New quantum atomic spectrum of Schrödinger equation with pseudo harmonic potential in both noncommutative three dimensional spaces and phases

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Abstract

In present work, we obtain the modified bound-states solutions for three dimensional pseudo harmonic potential in both non-commutative three dimensional spaces and phases. We show the non commutative new anisotropic Hamiltonian containing three terms, the first is the usual Hamiltonian in ordinary quantum mechanics; the first new term describes the spin-orbit interaction while the second new term describes the modified Zeeman effect. It has been observed that, the energy spectra in ordinary quantum mechanics was changed, and replaced by degenerate new states, depending on new discreet quantum numbers: \( m, l, j \) and \( s=\pm 1/2 \), in addition to four infinitesimal parameters: \( \alpha, \xi, \theta, \bar{\theta} \). The new symmetries were extended to include automatically new physics phenomena’s.

Keywords: Star product, Boopp’s shift method, three dimensional pseudo harmonic potential, Noncommutative space and Noncommutative phase.

Resumen

En este trabajo, obtuvimos las soluciones modificadas de los estados bound para tres potenciales dimensionales pseudoarmónicos tanto para tres espacios y fases dimensionales no conmutativos. Demostramos el nuevo hamiltoniano anisotrópico no conmutativo que contiene tres términos. El primero es el hamiltoniano usual en una mecánica cuántica ordinaria. El primer término nuevo describe la interacción spin-orbit; mientras que el segundo término nuevo describe el efecto modificado de Zeeman. Se ha observado que el espectro energético en una mecánica cuántica ordinaria fue cambiado y reemplazado por nuevos estados degenerados, dependiendo de los nuevos números cuánticos discretos: \( m, l, j \) y \( s=\pm 1/2 \). Además de cuatro parámetros infinitesimales: \( \alpha, \xi, \theta, \bar{\theta} \). Las nuevas simetrías fueron extendidas para incluir automáticamente nuevos fenómenos físicos.

Palabras clave: Producto Star, Metodo de derivada de Boopp, Potenciales dimensionales pseudoarmónicos, espacio no conmutativo, fase no conmutativa.

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I. INTRODUCTION

The central potentials have been studied in various fields of material sciences, nuclear physics, quantum molecules by using relativistic Klein-Gordon equation, Dirac equation and non-relativistic Schrödinger equation in commutative and noncommutative two and three-dimensional space and phase [1-44]. The Schrödinger equation rest major sources of principal information’s and it consider as a big revolution like general relativity and special relativity for describing physics phenomena at microscopic (Planks scales) and macroscopic scales (the planets movements). In another hand, the notions of noncommutativity of space and phase, will be another big modern revolution to gives the profound interpretations in different fields. Based essentially on Seiberg-Witten map and Boopp’s shift method and the star product, depended on the antisymmetric infinitesimal parameters (\( \theta^{\mu\nu} \) and \( \bar{\theta}^{\mu\nu} \)) and equals \( 1/2 \, \varepsilon^{ijk} \theta_k \) and \( 1/2 \, \varepsilon^{ijk} \bar{\theta}_k \), respectively, in first order of two infinitesimal parameters \( \theta_k \) and \( \bar{\theta}_k \), the new star product defined as [23-43]:

\[
\begin{align*}
  f(x)^* g(x) = f(x)^* g(x) - i \frac{1}{2} \theta^{\mu\nu} \left( \partial_{\mu} f(x) \right) \left( \partial_{\nu} g(x) \right) \\
  &- i \frac{1}{2} \bar{\theta}^{\mu\nu} \left( \bar{\partial}_{\mu} f(x) \right) \left( \bar{\partial}_{\nu} g(x) \right).
\end{align*}
\]

(1)

As a direct principal’s result of the above equation, the two new non nulls commutators \( [x_i, x_j] \) and \( [\hat{p}_i, \hat{p}_j] \) in the notion of star product:

