Understanding the stationary and transient state of a solar array: Model and simulation

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Abstract

The simulation of a solar array behavior is shown in this paper. In order to do this we first provide an overview of the physical processes that occur in a solar cell for the photovoltaic effect to take place, its equivalent electrical circuit is determined with an explanation of its components (photovoltaic current, diode, series and parallel resistance, capacitance). The next step involves the design of the mathematical model of a solar array, this one will be simulated in Simulink (from MATLAB) and the data we can obtain from it (currents, voltages, fill factor, power, efficiency, among others) along with its comportment under different conditions (the I-V curves of series, parallel and combined solar cells interconnections, the dependence with the intensity of the solar irradiance and the transient response considering different technologies) will be shown and discussed. From this paper the reader will obtain a high level in order to understand the operational principle of a solar array together with a practical notion of its electrical performance.

Keywords: Solar Array, Mathematical Model, Simulation, Stationary and Transient State.

Resumen

La simulación del comportamiento de un panel solar se muestra en este documento. Con el fin de hacer esto, primero mostramos una visión general de los procesos físicos que ocurren en una célula solar para que ocurra el efecto fotovoltaico, su circuito eléctrico equivalente se determina con una explicación de sus componentes (corriente fotovoltaica, diodos, series y resistencias en paralelo, capacitancia). El siguiente paso consiste en el diseño del modelo matemático de un panel solar, éste se simulará en Simulink (de MATLAB). Se muestran y discuten los datos que se puedan obtener (corrientes, tensiones, factor de llenado, energía, eficiencia, entre otros) de su comportamiento en diferentes condiciones (las curvas IV de la serie, las interconexiones paralelas y combinadas de las células solares, la dependencia con la intensidad de la radiación solar, y la respuesta transitoria considerando diferentes tecnologías). Con este artículo, el lector obtendrá un alto nivel de entendimiento del principio de funcionamiento de un panel solar, además de una noción práctica de su rendimiento eléctrico.

Palabras clave: Matriz solar, Modelo matemático, Simulación, Estado estacionario y transitorio.

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I. INTRODUCTION

In our society, the amount of people is increasing in time, because of this we require more industries, houses and transport systems and as a result, a higher demand of electrical energy is needed [1], mostly of this demand is covered by means of non-renewable energy, nevertheless this has a limit and we might reach a moment in the future in which we would not be able to count with it; here comes the need of focus in the research and develop of different renewable energy systems [2].

Photovoltaic appears as a very popular way to generate energy with a big potential to have a high development in the future [3] because of the advantages it presents like for example, it converts directly the energy from the light into electricity, it doesn't require any mechanical movement (like the eolic generators or in hydro plants) so there will not be losses due to friction, depending of our requirements we can have portable solar cells like in our bags or a small solar system in the roof of a home to provide energy to a house, or even a big one that could extend to kilometers to provide energy to a town (known as solar farms), due to this the solar cell production is increasing in time [4].

On this paper we describe how to get the mathematical model of a solar array considering different parameters of this system and then we will proceed to develop the simulation of this model in Simulink and explain the obtained results.

II. ELECTRICAL MODEL OF A SOLAR CELL

A solar cell is an element in which the photovoltaic effect [5] is present; it means that it can generate electricity from light. In order to represent it with equations, is necessary to consider different characteristics that it has, for example, we consider the solar cell to be a semiconductor material [6] (like Silicon) divided in two regions, one region doped by an n-material (like Phosphorous) in order to provide free electrons and the other region doped by a p-material (like Boron) in order to provide free holes. As we can understand until this point, having a semiconductor with a p-n region means that we are dealing with a diode *D* [7].

When the photons impact the solar cell, if they have enough energy, they will release and generate an electron-hole pair that will contribute to the photovoltaic current I_{ph} (current produced inside the solar cell due to the photovoltaic effect), so this means that if we increase the number of photons with enough energy we will increase in the same proportion the photovoltaic current (if we assume an ideal case), hence we can consider that there is a direct relation between the solar irradiance and the photovoltaic current.

There are also resistivity losses, for instance we can consider the resistances at the metallic contacts (fingers and busbars) of the solar cell needed to transport the electrons from the solar cell to the external circuit; the resistances at the intersection between the semiconductor and the metallic contacts of the solar cell and also the resistances that are presented in the semiconductor (electrons need to flow through the solar cell); all these losses are represented as the series resistance R_s.

Another resistance that is important to represent is the shunt resistance R_p, this one takes into account for the losses in which part of the photovoltaic current is not able to flow through the external circuit, by the fact that those free carriers are recombined inside the solar cell (this normally is due to manufacturing defects).

