A potentially meaningful teaching unit for the teaching of the concept of field in Physics

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Abstract

This work aims to describe the results of the implementation of a didactical proposal for teaching the concept of field in Physics. A lack of words in the literature is blatant as we could see based on a review of literature that found four other works that focused different goals, with just two aiming directly the teaching the concept of physical field. We propose the *Potentially Meaningful Teaching Units* (PMTU) of Marco Antonio Moreira as a teaching strategy that aims to facilitate Meaningful Learning through planning the instruction in eight steps that embrace premises of important Meaningful Learning Theory. We analyze data based on Greca and Moreira proposal that integrates *Assimilation Schemas* of Vergnaud and *Mental Models* of Johnson and Laird. We found two classes of schemas: a) Schemas for describing interactions without using the concept of field and b) Schemas for describing interactions using the concept of field. We could infer also five classes of Mental Models for framing the concept of field into the Schema of describing interactions: a) Mental Model to describe the ontology of the field in a general way, b) Mental Model to describe the field as an entity that transfers energy of a body to another, c) Mental Model to describe the field as possessing Energy, d) Mental Model to describe the fields as entities that transfer energy by colliding the other fields – Shock conception, e) Mental Model to describe the field based on the idea of interaction of the field with objects. We describe also two classes of Mental Models for representing geometrically the field: a) Mental Model to represent the field existing just outside the sources, b) Mental Model to represent the field existing both inside and outside the sources. It was possible to map four classes of Mental Models to represent mathematically the field: a) Mental Model to represent the field Mathematically without using the concepts of flux and circulation, b) Mental Model to represent the field Mathematically using the concept of flux as a normal projection of the field over a Gaussian Surface and the concept of Circulation as a tangent projection of the field over an Amperian Loop, c) Mental Model to represent the field Mathematically using the concept of flux as an indicator of the direction of the field and the concept of Circulation as an indicator of circular closed field Lines, d) Mental Model to represent the field Mathematically identifying the concept of flux to the concept of field. We could observe the Mental Model construction from prior knowledge-in-action for building internal representation of the concept of field. Great part of the students achieved adequate Meaningful Learning, some of the reached a close point of an adequate Meaningful Learning, and others (a minority) achieved inadequate meaningful learning. Implications are discussed.

Keywords: Meaningful Learning, fields, physics teaching.

Resumen

Este trabajo intenta describir los resultados de la aplicación de una propuesta didáctica para la enseñanza del concepto de campo en Física. Un bajo número de trabajos en la literatura es flagrante como podemos ver, basados en una revisión de la literatura que hicimos y que ha encontrado otros cuatro artículos que trataban diferentes objetivos, de los cuales, solamente dos discutían la enseñanza del concepto de *campo físico*. Nosotros proponemos las Unidades de Enseñanza Potencialmente Significativas o PMTU (por sus siglas en inglés) de Marco Antonio Moreira, como una estrategia didáctica que intenta facilitar el *Aprendizaje Significativo* a través del planeamento de la instrucción en ocho pasos que abarcan importantes premisas de la teoría del Aprendizaje Significativo. Analizamos los datos basados en la propuesta de Greca y Moreira que integra los *Esquemas de Asimilación* de Vergnaud y los *Modelos Mentales* de Johnson y Laird. Encontramos dos clases de Esquemas: a) Esquemas para describir las interaciones sin usar el concepto de campo, y b) Esquemas para describir las interaciones usando el concepto de campo. Ha sido posible inferir también cinco clases de Modelos Mentales para encuadrar el concepto de campo electromagnético el esquema para describir interaciones: a) Modelo Mental para describir la ontología del campo de forma general, b) Modelo Mental para describir el campo como una entidad que transfere energía de un cuerpo a otro, c) Modelo Mental para describir el campo como portador de energía, d) Modelo Mental para representar el campo como una entidad que transfere energía a través de la colisión con otros campos – concepción de choque, e) Modelo Mental para describir el campo como un ente que es responsable por la interacción entre objetos. Describimos también dos clases de Modelos Mentales para representar geométricamente el campo: a) Modelo Mental para representar el campo solamente en el exterior de las fuentes, b) Modelo Mental para representar el campo tanto en el interior como en el exterior de las fuentes. Há sido posible construir cuatro clases de Modelos Mentales para representar matemáticamente el campo: a) Modelo Mental para representar el campo matemáticamente sin usar los conceptos de flujo y de circulación, b) Modelo Mental para representar el campo matemáticamente usando el concepto de la projección normal sobre la superfície gaussiana y el concepto de circulación como la projección tangente del campo sobre una curva amperiana, c) Modelo Mental para representar el campo matemáticamente usando el concepto de flujo como un indicador de la dirección del campo y el concepto de circulación como un indicador de líneas de campo circulares cerradas, d) Modelo Mental para representar el campo matemáticamente identificando los conceptos de flujo. Observamos la construcción de Modelos a partir de conocimientos-en-acción prévios para construir una representación interna del concepto de campo. Gran parte de los estudiantes pudieran lograr un aprendizaje adecuado, algunos llegarán cerca de un aprendizaje científicamente aceptable, y otros (la minoría) lograrán Aprendizaje Significativo inadecuado o incompleto. Las implicaciones de esto son discutidas.

Palabras clave: Aprendizaje Significativo, campos, enseñanza de la física.

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I. INTRODUCTION

It is evident that research in Physics teaching points that a great amount of students don't succeed to learn physics basic concepts [1]. Furió and Guisasola [2] indicate two possible reasons associated to flaws in teaching-learning processes, which are the linear teaching and the abusive use of narrative.