Where \( a = D_e r_e^{-2} \), \( b = D_e r_e^{-2} \), and \( c = -2D_e \), the radial function \( \Psi(r) \) in 3-dimensional space for three-dimensional pseudo harmonic \( (h = c = 1) \) is [6]:

\[
\frac{d^2 \Psi(r)}{dr^2} + \frac{2}{r} \frac{d \Psi(r)}{dr} + \left[ E - V_{3dp}(r) \right] \frac{l(l+1)}{r^2} \Psi(r) = 0 .
\]

(6)

Where \( l \) denote to the orbital angular momentum quantum numbers. Furthermore, to remove the first derivative from the above equation, we introduce a new radial function:

\[
R(r) \left( \Psi(r) = \frac{R(r)}{r} \right),
\]

then, the eq. (6) reduced to the form:

\[
\frac{d^2 R(r)}{dr^2} + \left[ 2 \mu \left( E - V_{3dp}(r) \right) + \frac{l(l+1)}{r^2} \right] R(r) = 0 .
\]

(7)

The complete orthonormalization functions and energy eigenvalues in three dimensional spaces for fundamental state \( (E_0^\delta and \Psi^0(r)) \), first excited state \( (E_1^\delta and \Psi^1(r)) \) and \( p \) and excited state \( (E_p^\delta and \Psi^p(r)) \) are given by, respectively [6]:

\[
E_{0\delta} = c + \sqrt{2a \mu \delta} (\delta + 2),
\]

(8)

\[
\Psi^0(r) = a_0 r^{\delta+1} \exp(\sqrt{2a \mu r^2}).
\]

\[
E_1^\delta = c + \sqrt{2a \mu} (\delta + 4),
\]

(9)

\[
\Psi^1(r) = \left( a_0 + a_1 r^2 \right) r^{\delta+1} \exp(\sqrt{2a \mu r^2}).
\]

\[
E_p^\delta = c + \sqrt{2a \mu} (2p + \delta + 2),
\]

(10)

\[
\Psi^p(r) = \left( a_0 + a_2 r^2 + \ldots + a_p r^{2p} \right) r^{\delta+1} \exp(\sqrt{2a \mu r^2}).
\]

The constants \( \delta, a_0 \) and \( a_2 \) are given by [6]:

\[
\delta = -1 + \sqrt{2b \mu + \eta^2},
\]

(11)

\[
a_0 = \frac{2}{(\delta + 1)!} \sqrt{2a \mu} \frac{\delta + 1}{2},
\]

\[
a_1 = \frac{1}{(\delta + 2)!} \sqrt{2a \mu} \frac{\delta + 2}{2}.
\]
And
\[ \eta = l + \frac{1}{2}. \]

**III. NONCOMMUTATIVE THREE DIMENSIONAL SPACE-PHASE HAMILTONIAN FOR PSEUDO HARMONIC POTENTIAL**

**III.A Formalism of Boopp’s shift method**

The quantum noncommutative Schrödinger equation, can be determined by apply the following steps for pseudo harmonic potential [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44]:

Ordinary Hamiltonian: \( \hat{H} \left( p_i, x_i \right) \)

NC-Hamiltonian: \( \hat{H} \left( \hat{p}_i, \hat{x}_i \right) \)

Ordinary-complex wave function: \( \Psi \left( \vec{r} \right) \rightarrow NC \)-complex wave function: \( \hat{\Psi} \left( \vec{r} \right) \)

Ordinary-energy: \( E \rightarrow NC - Energy : E_{nc-ph} \)

Ordinary-product \( \rightarrow \) New star product-acting on phase and space: *

Which allow us to writing the three dimensional space-phase quantum noncommutative Schrödinger equations as follows:

\[
\hat{H} \left( \hat{p}_i, \hat{x}_i \right) \hat{\Psi} \left( \vec{r} \right) = E_{nc-ph} \hat{\Psi} \left( \vec{r} \right). \tag{13}
\]