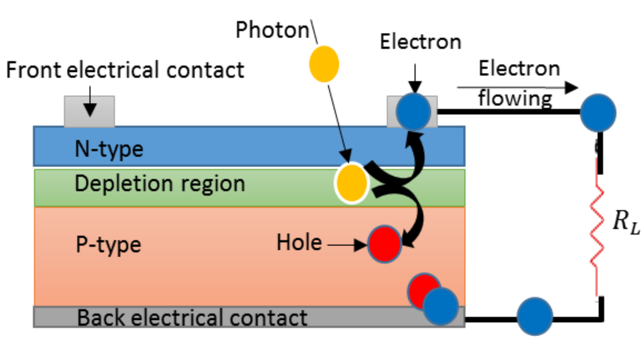

FIGURE 1. Schematic of photovoltaic effect.

As explained in the first paragraph of this section, the solar cell consist of a p and n region, at the junction between these two, we will find cations (positive ions found at the n side) and anions (negative ions found at the p side), this small region is named the "depletion region", so we can understand that at the junction we have positive charges in one side and negative in the other, that for simplicity we can consider are separated by an average distance of the depletion region, due to this we are having a capacitor with capacitance *C* (that can be described by the parallel plate capacitor). All these parameters explained are reflected in the following graph:

Hence, the electrical model of a solar cell is:

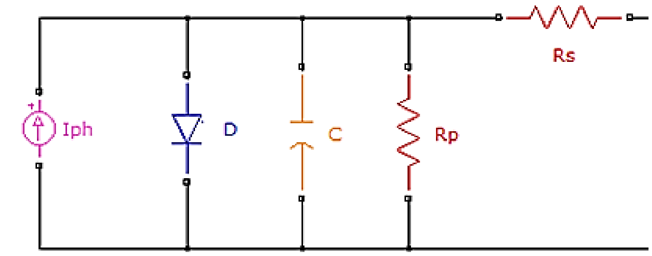

FIGURE 2. Electrical circuit of a solar cell.

III. ELECTRICAL AND MATHEMATICAL MODEL OF A SOLAR ARRAY

A solar array consists in a number of solar cells connected in series and/or parallel among them, from the electrical model of the solar cell it can easily be shown that the equivalent model of a solar array is (the output voltage and current will have exactly the same expression by using the following model or by using the previous model of a solar cell from section II and then connecting them in series and parallel to have the solar array):

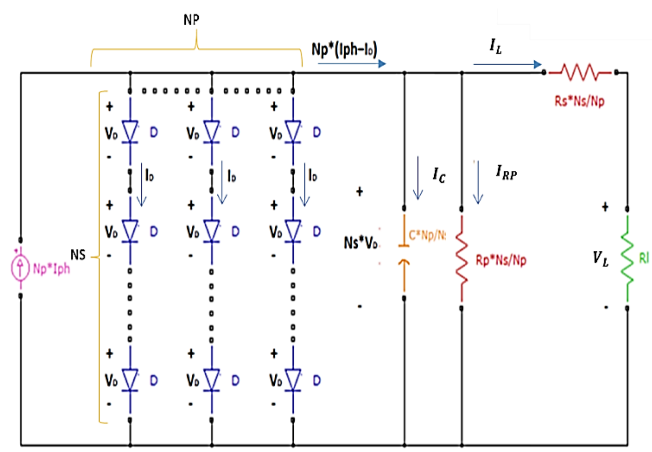

FIGURE 3. Electrical circuit of a solar array.

Where:

Ns: Number of solar cells in series.
Np: Number of solar cells in parallel.
V_D: Diode voltage.
I_D: Current trough a diode.
R_L: Load resistance.
V_L: Load voltage (output voltage).
I_L: Load current (output current).

V_L is the voltage that the solar array will produce when connected to an external circuit (in this case the resistance R_L), so in the following steps we will focus to find an expression of this voltage in terms of R_L, from Figure 3 we can solve this electrical circuit.

We can notice that the total photovoltaic current is divided among the diodes, capacitor (I_C), shunt resistance (I_{RP}) and the load, so by using Kirchhoff Nodal rule we can get the equation:

$$N_p * I_{ph} = N_p * I_D + I_C + I_{Rp} + I_L \,. \tag{1}$$

The relation between the current and voltage of a diode is expressed by the Shockley equation [8] with an extra factor known as the ideality factor "n" whose value reveals the kind of recombination mechanism that is imposing in the solar cell:

$$I_D = I_o * \left(e^{\left[\frac{e^- * V_D}{n * K_B * T}\right]} - 1 \right). \tag{2}$$

Where:

I_o: dark saturation current (constant of a solar cell that depends on parameters such as the free carrier's diffusivity and the doping level).

e^-: charge of the electron ($1.602176565\times10^{-19}$ C).

K_B: Boltzman constant (8.6173324×10^{-5} eV/K).

T: solar cell temperature.

The current of the capacitor I_C is related with its voltage V_C in the following way [9]:

$$\mathrm{I_c} = \left(C * \frac{N_P}{N_S} \right) * \frac{d(V_C)}{dt} = \left(C * \frac{N_P}{N_S} \right) * \frac{d(N_S * V_D)}{dt}. \tag{3}$$

Note: keep in mind that the total capacitance of the capacitor from Fig. 3 is: $C * \frac{N_P}{N_S}$.