As an attempt of solving this problem Moreira [3] proposes a new teaching methodology based on a set of learning theories due to great researchers in educational psychology as, for example, David Ausubel, Gèrard Vergnaud, Phillip Johnson-Laird, Bob Gowin, Joseph Novak and Lev Vygotsky. This proposal has a general structure that can fit to a large variety of teaching-learning contexts. Moreira calls it Potentially Meaningful Teaching Units (PMTU).

When treating Meaningful Learning, a so treacherous concept because of the myriads of meanings it carries with it, we must present our theoretical position on it. We understand *Meaningful Learning as the phenomelogical product of the non-random and non-literal interaction among prior cognitive structures and new knowledge to be learned* [4]. The concept can be tied both to the verbal-predicative form and to the operative form and to embrace the two forms of knowledge we will approach knowledge as being composed of Schemas (long term memory) and Mental Models (work memory) [5]. Intentionality, in this process, plays a fundamental role, because a deliberate behavior of establishing relations among new knowledge units and prior knowledge in cognitive structure is a necessary condition to achieve Meaningful Learning.

We are focused in this work in the processes of teaching and learning of the concept of field in physics. This concept is ubiquitous and valid in a great number of areas in Physics, from the Classical Physics (gravitational field, for example), to the Modern Physics (guclear field, for example). Authors as, for example, Llancacqueo *et al.* [6] assert that this concept is fundamental for the description and explanation of transport, electromagnetic and gravitational phenomena. To this concept can be subsumed a large number of specific fields, if the ontological differences among physics domains are respected. This status classifies the concept of field as a general concept in Physics.

Besides of this presence in a great number of domains in Physics, the concept of field has adequate and plausible symbolic representation for being taught in the undergraduate level if the student has conceptual basis in Vectorial Calculus.

Bringing back the discussion on ontological domains of validity of a concept (the conception of reality that underlies the concept), for example, the differences of fields associated to waves in classical physics (continuum) and to particles in Quantum Physics (discrete), it's possible to defend the Physics teaching based on an explicit view on nature of Science [7]. Ours conceive that Physics is a Science in which occurs conceptual leaps among research programs that overcome the opponents until the weaker ones (the ones that foresee a fewer number of new facts) starts to perish when compared to the stronger ones. Physics possesses also a set of particular methodologies that fits it in the definition of Science (for a great number of epistemologists), once it entails formulation and test of hypotheses, community judgment of inclusion of a new unit of knowledge in its' content [2]. Taking these ideas into classroom can avoid the linear aspect usually tied, both by students and sometimes by text-books, to scientific knowledge and fits perfectly to the structure of our proposal.

We shall enlighten that important evidences show us difficulties of the students when mastering this concept [8], [6]. Martin and Solbes [8] found evidence that freshmen in physics had epistemological difficulties (they didn't know the concept of field) or ontological difficulties (they understood field as force and didn't know the advantages of conceptualizing reality with fields). Llancacqueo et al. [6] propose a conceptual reference structure for the teaching of the concept of field and analyze the shifts of students' conceptions compared to it. Pocovi and Finley [9] and Krapas and da Silva [10] point a large amount of wrong accounts in textbooks related to the concept of field as, for example, historical/epistemological misuses of the concept [9] and polysemy in the concept of field [10].

The following questions we present are the ones that oriented the research:

- Do Potentially Meaningful Learning Units present evidence of facilitation in mastering/acquisition of Physics knowledge associated to the concept of field?
- How the construction of internal representations, attached to the concept of field, occurs?

To give answers to such questions, we analyzed students' responses to pencil and paper problems (including open problems, textbook problems and concept maps), as long as field notes taken right after the classes. Analyzes were carried out taking into account that students build Mental Models when tackling new problems. These models entail elements from students' prior knowledge in assimilation Schemas. In the same discussion the didactical proposal is empirically put to test. In the following section we describe the methodology used in this study.

II. METHODOLOGY

In this section we will discuss the relevant details about the methodology used in this study. For this purpose, it seems convenient to divide it into two parts: teaching methodology and research methodology.

A. Teaching Methodology – Potentially meaning teaching units

A Potentially Meaningful Teaching Unit is a new methododology proposed by Moreira [3] to make easier Meaningful Learning. It is based in a set of learning theories to which the construtivism underlies. It is a collection of ideas put in a coherent whole. That follows, basically, eight steps which we will briefly discuss. In italics we present how it fits to our proposal before we discuss them in more detail:

1. Definition of the topic to be taught and identification of the features and concepts related to this topic - *the concept of field in Physics*;
2. Create problem-situations (discussion, conceptual map, pencil and paper problems, V diagrams) that lead students to externalize their prior knowledge, correct or not, relevant for meaningful learning – *conceptual problems used with the students in each class;*
3. Propose problem-situations, in an introductory level, taking into account students' prior knowledge for introducing content – *Activity presented in the first class with four problems related to students' prior knowledge and that aimed to investigate their prior knowledge;*
4. Present knowledge to be taught/learned, taking into account progressive differentiation, starting with general knowledge and turning it more specific – *classes consisting in brief explanation in the topic (general explanation entailing general ideas in first place) followed by task resolution in groups;*
5. Present, one more time, the most general features structuring content in a new presentation (in a higher level of conceptualization – more complexity) – *New presentations in which the general features are differentiated and new (and more complex) problems are proposed after the first class.*
6. Present the content as a whole, stating again the most general features in a way that turns possible the visualization of the conceptual threads in the conceptual structure and making, therefore, the integrative reconciliation – *Presenting a review in which all the content can be subsumed.*
7. Evaluation – *Summative (at the end) and Formative (during the process) evaluation.*

For the implementation of the PMTU, we've written a basic text emphasizing the main features, but we do not consider it the only didactic resource. We complement it with articles, class notes or problems from different textbooks. We aim emphasizing produced knowledge instead of teacher's knowledge and research in opposition of narrative. The main objective is to stimulate questioning and critics of students.