The Boopp’s shift method permutes to reduce the above equation to the form:

\[
\hat{H} \left( \hat{p}_i, \hat{x}_i \right) \Psi^p \left( r \right) = E_{nc-ph} \Psi^p \left( r \right). \tag{14}
\]

Here the two \( \hat{x}_i \) and \( \hat{p}_i \) operators in (NC-3D) phase and space are given by [23-43]:

\[
\hat{x}_i = x_i - \frac{\hbar \eta}{2} \hat{p}_j \quad \text{and} \quad \hat{p}_i = p_i + \frac{\hbar \eta}{2} \hat{x}_j. \tag{15}
\]

On another hand, the two operators \( \hat{r}^2 \) and \( \hat{p}^2 \) in (NC-3D) spaces and phases are given by [37, 39]:

\[
\hat{r}^2 = r^2 - \hat{L} \hat{\Theta}, \quad \hat{p}^2 = p^2 + \hat{\Theta}. \tag{16}
\]

New quantum atomic spectrum of Schrödinger equation...

Where \( L \Theta \) and \( \hat{L} \hat{\Theta} \) are given by, respectively:

\[
L \Theta = L_x \Theta_{12} + L_y \Theta_{23} + L_z \Theta_{13} \tag{17}
\]

\[
\hat{L} \hat{\Theta} = L_x \hat{\Theta}_{12} + L_y \hat{\Theta}_{23} + L_z \hat{\Theta}_{13}.
\]

Where \( \Theta = \frac{\theta}{2} \), based, on the Eq. (17), in the first order of two infinitesimal parameters \( \Theta \) and \( \hat{\Theta} \), the two important terms which will be used to determine the noncommutative pseudo harmonic potential can be written explicitly, in (NC-3D) spaces and phases as:

\[
ar^2 = ar^2 - a\hat{L} \hat{\Theta},
\]

\[
b \frac{b}{r^2} = b + \frac{b}{4r^4} L \Theta.
\]

The modified pseudo harmonic potential operators \( V_{3dph} (\hat{r}) \) in both (NC-3D) phase and space will be written as:

\[
V_{3dph} (\vec{r}) = ar^2 + b \frac{b}{r^2} + c + \left( -a + \frac{b}{4r^4} \right) L \Theta + \frac{\hat{L} \hat{\Theta}}{2 \mu}. \tag{19}
\]

The three first terms in above equation are given the ordinary pseudo harmonic potential in 3D spaces, while the rest terms are proportional’s with two infinitesimals parameters \(( \Theta \) and \( \hat{\Theta} \)) and then gives the terms of perturbations for pseudo harmonic potential \( H_{ph-pert} (r, \Theta, \hat{\Theta}) \) in (NC-3D) real space and phase as:

\[
H_{ph-pert} (r, \Theta, \hat{\Theta}) = \left( - a + \frac{b}{4r^4} \right) L \Theta + \frac{\hat{L} \hat{\Theta}}{2 \mu}. \tag{20}
\]

**III.B The second part of noncommutative 3D: RSP**

It is possible to replace \( L \Theta \) and \( \hat{L} \hat{\Theta} \) by \( 2 \hat{\hat{S}} \hat{L} \) and \( 2 \hat{\Theta} \hat{S} \hat{L} \), respectively, to obtain the new forms of \( H_{ph-pert} (r, \Theta, \hat{\Theta}) \) for pseudo harmonic potential:

\[
H_{ph-pert} (r, \Theta, \hat{\Theta}) = \left( - a + \frac{b}{4r^4} + \frac{\hat{\Theta}}{2 \mu} \right) \hat{L} \hat{S} \hat{L}. \tag{21}
\]

