quad (41)$$
$$L_{so7}=a_0a_1\frac{\Gamma2\left(\delta+3,\beta r^2\right)}{\left(-2\sqrt{2a\mu}\right)^{\delta+3}}\quad\text{and}\quad L_{so8}=\frac{ba_0a_1}{4}\frac{\Gamma\left(\delta,\beta r^2\right)}{\left(-2\sqrt{2a\mu}\right)^{\delta}}$$

Which allow us to obtaining the exact noncommutative spin-orbital modifications of energy $E_{ph0-up}\left(\Theta_1,\bar{\theta}_1\right)$, $E_{ph0-d}\left(\Theta_1,\bar{\theta}_1\right)$ of fundamental states and first excited states $E_{ph0-up}\left(\Theta_1,\bar{\theta}_1\right)$ and $E_{ph0-up}\left(\Theta_1,\bar{\theta}_1\right)$ for the studied potential.

V.B Noncommutative magnetic spectrum

Furthermore, to obtain, the exact noncommutative magnetic modifications of energy $E_{ph0-mag}\left(\Theta_2,\bar{\theta}_2\right)$ and $E_{ph1-mag}\left(\Theta_2,\bar{\theta}_2\right)$ for three dimensional pseudo harmonic potential of fundamental states and first excited states, its sufficient to replace: (k_+, k_-), Θ_1 and $\bar{\theta}_1$ in the Equations (33) and (39) by the following parameters: m, α_2 and $\bar{\varepsilon}_2$, respectively:

$$E_{ph0-mag}\left(\Theta_2,\bar{\theta}_2\right)=\left|a_0\right|^2m\left\{\left(-\alpha_2a+\frac{\bar{\varepsilon}_2}{2\mu}\right)L_{so1}-\alpha_2\frac{b}{4}L_{so2}\right\}$$

$$E_{ph1-mag}\left(\Theta_2,\bar{\theta}_2\right)=m\left\{\begin{array}{l}\left(-\alpha_2a+\dfrac{\bar{\varepsilon}_2}{2\mu}\right)\left(L_{so3}+L_{so5}+L_{so7}\right)\\-\alpha_2\dfrac{b}{4}\left(L_{so4}+L_{so6}+L_{so8}\right)\end{array}\right\}. \quad (42)$$

Where $-l\le m\le +l$, then we can be fixed $2l+1$ values correspond the magnetic effects. Know, we can resumed the global spectrum for three dimensional pseudo harmonic potential of a particle fermionic with spin up, spin down and non polarized at first order of two infinitesimals parameters (Θ and $\bar{\theta}$) corresponding $\left(H_{nc-ph}\right)_{11}$, $\left(H_{nc-ph}\right)_{22}$ and $\left(H_{nc-ph}\right)_{33}$ in both (NC-3D) phase and space for fundamental state and first excited state, respectively:

$$E_{\text{nc0-up}} = c + \sqrt{\frac{2a}{\mu}}\left(\delta+2\right) + \left|a_0\right|^2 k_+ \begin{Bmatrix} \left(-\Theta a + \frac{\bar{\theta}}{2\mu}\right) L_{so1} \\ -\Theta \frac{b}{4} L_{so2} \end{Bmatrix} \quad (43.1)$$
$$+\left|a_0\right|^2 m\left\{\left(-\alpha_2 a + \frac{\bar{\varepsilon}_2}{2\mu}\right) L_{so1} - \alpha_2 \frac{b}{4} L_{so2}\right\}.$$

$$E_{\text{nc0-d}} = c + \sqrt{\frac{2a}{\mu}}\left(\delta+2\right) + \left|a_0\right|^2 k_- \begin{Bmatrix} \left(-\Theta a + \frac{\bar{\theta}}{2\mu}\right) L_{so1} \\ -\Theta \frac{b}{4} L_{so2} \end{Bmatrix} \quad (43.2)$$
$$+\left|a_0\right|^2 m\left\{\left(-\alpha_2 a + \frac{\bar{\varepsilon}_2}{2\mu}\right) L_{so1} - \alpha_2 \frac{b}{4} L_{so2}\right\}.$$