Diagnostic evaluation, the first stage of PMTU was executed individually to infer some of students' prior knowledge. We implemented formative evaluation to analyze students' knowledge construction during the instruction. And we implemented individual summative evaluation (we do not present the results of this task in this paper) in the end of the didactical approach to study if learners evidence conceptual evolution, the results of PMTU implementation are considered satisfactory.

The implemented PMTU had six classes (of two hours long each). Each class consisted in an exposition about the content during 40 minutes and a session of pencil and paper problem-solving during one hour and 20 minutes. It was conducted under the following schedule (accordingly to the one presented in the one above):

1. Choice of the concept of field
2. In the **first class** we proposed a set of four problems for the students. The answers were analyzed and so we could infer students' prior knowledge.
3. Every class started with one or two questions for the introduction of the content to be taught. Sometimes as, for example, in the second and fourth classes, we presented a historical approach for some conceptual problems solved by the introduction of the concept to be taught in those classes. In the first class, specially, we focused the

conceptual distinction between action-at-distance and action mediated by fields.

4. After the presentation of the problems, the main concept was introduced. In the **second class** it was the concept of field (in a general way). We discussed the difference between action-at-distance and action mediated by fields, the reality of the field (considering its' role in the propagation of Energy and momentum) and the interaction between particles and fields. Then we introduced and differentiated the concept of field after a problem. In the **third class**, we followed the same model, however using the more specific idea of *Sources of field* (we discussed the difference between sources and Test Charges).
5. In the **fourth class** we "deepened" the ideas presented in the **second** and **third classes**. In this class we discussed in a fair general way the mechanisms of electromagnetic, gravitational, strong and weak interactions.
6. In the **fifth class** we introduced the concepts of *arrow diagram, field lines* and we followed in the explanations of how we represent the fields. In the **sixth class** we introduced the ideas of *Flux* and *Circulation*. We raised complexity by working simultaneously with both conceptual and representational of the field. The heyday of the Potentially Meaningful Teaching Unity (that proceeds a Potentially Meaningful Teaching Unity concerning the concept of Electric field) is introducing mathematical representation of fields (**sixth class**).
7. Summative evaluation after the sixth class
8. Reporting of students' conceptions through Mental Models and Conceptual fields theories.

With the exposition we aimed to stimulate discussion, to guide knowledge acquisition, to facilitate knowledge organization (progressive differentiation and integrative reconciliation), to facilitate Schemas and Mental Models developing. It is a deliberate intervention of a teacher to present to the learner the subject in its final form. It is possible to use concept maps, verbal instruction, computer simulation aided class. We used both verbal instruction and conceptual mapping.

With problem solving we aimed to stimulate recursive and operational knowledge, to facilitate knowledge manipulation, to facilitate students' conscience taking on their knowledge-in-action and to facilitate Schemas and Mental Models developing. It is a deliberate intervention of a teacher to unbalance the student cognitively. It is possible to use manipulation of classic textbook problems, computer modelling, and resolution of open problems.

We shall now discuss briefly how the didactical proposal was implemented, namely, how the subject was structured.

The PMTU focusing the concept of field, in a general way, is presented in six classes. In the first of them, we aimed to study students' prior knowledge through a short activity of four problems. In the second class we introduced the concept of field in a general way and emphasizing its relation with the concept of interaction (conceived as Force in classical physics). In the third class, we discussed the concept of Source Charge of the field and the physical entities attached to the generation of distinct fields in nature. In the fourth class we've worked on the processes of differentiation and detailing of the relation between fields and Interactions, discussing the four interactions conceived by science in nature. In the fifth class, we introduced the representation of field (pictoric), namely, the field lines, arrow diagrams and graphs. In the last class, we introduced the concepts of Flux and Circulation to make easier the understanding of the field equations for the students.

In the activity presented in the first class, we tried to understand how students differentiate between action-at-distance and action mediated by the field as long as the ontology of the concept of field supported by the students. The second question aimed to analyze how students conceive differences and similarities between field in a point of the Space and Force exerted on a body, as long as the ontology of the field just like in the first question. The third question is an attempt of studying the ontological characteristics attributed to fields and field Lines by the students. And the fourth question emphasized the study of the ontological characteristics associated to Energy's localization in the field.

In the second class we introduced the concept of field departing from its relation with the idea of Force, taking the latter as prior knowledge in students' cognitive structure. We divided the exposition in three topics: 1) *fields and Forces* in which we presented ontological differences between action-at-distance and action mediated by the field, as long and we established the advantages of field theories in comparison to action-at-distance theories; 2) *field's reality* in which there is an explicit differentiation of the ideas of field and Force. With this topic we aimed to distinguish action-at-distance and action mediated by the field; 3) *Interaction between Particles and fields* in which we introduced the concept of field's source, the concept of field in a point and the idea that an interaction between a field and particle occurs Energy transfer between fields and Charges/Masses. A set of three problems was presented for the students to solve in groups of four or five people.

In the third class we introduced the concept of field's Source, namely, objects that possess special characteristics that are attached to the generation of fields in space. We divided this exposition in three topics: 1) *Particles create fields that interact with other particles* in which we discuss the creation of a field by a particle that has mass (gravitational) or electric charge (electromagnetic) in Classical Physics; 2) *Source Charges and Test Charges* in which we introduce the concepts of field Source and Test Charge emphasizing that both create fields in the space and the negligible value of the field created by Test Charges; 3) *Superposition Principle* in which we describe the linear character of field theories. In the latter topic we also differentiate between the application of the principle to discrete and to continuum charge distributions.