Here \( \hat{S} = \frac{1}{2} \) denote the spin of electron, in quantum mechanics, it is possible also to replace \( \frac{L \Theta}{2} \) by \( L \Theta \left( \hat{\hat{S}}^2 - \hat{\hat{L}}^2 \right) \), which allow us to writing the perturbative terms for pseudo harmonic potential:
\[ H_{ph-pen}(r, \Theta, \bar{\Theta}) = \left( -a + \frac{b}{4r^4} + \frac{\bar{\sigma}}{2\mu} \right) G \] (22)

Where \( G^2 \) denote to the \( \frac{1}{2} \left( \hat{J}^2 - \hat{L}^2 - \hat{S}^2 \right) \). As it is known, this operator traduces the coupling between spin and orbital momentum. The \( (\hat{J}^2, \hat{L}^2, \hat{S}^2, s_z) \) formed complete basis on quantum mechanics, then the operator \( \left( \hat{J}^2 - \hat{L}^2 - \hat{S}^2 \right) \) will be gives 2-eigenvalues \( k_j \equiv \frac{1}{2} \left( l \pm \frac{1}{2} \right) \) corresponding \( j = l \pm \frac{1}{2} \) (spin up-spin down), respectively. Then, one can form a diagonal matrix \( H_{nc-ph}(r, \bar{\rho}, \Theta, \bar{\Theta}) \) of order \( (3 \times 3) \) with non null elements \( (H_{nc-ph}(r, \bar{\rho}, \Theta, \bar{\Theta}))_{11}, (H_{nc-ph}(r, \bar{\rho}, \Theta, \bar{\Theta}))_{22}, \) and \( (H_{nc-ph}(r, \bar{\rho}, \Theta, \bar{\Theta}))_{33} \), for pseudo harmonic potential in both (NC-3D) phase and space:

\[
\begin{align*}
(H_{nc-ph})_{11} &= -\frac{\Delta}{2\mu} + ar^2 + \frac{b}{r^2} + c + k_+ \left( -a + \frac{b}{4r^4} + \frac{\bar{\sigma}}{2\mu} \right) \\
& \text{if } j = l + \frac{1}{2} \Rightarrow \text{spin up} \\
(H_{nc-ph})_{22} &= -\frac{\Delta}{2\mu} + ar^2 + \frac{b}{r^2} + c + k_- \left( -a + \frac{b}{4r^4} + \frac{\bar{\sigma}}{2\mu} \right) \\
& \text{if } j = l - \frac{1}{2} \Rightarrow \text{spin down} \\
(H_{nc-ph})_{33} &= -\frac{\Delta}{2\mu} + ar^2 + \frac{b}{r^2} + c \rightarrow \text{Non - polarised - electron}
\end{align*}
\] (23)

III.C The third part of noncommutative 3D: RSP

In this sub section, we draw another interpretation for the production of modified three dimensional pseudo harmonic potential, we consider the two at infinitesimals parameters \( (\Theta \text{ and } \bar{\Theta}) \) are the sum of two infinitesimals parameters to each one as [33-37]:

\[
\Theta = \Theta_1 + \Theta_2 \quad \text{and} \quad \bar{\Theta} = \bar{\Theta}_1 + \bar{\Theta}_2
\] (24)

The two parameters \( \Theta_2 \) and \( \bar{\Theta}_2 \) are arbitrary infinitesimals constants, then we can introduce an external magnetic field \( B \) as [33-41]:

\[
\Theta_2 = \alpha_z B, \quad \bar{\Theta}_2 = \bar{\alpha}_z B \quad \text{and} \quad \vec{B} = B \vec{k}.
\] (25)

Where \( \alpha_z \) and \( \bar{\alpha}_z \) are two new proportional constants, which allow us to obtain the following results:

\[
\left( -a + \frac{b}{4r^4} + \frac{\bar{\sigma}}{2\mu} \right) \mathcal{G} + \mathcal{L} \Rightarrow \left( \alpha_z -a + \frac{b}{4r^4} + \frac{\bar{\sigma}}{2\mu} \right) + \varepsilon_2 \right) \mathcal{G} = \mathcal{G} \] (26)

