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## Abstract

Putting quantum mechanics (Q. M) at the reach of high school students is a real challenge. Usually Q.M. is considered so far away from common experiences that many standard textbooks advice not trying any image of the phenomena involved. However, Q. M. emerged from objective experiences that have to do with heat radiation. So, our proposal to help students to grasp the main aspects of Q. M. is to familiarize them with basic experiences with heat radiation and with light emission by heated bodies. We are convinced that the history of the development of Q. M. may be very valuable to understand the basic concepts of the theory. Thus we use a historical approach to present the main concepts of Q. M.

**Keywords:** Quantum mechanics, black body radiation, wavelengths, frequencies.

## Resumen

Poner la mecánica cuántica (M.C.) al alcance de los estudiantes de la escuela secundaria es un verdadero reto. Usualmente, la M.C. se considera tan lejana de las experiencias cotidianas que muchos libros estándar sugieren no hacerse imagen alguna de los fenómenos involucrados. Sin embargo, la M. C. surgió de experiencias objetivas que tienen que ver con la radiación de calor. Por lo tanto, nuestra propuesta para facilitar a los estudiantes la comprensión de los principales aspectos de la M.C., es familiarizarlos con experiencias básicas de la radiación de calor y emisión de luz por cuerpos calientes. Estamos convencidos de que la historia del desarrollo de la M.C puede ser muy valiosa para entender los conceptos básicos de la teoría. Entonces utilizamos un enfoque histórico para presentar los principales conceptos de la M.C.

**Palabras clave:** La mecánica cuántica, la radiación del cuerpo negro, longitudes de onda, frecuencias.

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## I. INTRODUCTION

Q.M. emerged from the study of the interaction matter-radiation. This problem has many aspects that have been explored both, experimentally and theoretically since long ago. We have, for example, the discovery by Newton of dispersion of light through a prism. Refraction is another phenomenon that results from the interaction matter-radiation. From this it can be inferred that colors are associated with different frequencies, or equivalently, different wave lengths. Thus, as previous knowledge, it is required to have the concepts of wave and its main aspects as frequency, wavelength, phase velocity, etc., as well as to have some knowledge of wave phenomena, as diffraction, interference and resonance.

Glowing bodies are particularly interesting, as they emit visible light. Thus stars, burning coal, melting metals, fireworks, etc. are examples of radiation emitters. But we

have also that bodies absorb radiation. We have the experience of feeling the warming sunlight, or the warming felt as we stand near a fire. Then all material bodies at any temperature absorb and emit radiation. Usually there is a typical color, or frequency, of the light emitted by these glowing bodies. This dominant color corresponds to the frequency where more radiation energy is emitted, but the energy is distributed among all frequencies. That is, there is a typical frequency or wavelength at which these bodies radiate, and this frequency depends on the temperature and the chemical composition of the radiating body. Thus when a metal is heated, first becomes reddish, then yellow and then white as the temperature is raised. Blacksmiths had to have this empirical knowledge to know how to forge their metals. Also, the artisans that make fireworks know empirically what substances produce what colors when fireworks become glowing.

These observations lead Bunsen to think that there could be a method of chemical analysis based on the color of glowing substances. When he exposed this idea to Kirchhoff, he obtained a better method of chemical analysis: the spectrometric method. The spectrometer is an instrument developed by Fraunhofer based on the observation by Newton of the dispersion of sunlight through a prism. Fraunhofer observed in the spectrum of the sunlight some dark lines, corresponding to determined frequencies. Kirchhoff identified these lines discovered by Fraunhofer as corresponding to sodium, proving that Comte (founder of the positivistic philosophy) was wrong when he affirmed that to ask for the chemical composition of the stars was not a scientific question, since it was impossible to go to them to corroborate it. Thus the study of the spectrometric lines were one of the roads to quantum mechanics, and the experience of seeing these lines is very important to have a clear idea of how these lines reflect somehow the structure of atoms. The spectrometric method is still an important tool in astrophysics, as well as in any need of identifying a substance, as in criminology. It is interesting that the element helium was identified by the spectrometric method in the sunlight before it was found on the Earth.