In the fourth class we raised the depth of the discussion on the concept of field discussed yet in the second class. We divided the discussion in three topics: 1) *Situations involving interactions between particles and fields* in which we discuss where we can find the ideas of Force, Energy/momentum transport and fields related one to the other 2) *Energy transport in the field*, in which we discussed classic and quantum forms of momentum/Energy transfer by the field in a general way, 3) *Fundamental fields in Nature* in which we describe the four fundamental fields known in nature (Gravitational, Electromagnetic, Strong and Weak).

In the fifth class we aimed to discuss how to represent fields pictorially. We divided the discussion in four self-explicative topics: 1) *Representation of the field;* 2) *Representation of the field by field Lines;* 3) *Representation of the field by Arrow Diagrams;* 4) *Representation of the field by graphs*.

In the last class we aimed to discuss how to represent fields mathematically. We divided the discussion in three topics: 1) *Representing the field by equations* in which we discussed advantages and disadvantages of representing the field by equations and how we can use their meanings to infer the form of the field in space, 2) *Flux equation* in which we discuss the intuitive (first approach) to the Flux, and relate to the sources and to superposition through the integral equation of Flux (the projection of the resultant field Vectors in the Gaussian Surface), 3) *Circulation equation* in which we discuss the intuitive idea of Circulation, and relate Circulation to sources through the idea of superposition through the integral equation of Circulation (the projection of the resultant field Vectors in the "Amperian" loop).

In the following session we discuss the research methodology used to collect data and right after that we discuss the theoretical framework used to analyze collected data.

B. Research Methodology

In this section we must discuss some features in the research, for example, the context in which we applied the research and the instruments used for collecting data as long as the kind of evaluation we took along the way we followed.

In this research our basic concern is to analyze students' answers to pencil and paper problems, concept maps and field notes. From this analysis we shall infer some cognitive entities of students participating of the research.

We shall discuss also in which context the data collecting was made. We have worked with 18 undergraduate students during a semester in a Physics subject about electromagnetism. We basically collected data in classroom and this data is associated to the tasks we proposed to students as formative evaluation. The classes occurred three times a week (on Mondays, Wednesdays and Fridays). We must highlight that we emphasized both process and product when we established equal weights to students' grade, once we've balanced formative and summative evaluation. So 50% of the final grade was attributed to formative evaluation (in classroom with proposed tasks) and summative evaluation (three final exams at the end of each unity – there were 3 unities).

To analyze students' tasks to search for evidences of Schemas and Mental Models, we used the content analysis methodology as suggested by Bardin [11]. The author facilitates the improving of interpretation and diminishes uncertainty of inferences made in the "reading". Content Analysis has, therefore, a heuristic function in the process of extracting information from incoming messages in communication and it is an interesting alternative to spontaneous or intuitive interpretation [12].

For Bardin, Content Analysis is a set of technics that aims the inference of the conditions of production of a given knowledge. This search requires indicators that assure certainty to these *inferences* that follow a procedure of logical deduction that allows the link between description and interpretation. We must consider in logical deduction, factors tied to questioning of the causes of statements, and also the possible consequences of being produced [11].

Content Analysis, briefly describing, aims the study of *inferred variables*, that can be sociological, cultural or psychological (in the case of research), relative to communication or to the context of production of the message. Inferred variables in this study are attached to conceptualization, namely, Mental Models and Schemas.

Content Analysis is composed of three stages: pre-analysis, exploration of the material, treatment of results (in which the inferences and interpretations are done). Each stage is composed by others and they result in a cycle of research. The main point here is to emphasize the idea of cycle in which ideas are developed and reorganized all the time. The last stage of the research is the framing of Schemas and Mental Models, the ones we present in the result.

It's important to highlight the context and, thus, the specific content we've worked on. It's also important the discussion on the framework we used to analyze data. Our objective in this research was studying the teaching-learning processes occurring in a Potentially Meaningful Teaching Unit. For characterizing this we shall highlight the concepts we aim to teach accordingly to Vergnaud's theory (Situations to which it is useful and meaningful, Operatory Invariants, and Representations). The concepts we aimed to teach were the ones of: Source Charge, Test Charge, field, Force (which we aimed to differentiate, once students are supposed to have prior ideas on it), Flux and Circulation.

It is notorious that we have to delimitate our inquiry. The first delimitation is the concept of field. We must define the concepts previously mentioned:

- Source Charge: the object that has the property of generating a field in space that depends on the nature of this source (if it has electric charge, mass, for example);

- Test Charge: the particle that suffers the action of the Force on it. A Test Charge is understood as a Source Charge with very small charge value (tiny mass, for example)[1];
- field: the physical entity that is responsible for mediating interaction among objects, it carries momentum/Energy and can be located in space;
- Force: the physical quantity representing interaction between two particles mediated by a field;
- Flux: a physical quantity that represents the projection of a field on the normal direction of an arbitrary oriented (closed or open) surface chosen for its calculation; it composes part of the information about the form of the field;
- Circulation: a physical quantity that represents the projection of a field on the tangent direction of an arbitrary oriented "Amperian" loop; it composes part of the information about the form of the field.

It is obvious that these descriptions are fair general. Further research is being developed to describe the conceptual field of the concept of field to give a more organized structure of these concepts in terms of situations, operatory invariants and representations. We gave the first step towards that and we intent to give a more deep answer in the next works.