Using Ohm´s law we can obtain the following relations:

$$\mathrm{I_{Rp}} = \frac{\mathrm{N_S * V_D}}{\frac{\mathrm{N_S}}{\mathrm{N_P}} * \mathrm{R_P}}, \tag{4}$$

$$\mathrm{I_L} = \frac{\mathrm{V_L}}{\mathrm{R_L}}. \tag{5}$$

By replacing Eq. (2), (3), (4) and (5) in (1) we obtain, after arranging the equation:

$$\mathrm{N_p * I_{ph}} = \mathrm{N_p} * I_o * \left(e^{\left[\frac{e^- * V_D}{n * K_B * T}\right]} - 1 \right) + \mathrm{C * N_P} * \frac{\mathrm{d(V_D)}}{\mathrm{dt}} + \frac{\mathrm{N_P * V_D}}{\mathrm{R_P}} + \frac{\mathrm{V_L}}{\mathrm{R_L}} \tag{6}$$

Equation (6) is only in term of V_L and the constants of the system except by V_D that is not a constant, so we need to get an expression for the diode voltage in term of the constants and V_L.

By relating the voltage of the diodes with the ones of the series resistance and load (using the Kirchhoff Voltage rule) we get:

$$\mathrm{N_S * V_D} = \frac{\mathrm{N_S * R_S}}{\mathrm{N_P}} * \mathrm{I_L} + \mathrm{V}_L = \frac{\mathrm{N_S * R_S}}{\mathrm{N_P}} * \frac{\mathrm{V}_L}{\mathrm{R_L}} + \mathrm{V}_L. \tag{7}$$

Solving Equation (7) for V_D:

$$\mathrm{V_D} = \frac{\mathrm{V}_L}{\mathrm{N_S}} * \left(1 + \frac{\mathrm{N_S * R_S}}{\mathrm{N_P * R_L}} \right). \tag{8}$$

Finally, by replacing Equation (8) in (6) and solving for V_L we obtain:

$$\mathrm{V}_L = \frac{\mathrm{N_p * I_{ph} - N_p * I_o} * \left(e^{\left[\frac{e^- * \frac{\mathrm{V_L}}{\mathrm{N_S}} * \left(1 + \frac{\mathrm{N_S * R_S}}{\mathrm{N_P * R_L}}\right)}{\mathrm{K_B * T}}\right]} - 1 \right) - \frac{\mathrm{C * N_P}}{\mathrm{N_S}} * \left(1 + \frac{\mathrm{N_S * R_S}}{\mathrm{N_P * R_L}}\right) * \frac{\mathrm{d(V_L)}}{\mathrm{dt}}}{\frac{\mathrm{N_p}}{\mathrm{R_P * N_S}} * \left(1 + \frac{\mathrm{N_S * R_S}}{\mathrm{N_P * R_L}}\right) + \frac{1}{\mathrm{R_L}}}. \tag{9}$$

The load voltage is in both sides of the Eq. (9), so in order to obtain its value, iterative methods in MATLAB are used.

The constants of the solar array that in this simulation belong to the typical range of values such as:

$$I_{ph} = 30 \ \frac{mA}{cell\ area\ in\ cm^2},$$

at a solar irradiance of 1000 $\frac{W}{m^2}$ (I_{ph} depends on the solar cell area because the bigger its area the more photons it can absorb and is proportional to the irradiance as explained in section II), solar cells with:

an area of 239 cm^2,

R_p=1000 ohm,

R_s=0.01 ohm,

I_o=1×10^{-10} A,

T= 298 K (ambient temperature) will be considered.

The values of "N_S", "N_P" and "C" will depend on the considerations for each simulation.

With this, we can present the model created in Simulink:

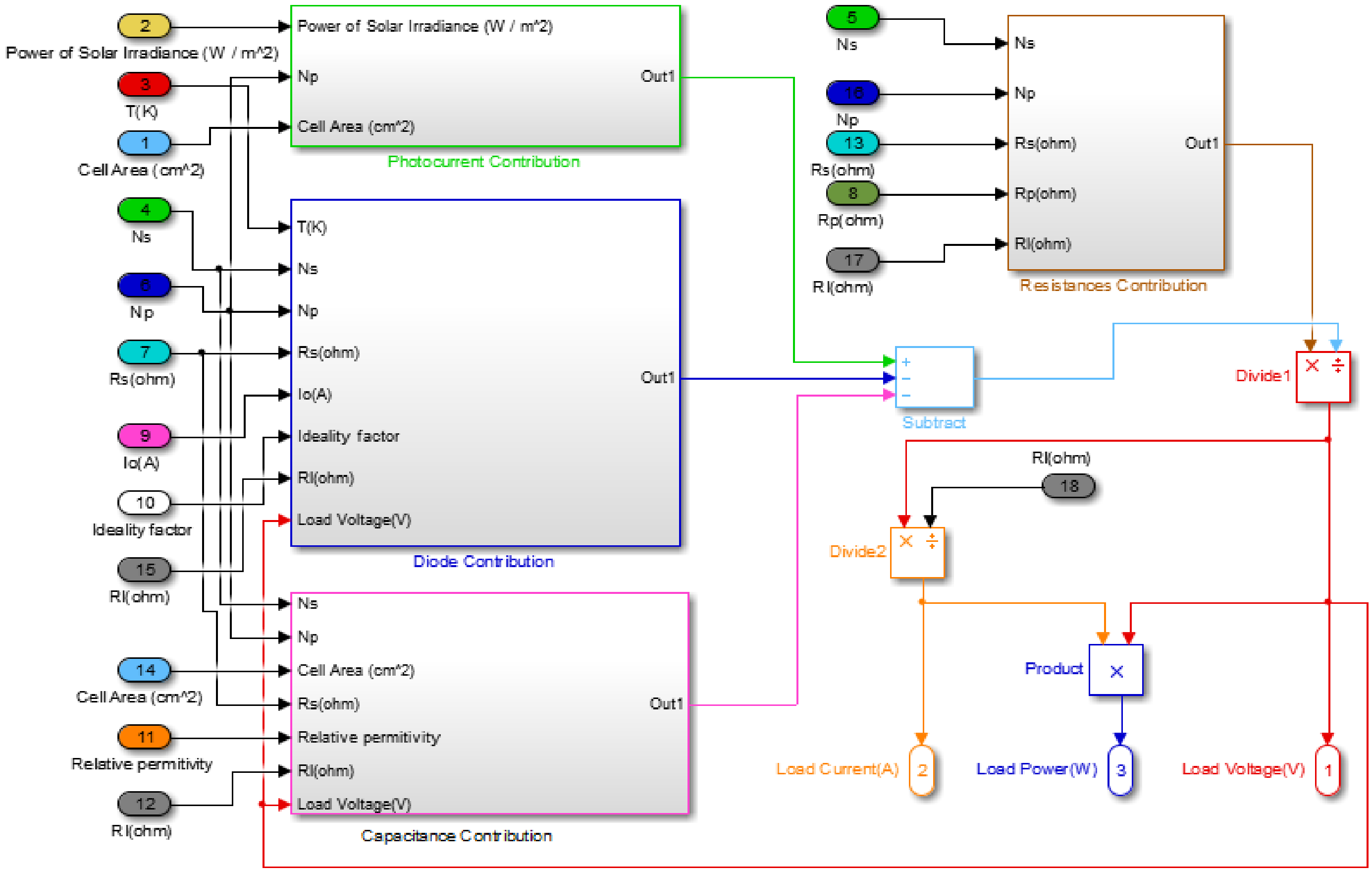

FIGURE 4. Solar array implemented in Simulink.

The Figure 4 represents in principle Equation (9), we can observe that there are four important blocks, the first block named "Photocurrent Contribution" corresponds to the first term of the numerator of Eq. (9), and it provides the relation between I_{ph} with the Power of Solar Irradiance (as explained in the last section):

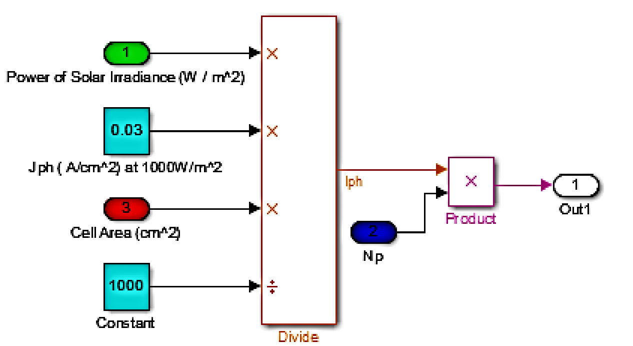

FIGURE 5. Relation between I_{ph} and the Solar Irradiance.

The second block named "Diode contribution" (D_{ic}) corresponds to the second term of the numerator of Eq. (9):

$$D_{ic} = N_p * I_o * \left(\exp\left(\frac{e^{-*\frac{V_L}{N_S}*\left(1+\frac{N_S*R_S}{N_P*R_L}\right)}}{K_B*T}\right) - 1\right). \qquad (10)$$

The third block named "Capacitance contribution" (C_{ac}) corresponds to the third term of the numerator of Eq. (9):

$$C_{ac} = \frac{C*N_P}{N_S} * \left(1 + \frac{N_S*R_S}{N_P*R_L}\right) * \frac{d(V_L)}{dt}. \qquad (11)$$

The fourth block named "Resistances contribution" (R_{ec}) corresponds to the denominator of Eq. (9):

$$R_{ec} = \frac{N_P}{R_P*N_S} * \left(1 + \frac{N_S*R_S}{N_P*R_L}\right) + \frac{1}{R_L}. \qquad (12)$$

Thus, from these four blocks we can obtain the value for V_L.

At the end of the simulation, we divided this voltage by R_L in order to obtain I_L and then multiply V_L with I_L to obtain the load power P_L; these are the three outputs we are interested to analyze from the solar system.