Thus, the second part of noncommutative Hamiltonians operator for three dimensional pseudo harmonic potential \( H_{ph-mag}(r, \bar{\rho}, \alpha_z, \bar{\alpha}_z, \bar{\varepsilon}_2) \) can be determined as follows:

\[
H_{ph-mag}(r, \bar{\rho}, \alpha_z, \bar{\alpha}_z, \bar{\varepsilon}_2) = \left( \alpha_z -a + \frac{b}{4r^4} + \frac{\bar{\sigma}}{2\mu} \right) \mathcal{G} = \mathcal{G} \] (27)

III.D The global noncommutative anisotropic Hamiltonian operator

The previous obtained results permitted to deduce the global diagonal noncommutative Hamiltonian matrices \( H_{nc-ph}(r, \bar{\rho}, \Theta, \bar{\Theta}) \) of order \( (3 \times 3) \), with non null elements:

\[
(H_{nc-ph})_{11}, (H_{nc-ph})_{22}, \text{ and } (H_{nc-ph})_{33}
\]

for three dimensional pseudo harmonic potential in both (NC-3D) phase and space:

\[
\begin{align*}
(H_{nc-ph})_{11} &= -\frac{\Delta}{2\mu} + ar^2 + \frac{b}{r^2} + c + k_+ \left( -a + \frac{b}{4r^4} + \frac{\bar{\sigma}}{2\mu} \right) \\
& \text{if } j = l + \frac{1}{2} \Rightarrow \text{spin up} \\
(H_{nc-ph})_{22} &= -\frac{\Delta}{2\mu} + ar^2 + \frac{b}{r^2} + c + k_- \left( -a + \frac{b}{4r^4} + \frac{\bar{\sigma}}{2\mu} \right) \\
& \text{if } j = l - \frac{1}{2} \Rightarrow \text{spin down} \\
(H_{nc-ph})_{33} &= -\frac{\Delta}{2\mu} + ar^2 + \frac{b}{r^2} + c \rightarrow \text{Non - polarised - electron}
\end{align*}
\] (28)

After a straightforward calculation, one can show that, the radial function \( R(r) \) satisfied the following equation, in (NC-3D: RSP) for three dimensional pseudo harmonic potential:

\[
\frac{d^2 R(r)}{dr^2} + \frac{2\mu}{r^2} \left( -a + \frac{b}{4r^4} + \frac{\bar{\sigma}}{2\mu} \right) \mathcal{G} = \mathcal{G} \] (29)

V. NONCOMMUTATIVE SPECTRUM

Know, we want to obtain the energies: \( E_{nc-up}, E_{nc-d} \) and \( E_{nc-up}, E_{nc-d} \) and
\( (E_{nc1-up}, \ E_{nc1-d}) \) of a particle fermionic with spin up-spin down at first order of two infinitesimals parameters \( (\Theta \ a \ \overline{\Theta}) \) corresponding \( \left( H_{nc-up} (r, p, \Theta, \overline{\Theta}) \right) \) and \( \left( H_{nc-ph} (r, p, \Theta, \overline{\Theta}) \right) \) for fundamental state and first excited state corresponding three dimensional pseudo harmonic potential in both (NC-3D) phase and space:

\[
\begin{align*}
E_{nc0-up} &= c + \sqrt{\frac{2\mu}{\hbar}} (\delta + 2) + E_{ph0-up} (\Theta_1, \overline{\Theta}_1) \\
&+ E_{ph0-mag} (\Theta_2, \overline{\Theta}_2) \\
E_{nc0-d} &= c + \sqrt{\frac{2\mu}{\hbar}} (\delta + 2) + E_{ph0-d} (\Theta_1, \overline{\Theta}_1) \\
&+ E_{ph0-mag} (\Theta_2, \overline{\Theta}_2).
\end{align*}
\]