## II. KIRCHHOFF LAW

Kirchhoff established a law about the absorption and emission of radiation by matter in thermodynamic equilibrium. The law says that, in thermodynamic equilibrium, the ratio of the power of the radiation emitted by a surface to the fraction of the power absorbed by the surface from the incident radiation, does not depend on the chemical composition of the body whose surface is emitting and absorbing radiation; depends only on the frequency and the temperature. Among other things, this law implies that, to maintain thermodynamic equilibrium, the more radiation absorbs a body the more radiation it emits. This is the reason why radiators in refrigerators and autos are black, while a reflecting sheet may maintain the temperature of a body more time. A simple way of corroborating these phenomena is by means of Leslie's experience: two recipients, one black and other with a reflecting surface are filled with hot water at the same temperature and let them cool. It will be observed that the black recipient cools first than the one with the reflecting surface.

When all radiation falling on the body is absorbed, it is said that its absorptivity or capacity of absorption is one, and then we have a black body. Then the radiation emitted by a black body, according to Kirchhoff's law, must be totally independent of the chemical nature of the body. Then there must be a general law of physics that describes how the radiation energy is distributed among the different frequencies, and the determination of this universal law became a very important problem at the time. This problem is known as the black body problem. Some of the

best physicists of this epoch tried to find this universal function, known as the spectral density of the black body.

## III. THE LIMITS OF CLASSICAL PHYSICS

As a model of a black body we have a cavity with a small hole, whose walls are at a given temperature. The black body is the hole, since all radiation falling on it is absorbed. Thus the radiation emitted by the hole has the quality of the black body radiation, which in thermodynamic equilibrium has the same characteristics as the radiation inside the cavity. Thus the black body radiation is equivalent to the cavity radiation. The cavity radiation is then a thermodynamic system which has volume (the volume of the cavity), temperature (the temperature of the walls of the cavity), and pressure (the radiation pressure deduced from the electromagnetic theory of Maxwell).

As a thermodynamic system, the cavity radiation was treated by the methods of classical thermodynamics by Wien, who could determine the spectral density only to an undetermined function:

$$\rho(\nu, T) = \nu^3 f(\nu/T). \quad (1)$$

Here  $\rho(\nu, T)$  is the spectral density of the cavity radiation, energy per unit volume, per unit frequency.  $\nu$  is the frequency of the radiation, and  $T$  is the absolute temperature.  $f$  is an unknown function. Wien proposed as a conjecture the function

$$f(\nu, T) = \alpha e^{-\beta \frac{\nu}{T}}, \quad (2)$$

with which the spectral density becomes

$$\rho(\nu, T) = \alpha \nu^3 e^{-\beta \frac{\nu}{T}}. \quad (3)$$

This is an empirical law, which was known to adjust well to the experimental data only for large frequency and low temperature. When Planck attacked this problem, there was another empirical law, which stated that at low frequencies and high temperatures the spectral density, the function  $\rho$ , was proportional to temperature and the square of the frequency, that is,

$$\rho(\nu, T) = \alpha \nu^3 \left(1 - e^{-\beta \frac{\nu}{T}}\right)^{-1}, \quad (4)$$

while the empirical law of Wien implied for low frequencies and high temperatures a proportionality to  $\nu^3$ . The law that establishes that the spectral density is proportional to the square of the frequency is equivalent to the Rayleigh-Jeans law, based on the classical principle of

equipartition of energy. This law, however, was proposed until 1905.

What Planck made was to try to find a function that for small frequencies and high temperatures would coincide with what now is known as the Rayleigh-Jeans law, and for high frequencies and low temperatures would coincide with Wien's empirical law. This intermediate function is an interpolation, and there can be many functions that satisfy the above conditions, but Planck used classical thermodynamic arguments that permitted him to find a particular function that resulted to fit all experimental data. See Figures 1, 2 and 3.

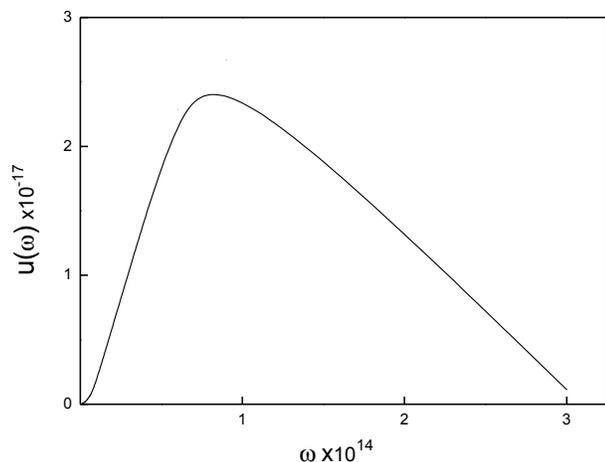


FIGURE 1. Planck spectral density of the cavity radiation vs. frequency.