After this description, we shall discuss the theoretical position we've used for interpreting data.

III. THEORETICAL FRAMEWORK

In this section we aim to describe Conceptual field Theory and Mental Model theory both to present and justify our "world view" on the problem we're tackling. We must differentiate both theories and tie them when necessary.

A. Vergnaud's conceptual field theory

Conceptual field theory is a theoretical framework that studies cognitive development taking into account not just the interaction between the subject that assimilates an object with it, but that approaches the problem regarding things as interactions between assimilation Schemas (cognitive entities) and Situations (problems to which Schemas are addressed). Vergnaud extends the original Schema's concept reach, introducing the concept of operatory invariant. This new component turned possible to integrate genetic epistemology with learning of specific content units [13].

Vergnaud conceives that cognitive development (and learning) occurs in large periods of time, through the "clash" among Schemas and a great number of formal or informal situations [13]. This approach allowed Vergnaud to perceive specific contents as playing important roles in development and learning processes. This perception motivated him to build an epistemological matrix called Conceptual field that, for him, is a set of situations.

Vergnaud understands a conceptual field as a set of problems and situations whose mastering requires concepts, procedures and representations of different kinds, but strongly related one to the other [14].

To define what a conceptual field is, we shall define what a concept is. For Vergnaud [13], a concept consists of triplet of three sets:

$$C = (S, I, R),$$

where

- S is a set of *situations* (or problems) which give rise to the sense of the concept. This set is referred as the *referent* of the concept;
- I is a set of invariants (objects, properties and relations) that assure the concept to be operational. This set can be understood as the group of invariants that can be recognized and used by individuals for analyzing and mastering situations. This set is referred as the *signified* of the concept.
- R is the set of *representations* (natural language, graphs and diagrams, formal sentences, etc.) that can be used to indicate and represent this invariants and situations as long as the procedures used to tackle them. This concept is referred as the significant of the concept.

After defining the concept as Vergnaud understands, we shall now show the meaning of the cognitive entities that interact with situations involving these concepts, the Schemas. For Piaget and for Vergnaud, knowledge is adaptation, in other words, assimilation and accommodation.

The Schemas, as Vergnaud preaches, are the invariant organization of actions taken by the individual in a certain class of situations [15]. For Vergnaud, the Schema is a functional dynamics totality [16] and not a rigid entity, once action is dependent on a large number of parameters of the situation to which it is addressed [16].

Vergnaud's idea of Schema includes elements of Piaget's, but includes also knowledge-in-action[2] of the

1 The distinction between Test Charge and Source Charge is artificial. The Test Charge is, actually, a Source Charge too, but with a very small (negligeable) field. So it becomes unnecessary to consider the effect of its self-interaction.

2 Vergnaud usually labels Schema's contents as operatory invariants or knowledge-in-action. We use operatory invariants when we talk about scientific concepts, once they are component of a Conceptual field and we use Knoweldge-in-action to describe content present is learners' Schemas. This distinction is artificial, but this way we can make reference to two things of the same nature, however totally different: scientific knowledge and private knowledge.

individual that, most of times, is implicit in action. These Schemas possess four elements:

- *Goals and anticipations*: goals and anticipations of the activity (partial goals in some cases). These cognitive entities allow to the individual the expectation of certain effects or phenomena [16];
- *Rules of action*: compose the generating part of the Schema, because they allow the continuous generation of actions that transform reality (psychologically, the situations). They also allow information storage and the controlling of results of these actions. This allows the guarantee of the success of activity in an evolving context [16].
- *Knowledge-in-action:* implicit or explicit conceptual basis that allows the acquisition of relevant information to infer, from this basis, from information and goals, the most pertinent action rules. They embrace pertinent categories (concepts-in-action) and propositions regarded as true about reality (theorems-in-action) [16];
- *Inferences:* all the activity requires "here and now" calculus that allow the evolution of the stages of thought operations [16].

We shall enlighten the importance of operatory knowledge in teaching-learning processes. The Knowledge-in-action are consistent of implicit (most of times) or explicit (in a smaller frequency) of conceptualization. In teaching, one of the hard problems to solve is that students' knowledge is largely implicit and it can contain a considerable number of epistemological obstacles, because implicit propositions cannot be discussed, in opposition of explicit knowledge [17]. The role of the teacher in Science teaching is changed and he/she becomes the one who proposes potentially meaningful problems through which students' progress in a conceptual field, by mastering these problems [15].

Another aspect to be highlighted and that is discussed in the didactical proposal we present is the one that puts the teacher as an agent of communication of an adequate representational language that aims to transform improper knowledge-in-action in true scientific concepts. This stage is very important for making shorter the difference between action and formalization.

We shall discuss why we planned teaching based in a conceptual field. We present some justification for this idea:

- The conceptual fields define the situations that give rise to concepts to be taught;
- The construction of the conceptual field makes easier the conscience of the teacher of his/her own Knowledge-in-action;
- Building a Conceptual field makes easier choosing the most accessible representations and the most potentially meaningful situations;

One of the highlight points in Vergnaud's theory is that it aims the studying of learning and development processes in long term. As a consequence of this theoretical position, we have relatively stable cognitive entities that, however, allow flexibility in long term memory. Vergnaud doesn't present a mechanism to describe knowledge acquisition in new situations. The authors only states that the subject manipulate prior Schemas, trying to adapt to the new situations, but he just emphasizes trial and error processes. In the other hand, Johnson-Laird presents us a concept that aims to describe ordinary reasoning in short-term or working memory. We can, now, present a model for describing how students tackle new situations that underlies Meaningful Learning, using the concept of Mental Model. In sequence we discuss the Mental Model theory.