And

\[
\begin{align*}
E_{nc1-up} &= c + \sqrt{\frac{2\mu}{\hbar}} (\delta + 2) + E_{ph1-up} (\Theta_1, \overline{\Theta}_1) \\
&+ E_{ph1-mag} (\Theta_2, \overline{\Theta}_2) \\
E_{nc1-d} &= c + \sqrt{2\mu} (\delta + 2) + E_{ph1-d} (\Theta_1, \overline{\Theta}_1) \\
&+ E_{ph1-mag} (\Theta_2, \overline{\Theta}_2).
\end{align*}
\]

Where:

\[
(E_{ph0-up} (\Theta_1, \overline{\Theta}_1), \ E_{ph0-d} (\Theta_1, \overline{\Theta}_1),
\]

\[
E_{ph0-mag} (\Theta_2, \overline{\Theta}_2) \text{ and } (E_{ph1-up} (\Theta_1, \overline{\Theta}_1),
\]

\[
E_{ph1-d} (\Theta_1, \overline{\Theta}_1), \ E_{ph1-mag} (\Theta_2, \overline{\Theta}_2),
\]

are the exact modifications of spin-orbital (up-down) and magnetic for the three dimensional pseudo harmonic potential corresponding fundamental state and first excited state, respectively.

\V.A Noncommutative spin-orbital spectrum

To obtain the exact noncommutative spin-orbital modifications of energy \( E_{ph0-up} (\Theta_1, \overline{\Theta}_1) \), \( E_{ph0-d} (\Theta_1, \overline{\Theta}_1) \) for three dimensional pseudo harmonic potential corresponding fundamental states, we apply the standard perturbations theory:

\[
\begin{align*}
E_{ph0-up} (\Theta_1, \overline{\Theta}_1) &= \left[ \int_0^{+\infty} r^{2\delta+1} \exp \left( \frac{2}{2a \mu} \right) \left( -\theta + \frac{b}{4r^2} + \frac{\overline{\Theta}}{2\mu} \right)^2 dr \right] a + \left[ \int_0^{+\infty} r^{2\delta+1} \exp \left( \frac{2}{2a \mu} \right) \left( -\theta + \frac{b}{4r^2} + \frac{\overline{\Theta}}{2\mu} \right)^2 dr \right] a
\end{align*}
\]

(32)

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It’s convenient to rewrite the two above equations as follows:

\[
E_{ph0-up} (\Theta_1, \overline{\Theta}_1) = \left| a \right|^2 \int_0^{+\infty} r^{2\delta+1} \left( -\theta + \frac{b}{2\mu} + \overline{\Theta} \right)^2 dr
\]

(33)

And

\[
E_{ph0-d} (\Theta_1, \overline{\Theta}_1) = \left| a \right|^2 \int_0^{+\infty} r^{2\delta+1} \left( -\theta + \frac{b}{2\mu} + \overline{\Theta} \right)^2 dr.
\]

(34)

Where:

\[
L_{so1} = \int_0^{+\infty} r^{m} \exp (-\beta r^2) dr
\]

(35)

With \( m_1 = 2\delta + 3 \), \( m_2 = 2\delta - 1 \) and \( \beta = -2\sqrt{2a \mu} \). Know we apply the special integral [38]:

\[
\Gamma \left( \frac{m+1}{n}, \beta x^n \right)
\]

(36)

Where \( \Gamma \left( \frac{m+1}{n}, \beta x^n \right) \) denote to the incomplete Gamma function. After straightforward calculations, we can obtain the results:

\[
L_{so1} = \frac{\Gamma \left( \delta + 1, -2\sqrt{2a \mu} \right)}{2 \left( -2\sqrt{2a \mu} \right)^{\delta+2}}
\]

(37)

