The Special Theory of Relativity

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Abstract

The Special Theory of Relativity has invariants, as the space-time interval, and relative quantities as space and time, or the electric and magnetic fields. "Relative" here means that, a space interval in one reference frame may be a space and time intervals in another. We have mentioned that a zero time interval in a reference frame, can be a finite time interval in other reference frame; that is, two events simultaneous in a reference frame are not simultaneous in other.

Keywords: Relativity, Invariants, Frame.

Resumen

La Teoría Especial de la Relatividad tiene invariantes, como el intervalo de espacio-tiempo, y cantidades relativas como el espacio y el tiempo, o los campos eléctricos y magnéticos. "Relativo" aquí, significa que un intervalo espacial en un marco de referencia puede ser un intervalo de espacio y tiempo en otro. Nosotros hemos mencionado que un intervalo de tiempo cero en un marco de referencia, puede ser un intervalo de tiempo finito en otro marco de referencia; es decir, dos eventos simultáneos en un marco de referencia no son simultáneos en otro.

Palabras clave: Relatividad, Invariantes, Marco.

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I. INTRODUCTION

Since its inception, the special theory of relativity presented "paradoxes" that indicated that its concepts were distant from common sense. The discussion of these paradoxes declined only because the concepts of quantum mechanics were still harder to understand.

In the present work we discuss some aspects that we consider are still the source of misconceptions. We take as examples statements as "space and time are the same entities", "energy and inertia are the same", and "the magnetic field is the electric field seen from another reference frame".

In discussing these examples, we can understand how the relativity theory of Einstein differs from that of Lorentz and Poincaré.

Our proposal for a better understanding of the special theory of relativity is to employ a historical perspective, which gives us a glimpse of the problems that occupied the interest of physicists at a time.

In particular, the concept of relativity of motion is clearly expressed by Galileo, who said that the motions of bodies in a ship which sails at constant velocity are the same as those motions with respect to land.

This relativity of motion implies that the velocity of a body with respect to the ship, v(ship), the velocity of the ship with respect to land, V (land), and the velocity of the body with respect to land v(land), are related by the law of addition of velocities,

$$v(land) = v(ship) + V\,(land)$$

This can be verified by common experiences, and it is hard to imagine experiences where this law does not hold. This concept of relativity can be expressed as "the laws of motion are the same in all inertial frames" Therefore there is not a frame which can be considered at absolute rest, since there are not mechanical experiments that can distinguish such frame.

However, the idea of absolute rest was considered again after Maxwell identified light as electromagnetic waves, since the hypothetical medium that sustain these waves could be regarded at absolute rest.

As is well known, at the end of the 19^{th} century most physicists believed that an electromagnetic medium, the ether, was indispensable for the propagation in space devoid of matter, of electromagnetic waves.

Since Maxwell´s equations are the equations governing such waves, and therefore also apply to optical phenomena, the question arose about the transformation of these equations from an inertial frame of reference to another.

The Maxwell equations acquired its simplest form with respect to the ether, considered at absolute rest.

For understanding relativity theory it is then necessary to know, as previous knowledge, what is an inertial system of reference, or inertial reference frame.

These frames can be thought as three orthogonal straight lines labeled as x, y, z, axes and a clock. It is assumed that the clocks of all inertial frames indicate the same time, that is, there is a universal, absolute time, for all these frames.

These frames most move with a constant velocity, i.e. constant in magnitude and direction, with respect to the farthest stars. Thus they move with constant velocity with respect to each other.

From the experiences of motion in a bus moving with constant velocity the classical law of addition of velocities can be drawn.

With the advent of Maxwell´s electromagnetic theory, which implied the existence of electromagnetic waves, experimentally shown by Hertz, arise the question of the medium which support these waves, as already mentioned.

It was assumed that this medium, the ether, was at absolute rest, and thus emerged the problem of detecting by optical means the motion with respect to the ether. This was the aim of the Michelson-Morley experiment that, as is well known, gave a negative result: no motion with respect to the ether could be detected. In order to "explain" this negative result Lorentz and Fitzgerald introduced the hypothesis of contraction in the direction of motion.

Another problem at that time was the question of the dragging of the ether by a transparent medium in motion.

Fresnel developed a formula which showed a partial dragging effect, and Fizeau obtained some empirical support of this formula by measuring the speed of light in running water. Thus the ether appeared as a very real medium, contrasting with the negative result of the Michelson-Morley experiment.