B. Johnson-Laird Mental Models Theory

The range of validity of Mental Models theory is the one of explaining the representation process of the external world, its objects, properties and relations (entangled in situations), by human beings, through common reasoning. Johnson-Laird tries to embrace in this problem:

- Explanation of mental representations build by common people;
- Understanding of mental processes underlying ordinary reasoning;
- Representation of meanings of words presented by individuals.

Events like these can be understood not only in cognitive psychology's world view, but also in Physics Teaching. Johnson-Laird bases his explanation using the concept of Mental Model.

In first place we must discuss the difference between the concepts of *external Representation* and *internal Representation*. *Representations can be understood as notations, signs or sets of symbols that re-present an aspect of the external world or imagination when this aspect is absent* [18]. External representations can be analogic (its' structure is similar to the one of the represented world) or symbolic (the relation between the sign and representation is arbitrary) and they are outside individuals' mind [18]. Internal representations are, however, built by people to represent, inside their cognitive system, the external world. The losses occurring in mapping external representation into external representation are notorious. In addition to this, as a mapping (that depends on subjects' perception), there is an infinity of forms for the occurrence of this procedure.

With the concept of Mental Model, Johnson-Laird [19] embraces, at the same time, problems as:

- A possible kind of representation built by human beings;
- Explanations for mental processes underlying ordinary reasoning as, for example, syllogistic reasoning;
- Figural Effect;
- The bias of not achieving a valid conclusion;

- A possible explanation for the representation of Meanings constructed by individuals, taking into account the extension (context) of primitive semantics.

We shall highlight that Mental Models don't have syntactic structure, once they are structural analogs of the state of affairs of the world. These models codify propositions and images in mental language, transforming them in internal representation. Those constructed by propositions are less analogical then the constructed by images or spatial layouts. Mental Models specificity, as states Moreira [20], depends on clarity of information that comes from the world. Mental Models are more specific if they're constructed from coherent information as, for example, unambiguous spatial description. Discourse, in other hand, lots of times, undetermined and it implies difficulty in construction of coherent Mental Models. In these situations, subject is forced to assimilate information propositionally [19].

Mental Models vary in content. They can contain nothing more elements representing individuals and relations among them, as in syllogistic reasoning or can even represent relations among events [20]. Mental Models are structures that share, however, a fundamental characteristic, the one of serving to the function they've been built, namely, predict, explain or control [20].

Moreira [20] highlights yet, recursiveness of Mental Models. A fundamental aspect in the construction of Mental Models is not just in the construction of adequate models for representing different states of affairs, but is also the capacity of criticizing the model to eliminate putative conclusions. It is possible that this aspect could be strongly linked to Meaningful Learning, once it is tied to the condition of modifying the cognitive structure.

As Moreira states [20] that there is a difference between Mental Models (private) and Conceptual Models (precise, complete and external representations of events or objects), that are projected as tools for making easier comprehension or teaching. Mental Models, even though functional, can be confusing, instable and incomplete. These models are limited by prior experience with similar features of situations tackled by the individual, by prior knowledge and reasoning/process capacity.

Conceptual differentiation is associated, thus, to the recursive character of Mental Models. A person can modify their Mental Models until their "functionality" is achieved. This way, interaction between prior knowledge and knowledge in external world (in situations) is given through the mediation of a Mental Model that is constructed (when there is no Schema to assimilate or to interact with the situation). Differentiating this concept consists, thus, in the review and improving of the Mental Model.

In the following section we describe how we can integrate both proposals, the one due to Johnson-Laird and the other due to Vergnaud, to develop the research, both in students' reasoning and teaching planning.

C. Integrating Mental Models and Schemas

Greca and Moreira [5] proposed a theoretical integration between Vergnaud's and Johnson-Laird's frameworks for studying students' conceptions in physics. This proposal consists, basically, in the supposition that when students experience a cognitive disequilibrium (when tackling a situation for which the student doesn't have Schemas for solving it) the individual would build a functional model of the situation that tries to solve in his/her working memory. After the mastering of the situation, the model would be "thrown away" and the tackling with other situations would induce the students to build new Mental Models that would become stable in the long term memory when those students constantly deal with these situations.

According to Greca and Moreira [5], this Model of Learning could explain the nature of new Schemas in long term memory, the processes of accommodation arising from cognitive disequilibrium that occur when the student can master the situation aided by someone more experienced and fast obliteration tied to rote learning (associated to random and literal assimilation of new knowledge – when the person doesn't build a Mental Model to tackle a situation). The Mental Models making is an evidence of Meaningful Learning and is guided, in parts, by operatory knowledge for the search and selection of information. Therefore, the role of a Mental Model is similar of the Schema's role, but it operates in the working memory and, thus it's much more changeable.

In the presentation of findings we interpret students' cognition through their cognitive action. We present some Schemas and Mental Models labeled with the action they mediate. We think it is important to express that these Schemas and Mental Models are situation-dependent and that a class of Schemas/Mental Models are addressed to a certain class of problems. Working this way we shall show that the Schemas and, even more, the Mental Models are dynamical totalities [16] that are built or used to tackle situations. This choice allows us a process analysis instead of a product analysis, something that is more closely related to our theoretical framework and that can contribute in a different way with Physics Education research.

We present below two pictures that make clearer the mechanism of conceptualization we considered for interpreting data: if the student is tackling a situation that can be mastered with its' prior knowledge (*Schemas*), then the student at first recognizes a goal for the activity (consciously or unconsciously[3]), when learners recognize a goal, then the search for knowledge-in-action to solve the activity is started. To give an answer, the students must infer it and this process of inference is generated by action rules.