IV. SYSTEM SIMULATION

The results of the simulations given in the following pages can be separated into two categories, first we analyze only the stable state behavior of the solar array (no effect of the capacitor) and then we will simulate the system with a transient behavior including its capacitive effect to observe how much this one affect its response.

The parameters in the following simulations correspond to the ones indicated in section III.

IV.A Single solar cell behavior: stationary state

The simulations on this section correspond to a single solar cell and will provide a good understanding of how it works under different conditions.

IV.A.1 I-V Curve of a solar cell

Fig. 6 shows the behavior of a single solar cell, we can appreciate how it is formed from the contribution of the diode IV curve and the photovoltaic current [10]:

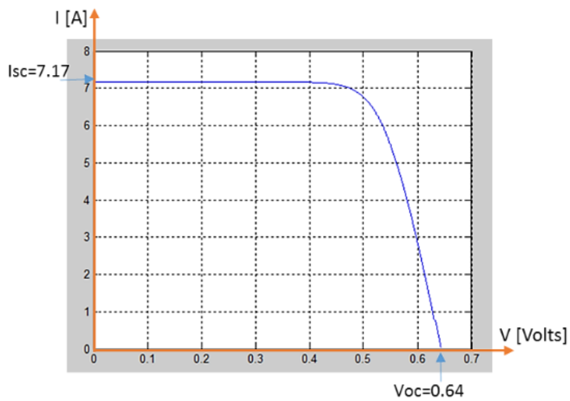

FIGURE 6. I-V curve of a single solar cell.

In the last figure, we can check two important parameters, the first one is the Open Circuit Voltage (V_{OC}), voltage obtained from the solar cell when the external resistance is set to infinitive and the obtained value is 0.64 V. The second parameter is the Short Circuit Current (I_{SC}), current generated at the output when the load is set to zero, 7.17 A is the value gotten in the simulation.

Notice how this value seems to be the same as the photovoltaic current (I_{ph} =30 $\frac{\text{mA}}{\text{cell area in cm}^2}$*239 cm^2), however from the electrical circuit in Fig.2 we can understand that I_{ph} will be a little higher that I_{SC}, being R_S a factor that will determine how big is this difference (the bigger R_S the smaller the I_{SC} and viceverse); in this case we obtained the same value because the quantities are represented up to two decimals and the set value of R_S is very small.

IV.A.2 Efficiency and power curve of a solar cell

From Fig. 6 we can calculate the power at the external load by multiplying the external current into the external voltage obtaining the Power Vs Voltage curve as shown in Fig. 7.

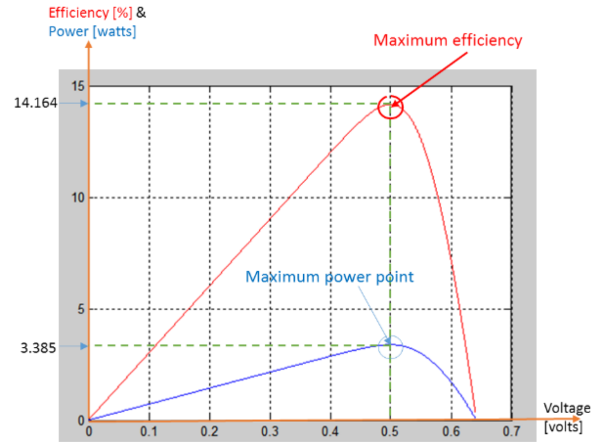

FIGURE 7. Efficiency and Power Curve of a Solar Cell.

By definition the efficiency is defined as the relation between the Output Power (the power calculated from the previous paragraph) with the Input Power (the power coming from the sun in the solar cell).

$$\eta = \frac{\text{OutPower}}{\text{Input Power}} * 100 = \frac{\text{Output Voltage}*\text{Output Current}}{1000\,\frac{\text{W}}{\text{m}^2}*239\ \text{m}^2} * 100. \quad (13)$$

The behaviour of the solar cell efficiency vs the external voltage is also shown in Fig. 7.

We notice in both graphs, when there is no voltage or current produced at the output, the output power and efficiency are zero; this means that there is a point in which these values reach a maximum, in this case these are found at a load voltage of 0.50 V, then by looking at the graph we find that the Maximum Power Point (MPP) is 3.39 W and the Maximum Efficiency is 14.16%. By assuming in our simulation a better metallization quality, antireflection coating, passivation layers, among others [11], we would have obtained a higher efficiency; currently the highest efficiency of Si solar cells is reported to be 25.6 % [12] and the maximum theoretical one is about 30% [13].

An advantage of the photovoltaic system in comparison with other renewable energy systems (like the ones obtained from the eolic or hydroelectric power plants) is that the maximum power and efficiency are both at the same operational point, so in normal conditions the solar arrays are set to work at this point.