For Planck, in his own words, this was “a fortunate conjecture”, which let him obtain his famous formula. However, Planck was unsatisfied with *the fact that the interpolation of the two empirical laws is also an empirical law*, even if it adjusts well to the experimental data, as was the case. So he searched for a deeper foundation of his empirically correct formula, and he turned to the atomistic and statistical ideas of Boltzmann, initially rejected by Planck and many other physicists of the time. Since atoms are usually neutral, Planck imagined the walls of the cavity containing radiation as composed by atoms consisting of a positive electrical charge joined by a spring to an equal negative charge, and oscillating, absorbing and emitting radiation, which in thermodynamic equilibrium, would emit on average as much radiation energy as it absorbed. The method of Boltzmann required counting the number of microstates of the atoms compatible with the macroscopic state of the system, that is, with the energy, volume, temperature and pressure of the radiation in the cavity. The number of these microstates is the number of different ways in which the total energy can be distributed among the oscillators representing the atoms. In order to do this counting Planck divided the total energy, shared by oscillators and radiation, into a finite number of identical energy packets. In this way Planck could see his formula from the atomistic and statistical point of view of Boltzmann, and

found that the possible energies of the oscillators had to be of the form  $\epsilon = h\nu$ . A surprise turned out: he obtained the desired formula but when the limit of infinitesimal energy packets was taken, then the Rayleigh-Jeans law was obtained, that is, the classical result. **So Planck was obliged to postulate a quantized exchange of energy between oscillators and cavity radiation. [Only the exchange of energy was discrete!]** This is the new law of nature, the quantum postulate. [It must be remarked that the idea of quantization of energy, and not only its exchange between oscillators and radiation, is not in Planck's work; it will come later, in 1905, with Einstein].

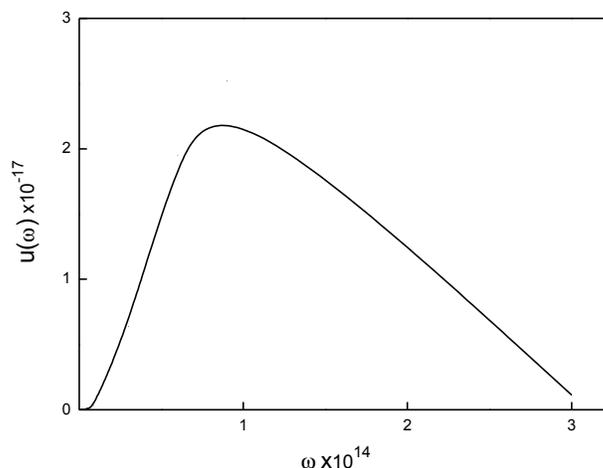
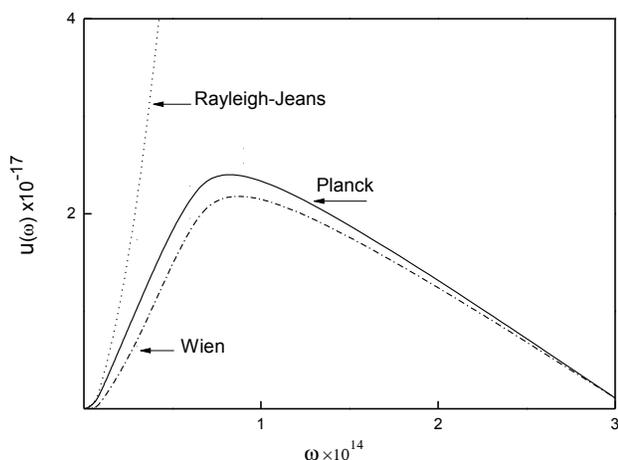


FIGURE 2. Wien spectral density of the cavity radiation vs. frequency.