[3] This discussion is a topic of research we will describe further. By now we are studying study's conceptualization and putting the level of implicitness as a second-order factor.

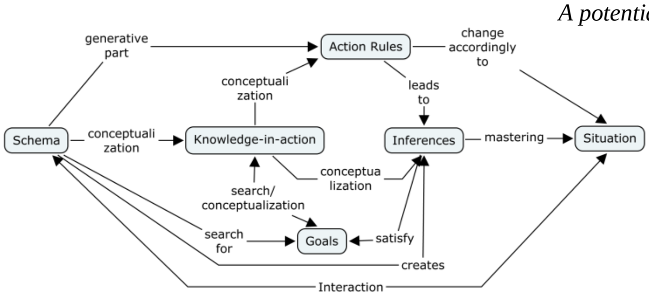

FIGURE 1. A diagram for the presentation of the mechanism of conceptualization using Schemas.

In the case of a new situation, the process is similar, but goals and recursive operations are more volatile and can change drastically in the learning process. The main difference is that prior knowledge is borrowed from the Schemas for conceptualization.

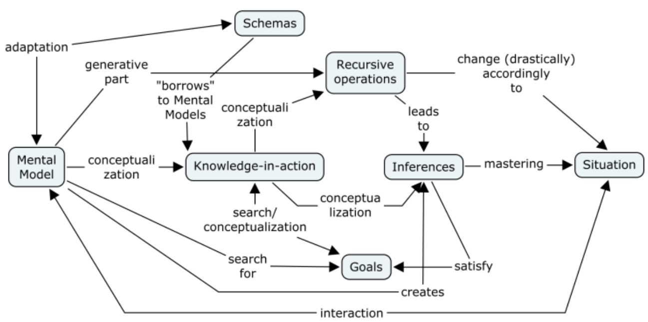

FIGURE 2. A diagram for the presentation of the mechanism of conceptualization using Mental Models.

IV. FINDINGS

We'll present below the findings following the research questions. We expose our reasons to adopt the following order:

1) How do representation construction occurs for the concept of fields during the PMTU?
2) Can we present evidence of facilitation in the mastering of the concept of field in a PMTU?
3) What is the explanatory capacity of Greca and Moreira [18] proposal that integrates Mental Models and Assimilation Schemas?

The ***first question***, aims to understand representations construction of the concept of field in a PMTU and it drives us to the adopted inquiry mechanism. We worked on the data analysis based on the theories of Gèrard Vergnaud and Phillip Johnson-Laird. Both authors define distinct, but complementary, forms of *internal representation* of the external world. While Vergnaud emphasizes representations associated to long-term memory actions, Johnson-Laird discusses the problem of representation tied to working memory actions.

Following the idea that our study can be identified as a concept representation (built by learners) study, we shall take into account that this representation is merged on physics content, correct or not, once it is private. These primary components are foundational to conceptualization and are known as knowledge-in-action. The representation of these aspects requires fundamentally reference to some problematic aspects of "reality", namely, situations. Thus, out fundamental task is the study of cognitive conducts patterns presented by students when they tackle problem-situations which they shall master. It turns clear, therefore, the necessity of working with a *Conceptual field* of the concept of *field* in Physics.

$$C = (S, I, R),$$

where S is the set of *situations* that make the concept of field useful and meaningful, in other words, the situations that make sense when we use the concept of field. I are the *operatory invariants* attached to the concept of field (scientifically accepted, because here we emphasize thought operations proposed by scientists). R is the *external representation* of the concept of field.

When we say *internal representation*, we refer to a private representation built internally by subjects and that can eventually be put out their cognition. These representations are mostly implicit in action, but we can express them through assimilation Schemas and Mental Models that process them when leaners tackle a situation. We shall distinguish these *internal representations* from *external representations*, which are referential forms of understanding the concepts.

Internally, people represent both operatory invariants and situations and, because of that, we draw the discussion on the interaction between Schema-Situation or between (Mental) Model-Situation. Schema brings with it an internal representation that is merged in conceptualization. Schema also adapts to situation accordingly to its Knowledge-in-action. The Schema, thus, represents actions that are "drenched" with conceptualization, represents the situations learners have to tackle and adapts to situations once it is a dynamical totality, in other words, it is changeable.

When we think in working memory, the discussion changes, but it remains almost in the same path. We draw the discussion between (Mental) Model-Situation. Mental Model is an internal representation that brings with it the conceptualization present in prior Schemas and adapts itself to situations in a more flexible way than Schemas. Mental Models allow inference making and the production of objectives to certain activities. The main difference in Mental Models and Schemas are the Recursive Operations (processing) that rebuild the model when there is inconsistency.

The major problem of these recursive operations is that they're implicit as long as the action rules, however, action rules are flexible when we change parameters of situations or when we change the situations slightly. Action rules cannot adapt a Schema to a situation in which there is not knowledge-in-action to solve it. Thus, great part of our work was the rebuilding of steps given by students when they mastered situations. We tried to identify fundamental gaps in students' (cognitive) actions to understand how they built or worked with Mental Models or Assimilation Schemas.

The ***second question*** is nothing more than a time organization of the first one. With it we shall analyze the development of the part of students' cognitive structure tied to the concept of field when they participated in a PMTU on the concept of field. This kind of analysis is important, because it allows us to evidence if PMTU facilitate Meaningful Learning or not. We seek to understand how students develop cognitive structures through Schemas improvement (for known situations) and the construction of Mental Models (for new situations). We're also interested in the construction of Mental Models for known situations that require the introduction of a new concept as, for example, the framing of the concept of field in the explanation of interactions to explain the *ontological shift* between action at distance and action mediated by fields.