Another important parameter that can be find from this graph is the Fill Factor (FF), defined as:

$$\text{FF} = \frac{\text{MPP}}{\text{Voc}*\text{Isc}} * 100\% \,. \quad (14)$$

By replacing the values in the previous equation:

$$\text{FF} = \frac{3.39\text{W}}{0.64\text{V}*7.17\text{A}} * 100\% = 73.88\%. \quad (15)$$

We are interested to have a high FF because it gives us the proportion of internal rectangle formed by the voltage and current at the maximum power point (colour orange) compared with the external rectangle formed by the V_{OC} and I_{SC} (colour green) as shown in the following image:

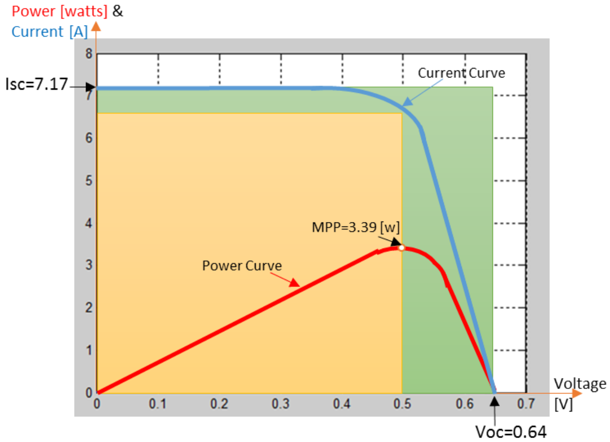

FIGURE 8. Rectangle proportion for Fill Factor.

IV.A.3 Irradiation dependence on a solar cell

As explained in section I, the amount of solar irradiance that the cell receives affects in a proportional way the I_{ph}, in the following graph the I-V curve of a solar cell is obtained from different values of solar irradiances of 600, 800, 1000 and 1200 $\frac{W}{m^2}$:

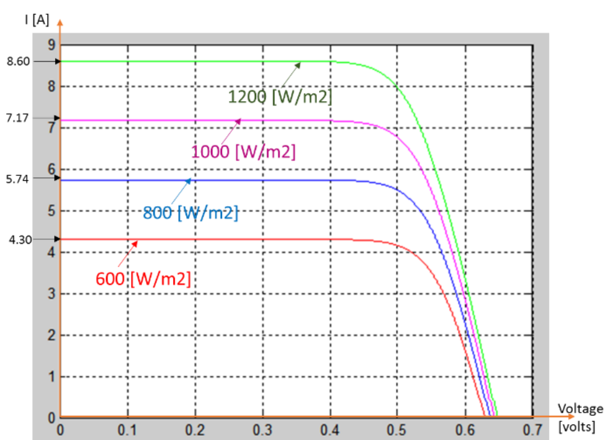

FIGURE 9. I-V curve at different irradiance level

By analysing the previous graph we observe that the proportion in which the power of the solar irradiance is increased, the I_{SC} increases in the same way. At this point we have to keep in mind what was explained in section i (I_{SC} is very similar that I_{ph}).

Another interesting fact is that the V_{OC} has a small increase with the increase of the power of solar irradiance, but not in a linear way, because the relation of V_{OC} with the I_{ph} is a logarithmic one [14].

IV.B Multiple solar cells behavior: stationary state

From section A, we got an idea of how much power we can obtain from a single solar cell, so if our load demands a higher power (like a TV, refrigerator, etc.) we need to use more solar cells that could be connected in series or in parallel depending on what we want to increase: the voltage, the current or both.

All solar cells will have the same parameters and will be identical among them (as we defined in section III).

IV.B.1 I-V Curve of solar cells in series

In mostly of the cases, solar panels are formed by solar cells that are connected in series, and the number of the solar cells will depend on how much power we are interested to generate with them.

The Fig. 10 shows the I-V behaviour of solar cells connected in series.

Note: for a single cell we declare 1SC, for 2 solar cells we declare 2SC, for 3 solar cells we declare 3SC, and so on.

Then if they are connected in series we declare first the number of solar cells followed by an S and finally we add SC. (For example if we want to connect 10 solar cells in series, we write 10S SC):

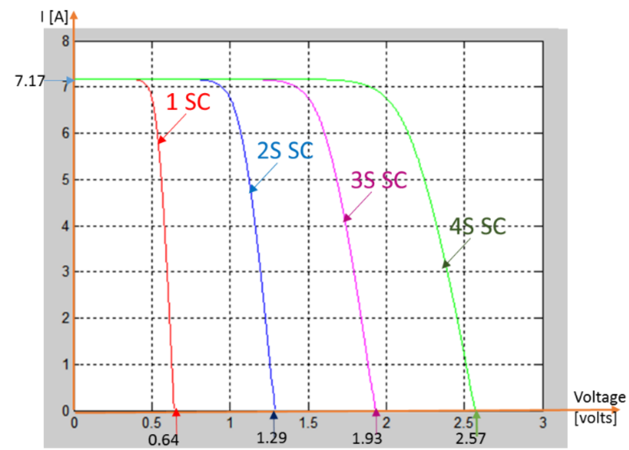

FIGURE 10. I-V curve of solar cells connected in series.