$$E_{\text{nc0-d}} = c + \sqrt{\frac{2a}{\mu}}\left(\delta+2\right), \quad (43.3)$$

and

$$E_{\text{nc1-up}} = c + \sqrt{\frac{2a}{\mu}}\left(\delta+4\right) + k_+ \begin{Bmatrix} \left(-\Theta a + \frac{\bar{\theta}}{2\mu}\right)\left(L_{so3}+L_{so5}+L_{so7}\right) \\ -\Theta \frac{b}{4}\left(L_{so4}+L_{so6}+L_{so8}\right) \end{Bmatrix} \quad (44.1)$$
$$+m\begin{Bmatrix} \left(-\alpha_2 a + \frac{\bar{\varepsilon}_2}{2\mu}\right)\left(L_{so3}+L_{so5}+L_{so7}\right) \\ -\alpha_2 \frac{b}{4}\left(L_{so4}+L_{so6}+L_{so8}\right) \end{Bmatrix}.$$

$$E_{\text{nc1-d}} = c + \sqrt{\frac{2a}{\mu}}\left(\delta+4\right) + k_- \begin{Bmatrix} \left(-\Theta a + \frac{\bar{\theta}}{2\mu}\right)\left(L_{so3}+L_{so5}+L_{so7}\right) \\ -\Theta \frac{b}{4}\left(L_{so4}+L_{so6}+L_{so8}\right) \end{Bmatrix} \quad (44.2)$$
$$+m\begin{Bmatrix} \left(-\alpha_2 a + \frac{\bar{\varepsilon}_2}{2\mu}\right)\left(L_{so3}+L_{so5}+L_{so7}\right) \\ -\alpha_2 \frac{b}{4}\left(L_{so4}+L_{so6}+L_{so8}\right) \end{Bmatrix}.$$

$$E_{\text{np1-ph}} = c + \sqrt{\frac{2a}{\mu}}\left(\delta+4\right). \quad (44.3)$$

As well known, we have two possible values of total momentums $j = l \pm \frac{1}{2}$ (spin up-spin down), thus every state in usually 3D of energy for three dimensional pseudo harmonic potential in NC (3D-SP) phase and space will be: $2(2l+1)$ sub-states.

VI. CONCLUSIONS

In both noncommutative three dimensional spaces and phases, the Schrödinger equations with three dimensional pseudo harmonic potential has been solved by using the Boopp's shift method and standard perturbation theory. The noncommutative modification of energy eigenvalues for three dimensional pseudo harmonic potential have been obtained exactly corresponding three modes (spin: up-down at high energy) depended with discreet quantum numbers *m*, *l*, in addition to four infinitesimal parameters α_2, $\bar{\varepsilon}_2$, Θ_1 and $\bar{\theta}_1$, while the third mode unchanged energy (in low energy). The obtained quantum atomic spectrum changed totally, every state in usually 3D space replaced by $2(2l+1)$ sub-states for three dimensional pseudo harmonic potential.

ACKNOWLEDGEMENTS

This work was supported with search Laboratory of: Physique et Chimie des Matériaux, in University of M'sila, Algeria.