For the first excited states, the exact noncommutative spin-orbital modifications of energy \( E_{ph1-up} (\Theta_1, \overline{\Theta}_1) \), \( E_{ph1-d} (\Theta_1, \overline{\Theta}_1) \) of three dimensional pseudo harmonic potential:

\[
E_{ph1-up} (\Theta_1, \overline{\Theta}_1) = k \left( \int_0^{+\infty} a + a r^2 \right)^2 r^{2\delta+1} \exp (2\sqrt{2a \mu r^2})
\]

\[
E_{ph1-d} (\Theta_1, \overline{\Theta}_1) = k \left( \int_0^{+\infty} a + a r^2 \right)^2 r^{2\delta+1} \exp (2\sqrt{2a \mu r^2})
\]

(38)
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A direct simplification gives:

\[
E_{\text{phi-up}}(\Theta_1, \bar{\Theta}_1) = k_+ \int_0^{e^{x+1}} \exp\left(\frac{-2a \mu r^2}{2\mu} + \frac{b}{4r^2}\right) dr
\]

\[
E_{\text{phi-down}}(\Theta_1, \bar{\Theta}_1) = k_- \int_0^{e^{x+1}} \exp\left(\frac{-2a \mu r^2}{2\mu} + \frac{b}{4r^2}\right) dr.
\]

(38)

It’s convenient to rewrite the two above equations as follows:

\[
E_{\text{phi-up}}(\Theta_1, \bar{\Theta}_1) = k_+ \left\{ -\Theta a + \frac{\bar{\Theta}}{2 \mu} \left[ L_{s03} + L_{s05} + L_{s07} \right] \right\}
\]

\[
E_{\text{phi-down}}(\Theta_1, \bar{\Theta}_1) = k_- \left\{ -\Theta a + \frac{\bar{\Theta}}{2 \mu} \left[ L_{s04} + L_{s06} + L_{s08} \right] \right\}.
\]

(39)

Where:

\[
L_{s03} = a_0 \int_0^{e^{m_3}} \exp\left(-\beta r^2\right) dr,
\]

\[
L_{s04} = \frac{bd_4}{4} \int_0^{e^{m_4}} \exp\left(-\beta r^2\right) dr,
\]

\[
L_{s05} = a_1 \int_0^{e^{m_5}} \exp\left(-\beta r^2\right) dr,
\]

\[
L_{s06} = \frac{b a_5}{4} \int_0^{e^{m_6}} \exp\left(-\beta r^2\right) dr,
\]

\[
L_{s07} = 2 a_0 a_1 \int_0^{e^{m_7}} \exp\left(-\beta r^2\right) dr
\]

and

\[
L_{s08} = \frac{b a_0 a_1}{2} \int_0^{e^{m_8}} \exp\left(-\beta r^2\right) dr.
\]

Which allow us to obtaining the exact noncommutative spin-orbital modifications of energy \(E_{\text{phi-up}}(\Theta_1, \bar{\Theta}_1)\), \(E_{\text{phi-down}}(\Theta_1, \bar{\Theta}_1)\) of fundamental states and first excited states \(E_{\text{phi-up}}(\Theta_1, \bar{\Theta}_1)\) and \(E_{\text{phi-down}}(\Theta_1, \bar{\Theta}_1)\) for the studied potential.

V.B Noncommutative magnetic spectrum

Furthermore, to obtain, the exact noncommutative magnetic modifications of energy \(E_{\text{phi-up-mag}}(\Theta_2, \bar{\Theta}_2)\) and \(E_{\text{phi-down-mag}}(\Theta_2, \bar{\Theta}_2)\) for three dimensional pseudo harmonic potential of fundamental states and first excited states, it’s sufficient to replace: \(k_+, k_-\), \(\Theta_1\) and \(\bar{\Theta}_1\) in the Equations (33) and (39) by the following parameters: \(m\), \(a_2\) and \(\bar{e}_2\), respectively:

\[
E_{\text{phi-up-mag}}(\Theta_2, \bar{\Theta}_2) = \left\{ a_0 \int_0^{e^{m_3}} \exp\left(-\beta r^2\right) dr \right\}
\]