After some pioneering trials by others, Bohr applied the quantum postulate to the hydrogen atom, obtaining the correct frequencies of its spectrum. His theory of the hydrogen atom gave the clue to the atomic structure and their corresponding spectra. It also gave a new vision of the interaction of matter and radiation. While the classical theory of electromagnetism predicts that a revolving electron around the proton in the hydrogen atom would collapse quickly radiating off all its energy, **Bohr's theory postulated that the electron in the orbits that comply with the quantum postulate does not radiate.** Radiation occurs when the electron jumps from a permitted orbit of higher energy to another permitted orbit of lower energy. Absorption is the opposite process. Bohr's theory also provided theoretical support to the empirical combination rule of Ritz, which states that in the spectra of atoms the observed frequencies obey a rule of differences, and not a rule of multiplication, as in the classical waves that obey the harmonic principle: the frequencies of a macroscopic vibrating body are such that all are multiples of a fundamental frequency. **This difference between classical and quantum frequencies is what lead Heisenberg to the discovery of a new kinematics, in which the physical quantities like position, momentum, and energy, are expressed in terms of matrices.** This kinematics gave rise, in a collaboration of Heisenberg, Born and Jordan, to

matrix mechanics, the first published version of quantum mechanics



**FIGURE 3.** R-J, Planck and Wien spectral density of the cavity radiation vs. frequency.

But there was a second version of quantum mechanics, proposed by Schroedinger in 1926, based on what is called the wave-particle duality. This conception was initiated by Einstein in 1905, when he proposed that radiation itself consisted of energy packets, later known as photons, but it was further developed by de Broglie in 1924. Einstein postulated that electromagnetic waves present aspects of particles in the limit of high frequencies, while de Broglie associated a wave to particles. In this way de Broglie could interpret the stationary electron orbits postulated by Bohr as those orbits in which the associated wave interferes constructively with itself. Schroedinger found the wave equation that the de Broglie waves obey, and solving with this theory the hydrogen atom he found the correct frequencies of the hydrogen spectrum. Later Max Born, based on the fact that in atomic collisions only particles are detected in photographic plates or fluoroscopic screens, proposed the interpretation of these waves in terms of probability: the square of the wave functions, solutions of the Schroedinger equation, is a probability density.

***These wave aspects of particles and its interpretation in terms of probability are the most difficult aspects of Q.M.***

Besides, there are different interpretations of probability. The most accepted interpretation of Q.M. conceives probability as degree of knowledge of some “observer”, and in this way the quantum system and the observer become united, giving rise to some paradoxes. The proponents of this interpretation say that quantum systems constitute an observer-created reality, and what is not observed simply does not exist. Here we find the positivistic philosophy proposed by Compton, which, as we saw, denied the possibility of knowing the chemical composition of stars.

There is, however, other interpretation of probability which refers to a large collection of experiments done

under identical conditions, and the probability gives the fraction of experiments that give a particular result. This is the interpretation of probability used by insurance companies or in quality control in industry, and was proposed by Einstein for the interpretation of quantum phenomena. There are, on the other hand, two paradigmatic phenomena that show these probabilistic wave aspects: the interference patterns that appear when particles, for example electrons, are fired to a double slit with a separation of about one de Broglie wavelength, and the tunnel effect, in which particles behave as waves in the sense that they can go around objects, as in wave diffraction. The operation of transistors and chips is based on this phenomenon. The quantum phenomena are thus interpreted in terms of particles and photons that in a probabilistic and statistical sense present wave-like behavior, as diffraction, interference, and resonance. Our modern electronic technology is based on these quantum phenomena. Understanding these aspects is still the object of controversy, and research. ***As Feynmann once said, nobody understands Q. M. to date.***

#### IV. CONCLUSIONS

Q. M. is based on the quantum postulate discovered by Planck and generalized by Einstein: the exchange of energy between matter and radiation is not continuous, as in classical physics, but occurs in finite packets called quanta of energy, and the energy of radiation is also constituted by finite packets of energy called photons. A manifestation of this quantization of energy is the wave-particle duality, with the wave aspect interpreted in terms of probability. There is still a controversy about what this probability refers to: a single particle, as the orthodox view affirms, or to an ensemble of particles prepared under the same macroscopic conditions, as Einstein proposed. The search for an understanding of quantum phenomena continues.

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