If Maxwell´s equations acquire its simplest form with respect to the ether, regarded at absolute rest, then it appears the problem of obtaining the form of Maxwell´s equations in inertial frames that move with respect to the ether which is precisely the title of Einstein´s article of 1905: "On the electrodynamics of moving bodies".

II. THE THEORY OF RELATIVITY OF EINSTEIN

Some authors (notably Whittaker) consider that the special theory of relativity was developed by Lorentz and Poincaré, since these authors obtained many of the equations obtained after by Einstein.

So, the question arises, what was the contribution of Einstein, if any? Also, we can inquire about the differences in their viewpoints. To this end we can pay attention to what Einstein exposed several times: He developed a new theory of space and time, this point is clearly expressed by Einstein in a letter to his friend Habitch, thus the theory implied a new kinematics , but the form of the laws of motion do not change.

Einstein based his theory on two postulates: the principle of relativity, which says that the general laws of physics are the same in all inertial frames of reference, and the principle of the constancy of the speed of light in every inertial frame.

From this second postulate Einstein deduced the transformations called by Poincaré "Loretz transformations", though these transformations had been also deduced by Larmor, and Voight, who searched for the transformations under which the wave equation maintain its form.

This last postulate means that the speed of light is independent of the state of motion of the emitter and the receiver. Therefore the Galilean law of addition of speeds of inertial frames must be modified. Also the speed of light results a limit to any other speed, that is, no material body can reach the speed of light.

This means that as the velocity of light is approached, the kinematics implies that it is more difficult, that it is, is required more energy, to change the velocity. This postulate is the most difficult to accept and was impugned by many physicists.

However, this postulate permits Einstein amalgamate the concepts of space and time. From this postulate Einstein deduces the relativity of simultaneity, as he describes in a popular exposition of his theory.

The constancy of the speed of light in any inertial reference frame (S) implies that if a spherical electromagnetic wave is produced when two reference frames, one moving with respect to the other, have their origins coincident at a time which is set to zero, at any other time t in one reference frame (S) and $t´$ in the other (S), in both reference frames the spherical wave moves with speed c. That means

$$x^2 + y^2 + z^2 = ct^2 \qquad \text{(S)}$$

and

$$x'^2 + y'^2 + z'^2 = ct'^2 . \qquad \text{(S')}$$

That is:

$$x^2 + y^2 + z^2 - ct^2 = x'^2 + y'^2 + z'^2 - ct'^2 = \text{Invariant.}$$

This invariant is called the space-time interval. Here the unprimed coordinates and time refer to one reference frame, while the primed ones refer to the other.

Note that the spatial coordinates have the contrary sign that the variable *t*, so that they are not identical and have a different role in the invariant.

It is clear that space and time are not the same entities, but form a kind of mathematical object, space-time, where each part is different, but unified in the invariant.

Einstein then investigates the transformations from one reference frame to other that satisfy the invariant. These transformations are the Lorentz transformations, the same that Lorentz and Poincaré investigated to show the invariance of the Maxwell equations. This implies a new kinematics.

The implications of the new kinematics are surprising: time intervals are dilated and lengths are contracted when compared from different inertial frames in relative motion.

Thus time and space become relative to inertial frames, while in Newtonian mechanics space and time are absolute.

The special theory unifies several physical concepts, as space and time. Other concepts so unified are the electric field and the magnetic field, as already suspected from Faraday´s law of induction. But this unification does not mean that space and time are the same entities, or that the electric and magnetic fields are the same.

What the special theory says is that space and time, or the electric and magnetic fields, are parts of more general conceptual objects, space-time and the electromagnetic field. What parts of these objects are present in a determined inertial frame depend on the state of motion of this frame. It is in this sense that space and time, or electric and magnetic fields, become relative to inertial frames.

After the publication of his theory of relativity, Einstein discovered another consequence of the theory: the equivalence of energy and mass. This consequence is deduced by Einstein considering that the conservation of the center of mass when radiation is exchanged between bodies requires assign mass to radiation. Then he generalizes this to any kind of energy. In this way it is established an equivalence, not an identity, between energy and mass.

We can see that the special theory of relativity has invariants, as the space-time interval, and relative quantities as space and time, or the electric and magnetic fields.

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III.CONCLUSIONS

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