From Fig. 10 we can notice that regardless of the number of cells in series, the short circuit current (I_{SC}) remains constant and with the same value found with a single solar cell; second, the number of cells in series is proportional to the increase of the open circuit voltage (V_{OC}) of this system compared with the V_{OC} of one solar cell (ex: three solar cells in series will provide a *Voc* three times higher than the one a solar cell).

Hence, we can understand that a series connection of solar cells increases to total power by increasing the external voltage.

IV.B.2 I-V Curve of solar cells in parallel

Connecting cells in parallel will generate an increase in the current of the total system, as is shown in Fig. 11.

Note: if the solar cells are connected in parallel we declare first the number of solar cells followed by a P and finally we add SC. (For example if we want to connect 10 solar cells in parallel, we write 10P SC):

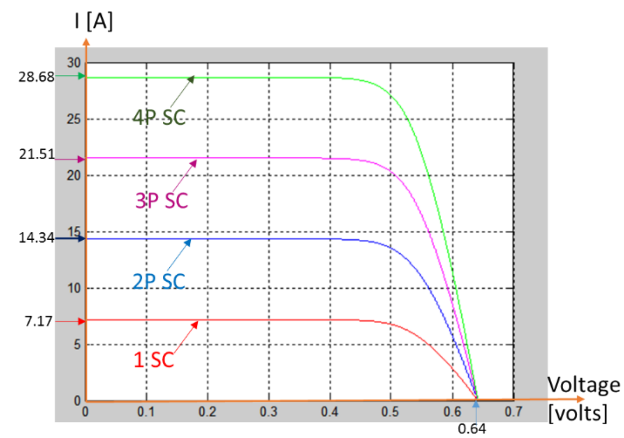

FIGURE 11. I-V curve of solar cells connected in parallel.

Notice how the *Voc* remains the same as the one from a single solar cell regardless of the amount of cells in parallel.

The number of the parallel resistances will indicate us how much the *Isc* of the all system is increased (ex: three solar cells in parallel will provide a short circuit current three times higher than one solar cell).

IV.B.3 I-V Curve of solar cells connected in series and parallel

This simulation shows the behaviour of a system, which has both, parallel, and series connection.

Note: for a single solar cell we declare 1SC; for a system composed of 2 solar cells connected in series and 1 extra cell in parallel to each one we declare 2S2P SC having 4 cells in total; for a system composed of 3 solar cells in series and 2 extra cells in parallel to each one we declare 3S3P SC having 9 cells in total; for a system composed of 4 solar cells in series and 3 extra cells in parallel to each one we declare 4S4P SC having 16 cells in total; and so on.

As expected, our results in Fig. 12 shows the series and parallel contribution in which the system of 2S2P SC, 3S3P SC, 4S4P SC has two, three and four times respectively the Voc and Isc of a solar cell.

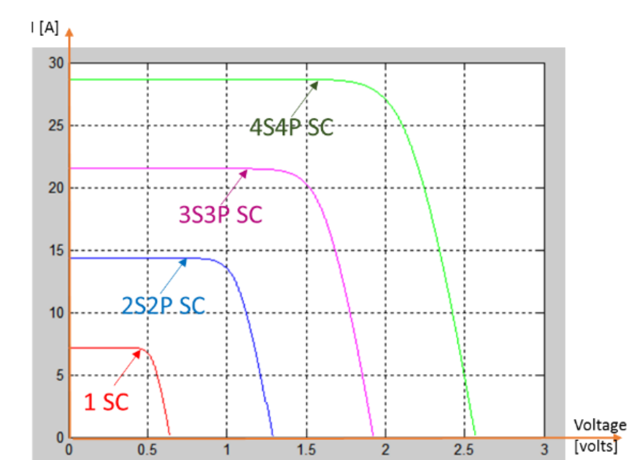

FIGURE 12. I-V curve of series-parallel solar cell.

IV.C Simulation of a transient behaviour

In this section we analyse the capacitor effect of a solar cell that we find in the equivalent circuit from Fig. 2.

An abrupt change in the power of the solar irradiance (change from $700\frac{W}{m^2}$ to $1000\ \frac{W}{m^2}$ in less than 0.0001 seconds) will be used in order to observe how strong is the capacitive behaviour of the system (a high capacitive behaviour means that regardless of the increase in the power of the solar irradiance, the solar cell will try to oppose to the change in voltage as much as possible).

From section I, we obtained the reason why the capacitor is presented, in which this is modelled as a parallel plate capacitor with the equation [9]:

$$C = \frac{\epsilon_r * \epsilon_0 * A}{d}. \tag{16}$$

In which ϵ_r represents the relative permittivity of the material (for silicon is 11.8), ϵ_0 the vacuum permittivity ($8.85 * 10^{-12}\ \frac{F}{m}$), *A* is the area of the solar cell and *d* is the width of the depletion region.

Depending on the technology of the solar cell we are dealing with, the depletion region can have a different value on its width and because of this we will analyse two cases, crystalline silicon solar cells and thin film solar cells.