\[
E_{\text{phi-down-mag}}(\Theta_2, \bar{\Theta}_2) = \left\{ \frac{bd_4}{4} \int_0^{e^{m_4}} \exp\left(-\beta r^2\right) dr \right\}
\]

\[
E_{\text{phi-up-mag}}(\Theta_2, \bar{\Theta}_2) = \left\{ \frac{b a_5}{4} \int_0^{e^{m_6}} \exp\left(-\beta r^2\right) dr \right\}
\]

\[
E_{\text{phi-down-mag}}(\Theta_2, \bar{\Theta}_2) = \left\{ \frac{b a_0 a_1}{2} \int_0^{e^{m_8}} \exp\left(-\beta r^2\right) dr \right\}.
\]

(40)

(41)

With \(m_3 = 2 \delta + 3\), \(m_4 = 2 \delta - 1\), \(m_5 = 2 \delta + 7\), \(m_6 = 2 \delta + 1\), \(m_7 = 2 \delta + 5\), \(m_8 = 2 \delta + 1\) and \(\beta = -2\sqrt{2a \mu}\). Know we apply the special integral (35), we obtains:

\[
L_{s03} = a_0 \frac{\Gamma(\delta + 2, \beta r^2)}{2 \left(-2\sqrt{2a \mu}\right)^{\delta + 2}} L_{s04} = \frac{ba_0}{8} \frac{\Gamma(\delta + 1, \beta r^2)}{\left(-2\sqrt{2a \mu}\right)^{\delta + 1}}
\]

and

\[
L_{s05} = a_1 \frac{\Gamma(\delta + 2, \beta r^2)}{2 \left(-2\sqrt{2a \mu}\right)^{\delta + 2}} L_{s06} = \frac{b a_5}{8} \frac{\Gamma(\delta + 1, \beta r^2)}{\left(-2\sqrt{2a \mu}\right)^{\delta + 1}}
\]

\[
L_{s07} = \frac{2 b a_0 a_1}{2} \frac{\Gamma(\delta + 1, \beta r^2)}{\left(-2\sqrt{2a \mu}\right)^{\delta + 1}}
\]

\[
L_{s08} = \frac{b a_0 a_1}{2} \frac{\Gamma(\delta + 1, \beta r^2)}{\left(-2\sqrt{2a \mu}\right)^{\delta + 1}}
\]

(42)

Where \(-l \leq m \leq + l\), then we can be fixed \(2l + 1\) values correspond the magnetic effects. Know, we can resumed the global spectrum for three dimensional pseudo harmonic potential of a particle fermionic with spin up, spin down and non polarized at first order of two infinitesimal parameters \(\Theta\) and \(\bar{\Theta}\) corresponding \(H_{nc-ph}\) and \(H_{nc-ph}\) in both (NC-3D) phase and space for fundamental state and first excited state, respectively:
VI. CONCLUSIONS

In both noncommutative three dimensional spaces and phases, the Schrödinger equations with three dimensional pseudo harmonic potential has been solved by using the Boopp’s shift method and standard perturbation theory. The noncommutative modification of energy eigenvalues for three dimensional pseudo harmonic potential have been obtained exactly corresponding three modes (spin: up-down at high energy) depended with discreet quantum numbers \( m \), \( l \), in addition to four infinitesimal parameters \( \alpha_2 \), \( \beta_2 \), \( \Theta_1 \) and \( \bar{\Theta}_1 \), while the third mode unchanged energy (in low energy). The obtained quantum atomic spectrum changed totally, every state in usually 3D space replaced by \( 2(2l+1) \) sub-states for three dimensional pseudo harmonic potential.

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As well known, we have two possible values of total momentum \( j = l \pm \frac{1}{2} \) (spin up-spin down), thus every state in usually 3D of energy for three dimensional pseudo harmonic potential in NC (3D-SP) phase and space will be: \( 2(2l+1) \) sub-states.
Abdelmadjid Maireche


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