REFERENCES

[1] Child, M. S., Dong, S. -H. & Wang, X.-G., *Quantum states of a sextic potential: hidden symmetry and quantum monodromy*, Journal of Physics **A33**, 5653-5661 (2000).
[2] Bose, S. K., *Exact bound states for the central fraction power singular potential V(r) = αr2/3+βr−2/3+γr−4/3*, Il Nuovo Cimento **109**, 1217 (1994).
[3] Buragohain, L. & Ahmed, S. A. S., *Exactly solvable quantum mechanical systems generated from the anharmonic potentials*, Lat. Am. J. Phys. Educ. **4**, 1 (2010).
[4] Raushal, S. A., Ahmed, S.A., Borah, B. C. & Sarma, D., *Generation of exact bound state solutions from solvable non-power law potentials by a transformations method*, Eur. Phys. J. **D17**, 335-338 (1992).
[5] Ikhdair, S. M. & Sever, R., *Exact solutions of the radial Schrödinger equation for some physical potentials*, CEJP **5**, 516-527 (2007).
[6] Ikhdair, S. M. & Sever, R., *On the solutions of the Schrödinger equation with some molecular potentials: wave function ansatz*, arXiv:quant-ph/0702052v2 12 Feb 2007.
[7] Tapas Das, S. R. & Altug, A., *Exact solution of n-dimensional radial Schrödinger equation with pseudo harmonic potential via Laplace transforme approach*, arXiv: 1308.5295v1 [math-ph] 24 Aug 2013.
[8] Nieto, M. M. *Hydrogen atom and relativistic pi-mesic atom in N-space dimension*, Am. J. Phys. **47**, 1067-1072 (1979).
[9] Ikhdair, S. M. & Sever, R., *Exact polynomial eigensolutions of the Schrödinger equation for the pseudo harmonic potential*, J. Mol. Struc.-Theochem **806**, 155-158 (2007).