IV.C.1 Transient behaviour of a crystalline silicon solar cell

The standard silicon substrate thickness is in the range of 200µm-240µm [15] and we will consider a depletion width of 0.35 μm for this simulation (this value was obtained by assuming a built in potential of 0.75 ev, acceptor concentration in the "p" material of 10^{16} per cm^3 and donor concentration in the "n" material of 10^{16} per cm^3) [16]. By using the eq. 16 we obtain a capacitance of 7.13 µF.

Using the conditions explained before in the simulation, we present in the following graph the irradiance, output voltage and output current of this solar cell:

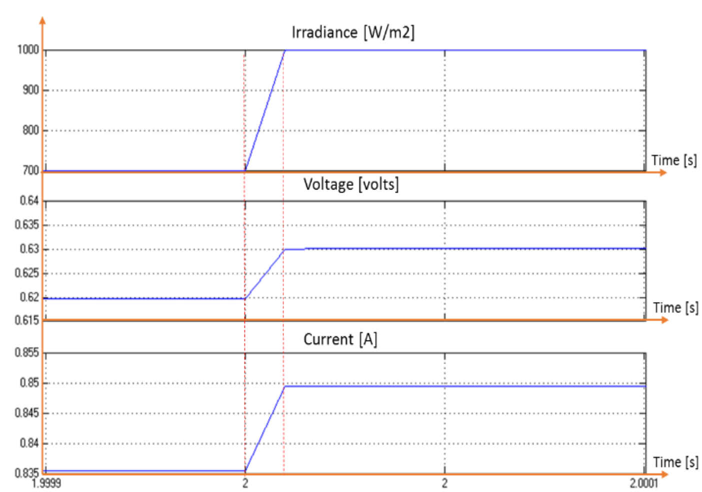

FIGURE 13. Crystalline Si solar cell transient behavior.

From Fig. 13 we can observe that the capacitor of the solar cell did not seem to oppose to the change in voltage because when the power of the solar irradiance is changing linearly, the voltage and the current were changing linearly and the same relation when it was constant.

IV.C.2 Transient behaviour of thin film technologies

Thin film technologies, as we can deduce from its name, is a technology thinner (can be less than 5 µm [17]) compared with the crystalline silicon, this is why for our simulation we will assume a depletion width of 10 nanometers and by using eq. 16 we obtain a capacitance of 249.59 µF. The simulation shows the following results:

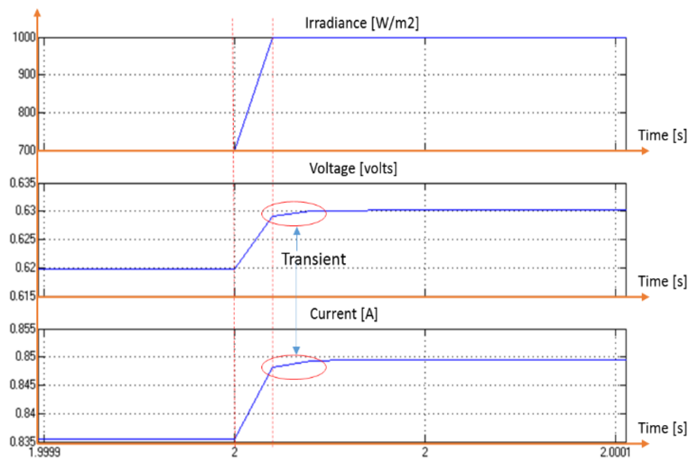

FIGURE 14. Thin film solar cell transient behavior.

In figure 14 we notice that just after the power of the solar irradiance reaches the 1000 $\frac{W}{m^2}$ and stays constant, the output voltage and current are still changing on time (they require more time to stabilize as compared with the crystalline silicone solar cells), this means that the capacitance was high enough in order to appreciate in the simulation the transient behaviour.

V. CONCLUSIONS

1. A solar array has different power points depending on the value of the external resistance, for practical application this resistance has to be fixed in order to always work in the Maximum Power Point so that most of the solar energy is converted into useful electricity.
2. By connecting solar cells in series, a lineal increase in the Voc is obtained and a similar result is occurs by connecting them in parallel with respect of the Isc; we also showed how the photovoltaic current is proportional to the light intensity.
3. If one of the cells connected in series cannot receive the same amount of light, for example an object might cause a shadow on the surface of the cell (known as the shadowing effect), this cell will not be able to absorb the same number of photons compared with the other ones, due to this it will not generate the same current (will be lower), as a result the total current of the system is reduced and also the power, so this is why solar panels should not be installed in places where they could receive some shadows by threes or buildings.
4. If in a parallel connection one of the cells, due to a defect or a different quality level, cannot generate as much as voltage as the other ones, this cell will force the rest to reduce their generated voltage (in order all to have the same voltage as expected from elements connected in parallel) and by this to reduce the all power of the system.
5. The capacitance of the crystalline silicon solar cell is too small in order to show a big response to the abrupt change in the simulated irradiance.
6. Thin film solar cells with such a short depletion width have a higher transient response in the abruptly changes of the solar irradiance than the crystalline silicone solar cells.

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