[10] Ahmed, A. S. & Buragohain, L., *Generation of new classes of exactly solvable potentials*, Phys. Scr. **80**, 1-6 (2009).
[11] Bose, S. K., *Exact solution of non-relativistic Schrödinger equation for certain central physical potentials*, Nouvo Cimento **B113**, 299-328 (1996).
[12] Flesses, G. P. & Watt, A., *An exact solution of the Schrödinger equation for a multiterm potential*, J. Phys. A: Math. Gen. **14**, L315-L318 (1981).
[13] Ikhdair, S. M. & Sever, R., *Exact polynomial solutions of the Mie-type potential in the N- dimensional Schrödinger equation*, Preprint: arXiv:quant-ph/0611065.
[14] Ikhdair, S. M. & Sever, R., *Exact solution of the Klein-Gordon equation for the PT symmetric generalized Woods-Saxon potential by the Nikiforov-Uvarov method*, Ann. Phys. (Leipzig) **16**, 218-232 (2007).
[15] Dong, S.-H., *Schrodinger equation with the potential V(r) = r*−4 + r*−3 + r*−2 + r*−1*, Physica Scripta **64**, 273-276 (2001).
[16] Dong, S.-H. & Ma, Z.-Q., *Exact solutions to the Schrodinger ¨ equation for the potential V(r) = r*2 + r*−4 + r*−6 in two dimensions*, Journal of Physics **A31**, 9855-9859 (1998).
[17] Dong, S.-H. *A new approach to the relativistic Schrödinger equation with central potential: Ansatz method*, International Journal of Theoretical Physics **40**, 559-567 (2001).
[18] Akder, A. *et al*., *A new Coloumb ring-shaped potential via generalized parametetric Nikivforov-Uvarov method*, Journal of Theoritical and Applied Physics **7**,17 (2013).
[19] Ikhdair, S. M. & Sever, R., *Relativistic two-dimensional harmonic oscillator plus cornell potentials in external magnetic and AB fields*, Advances in High Energy Physics **11**, (2013).
[20] Dong, S.-H. & San, G. -H., *Quantum spectrum of some anharmonic central potentials:wave functions ansatz*, Foundations of Physics Letters **16**, 357-367 (2003).
[21] Buragohain, L. & Ahmed, S. A. S., *Exactly solvable quantum mechanical systems generated from the anharmonic potential*s, Lat. Am. J. Phys. Educ. **4**, (2010).
[22] Ikhdair, S. M., *Exact solution of Dirac equation with charged harmonic oscillator in electric field: bound states*, Journal of Modern Physics **3**, 170-179 (2012).
[23] Connes, A., *Noncommutative geometry*, (Academic Press, Paris, 1994).
[24] Snyder, H., *The quantization of space time*, Phys. Rev. **71**, 38-41 (1947).
[25] Dossa, A. F. & Avossevou, G. Y. H., *Noncommutative phase space and the two dimensional quantum dipole in background electric and magnetic fields*, Journal of Modern Physics **4**, 1400-1411 (2013).
[26] Hassanabadi, H. *et al*., *Exact solution Dirac equation for an energy-depended potential*, Tur. Phys. J. Plus **127**, 120 (2012).
[27] Jacobus, D. T., *Calculating the moment of inertia of neutron stars*, (Department of Physics, Stellenbosch University, South Africa, 2010).
[28] Smailagic, A. *et al*., *New isotropic versus anisotropic phase of noncommutative 2-D harmonic oscillator*, Phys. Rev. **D65** 107701 (2002).
[29] Zu-Hua. Y. *et al*., *DKP oscillator with spin-0 in three dimensional non-commutaive phase-space,* Int. J. Theor. Phys. **49**, 644-657 (2010).
[30] Yuan, Y. *et al*. *Spin ½ relativistic particle in a magnetic field in NC Ph*, Chinese Physics **C34**, 543 (2010).
[31] Wang, J., Li, K. & Dulat, S., *Klein-Gordon oscillators in noncommutative phase space,* Chinese Physics **C10**, 803 (2008).
[32] Maireche, A., *Quantum Schrödinger equation with octic potential in non-commutative two-dimensional complex space*, Life Sci. J. **11**, 353-359 (2014).
[33] Maireche, A., *Spectrum of Schrödinger equation with h.l.c. potential in non-commutative two-dimensional real space*, The African Rev. Phys. **9**, 479-483 (2014).
[34] Maireche, A., *Deformed quantum energy spectra with mixed harmonic potential for nonrelativistic Schrödinger equation*, J. Nano-Electron. Phys. **7**, 02003 (2015).
[35] Maireche, A., *A study of Schrödinger equation with inverse sextic potential in 2-dimensional non-commutative space*, The African Rev. Phys. **9**, 185-193 (2014).
[36] Maireche, A., *Deformed bound states for central fraction power potential: non relativistic Schrödinger equation*, The African Rev. Phys. **10**, 97-103 (2015).
[37] Maireche, A., *Nonrelativistic atomic spectrum for companied harmonic oscillator potential and its inverse in both NC-2D: RSP*, International Letters of Chemistry, Physics and Astronomy **56**, 1-9 (2015).
[38] Gradshteyn, I. S. & Ryzhik, I. M., *Table of integrals, series and products*, 7th. Ed. (Elsevier, Ámsterdam, 2007).
[39] Maireche, A., *New exact bound states solutions for (C.F.P.S.) potential in the case of non-commutative three dimensional non relativistic quantum mechanics*, Med. J. Model. Simul. **4**, 60-72 (2015).
[40] Maireche, A., *New exact solution of the bound states for the potential family V(r) = A/r2-B/r+Crk (k=0,-1,-2) in both noncommutative three dimensional spaces and phases:non relativistic quantum mechanics*, International Letters of Chemistry, Physics and Astronomy **58**, 164-176 (2015).
[40] Maireche, A., *deformed energy levels of a pseudo harmonic potential: nonrelativistic quantum mechanics (two dimensional space)*, Yanbu Journal of Enginering and Science (accepted in 2015).
[42] Djemei, A. E. F. & Smail, H., *On quantum mechanics on noncommutative quantum phase space*, Commun. Theor. Phys. **41**, 837-844 (2004).
[43] Gamboa, J., Loewe, M. & Rojas, J. C., *Non-commutative quantum mechanics*, arXiv:hep- th/0010220v4 (2001).
[44] Mezincescu, L., *Star product in quantum mechanics,* arXiv:hep-th/0007046v2 (2000).