For some situations students possess the Schema of *explanation of interaction between two objects* that is based in the interpretation of action-at-distance. For understand situations accordingly to field Theory the student must not develop a new Mental Model for interactions or abandon completely its Schema, but he/she needs to build a Mental Model for the concept of field, that fits to the Schema of describing interactions. This model has to make possible the adaptation to the new Schema for a given class of situations.

We, therefore, focused our study in: Schemas and in its dynamics; the construction of Mental Models; the adaptation of Schemas from the assimilation and accommodation of Mental Model associated to actions tied to a concept learned by the student.

We shall enlighten that the three kinds of conduct embrace the concept of Meaningful Learning. The slight variation of the Schema is associated to prior situations mastered by the students. The relatively abrupt change occurring from assimilation and accommodation of a Mental Model to the Schema demonstrates the tendency of adaptation of the student to a different view for explaining known situations in a new way. The radical construction of Mental Models evidences high capacity of adaptation to new situations.

We divided the situations in two great groups, namely, the situations of *description of interactions between objects mediated by fields* and the situations of *field representation (geometrical/mathematical)***.**

The first class embrace any situations for which it is necessary: *identifying of interacting objects*, *identifying of the interaction*, *using the law of interaction*, and *description of objects' dynamics.* We supposed that this class of situation is partially known for the students, once they learned different kinds of interactions between bodies in prior Physics courses they've worked on (Mechanics, Thermodynamics, Gravitation, Wave Mechanics, Fluid Mechanics). One of the interactions studied by them is the Gravitational one that was used for us as a starting point, once it is associated to students' prior knowledge, and that facilitated the introduction of the concept of Gravitational field.

The second class of situations embrace the problems in which students have to: *identify sources of field; establish the symmetry relations in charge/mass distribution; attach a vector to each point of the space; relate the draw of arrows and the sketch of the field that respects the symmetry of charge/mass distribution; show coherence between field's sketch and Flux Law (Gauss's law); show coherence between field's sketch and Circulation Law (Ampère's law).* We supposed that this class of situations was completely new for the students, once they never learned how to the represent fields (in Fluid Mechanics, professors almost never discuss velocity as a Vector field). When we introduced the concept of field, we observed a shorter rupture between prior knowledge and new knowledge, once we emphasized at first, conceptualization and then we discussed representation.

A Schema or a Mental Model is addressed to a situation, so we dubbed student's Schemas with a very general "label" and then we distinguished among them. There are very similar Schemas and very distinct ones. Some of the Schemas approach features of reality[4] using one set of invariants and rules of action, while others use another set of invariants and rules of action, but both were described accordingly to scientifically accepted knowledge. Others used alternative knowledge, that were built from Meaningful Learning, but that was incorrect from science's point of view.

The ***third question*** brings elements of the first and the second questions in its' answer. If we aim to analyze the strength of Greca and Moreira [18] proposal, we shall study if it presents clearly the representations that learners build and if it fits to the description of their long-term memory and working memory.

To answer the *first question*, we will draw the description of the Schemas and Mental Models found in the research. We describe the Schemas as an *invariant organization of the action for a given class of situations* and label the steps taken by the students in conceptualization. We them distinguish among them, once there are parts in which they radically distinguish either epistemologically or ontologically. This is sufficient to describe how students internally represent knowledge. So the following section will describe these Schemas and Mental Models found.

To answer the *second question*, we will draw the evolution with time of the number of students that adhere to a Schema or build a Mental Model for mastering a

4 The reader might be shocked with the expression "reality", because it's usually triggered to a meaning of objectiveness, independence of an observed and attached to a ingenuous realism. In fact, we used the expression "reality" as a synonym for problems. For Vergnaud, the situations (referents) are, psychologically, the reality, while the operatory invariants (signified) and the representation (signified) are, psychologically, the representation. So we do distinguish between both definitions of reality.

situation. We show how the frequency of students, adopting a Schema, increase or decrease along time. This is necessary for us to identify if PMTU facilitates learning of accepted scientific knowledge. The section following the next will describe this "time-evolution".

To answer the *third question*, we shall analyze both sections and analyze the number of Models in transition to Schemas. It's a complicated answer to give, once we don't have in literature a definite number to guarantee that a Model has turned into a Schema. In the next studies we've been carrying out, we shall answer to this question with greater clarity. For now we will describe the construction of Mental Models and interpret some of them as a mechanism used to understand situations and that are assimilated by a well-established Schema and others as a new construction that will transform into a new Schema.

In the first section below, we present both Schemas and Mental Models developed by students. We distinguish them in four classes: *Schemas for describing interactions between/among bodies (S.I.), Mental Models for framing the concept of field in the Schemas for explaining interactions (M.F.), Mental Models for representing the field geometrically (M.G.) and Mental Models for representing the field mathematically (M.M.).*

In the second section below, we present students conceptions during time. That's important to see how conceptualization is carried out.

A. Description of students Schemas and Mental Models

We present below the Schemas students used to solve the situations presented to them right after the explanation made by the professor. We shall expose a convention: *the theorems-in-action will be stated in italic letters and the concepts-in-action in italic and bold letters*. We also put in bold letters the "**if → then**" of Action rules and Recursive Operations.

Schemas used for describing interactions between/among bodies (S.I.)

This Schema is related to the description of interactions occurring among physical objects, however, it is excessively general and, based in the differences in students' conceptualizations (knowledge-in-action, action rules, inferences, and goals), we could divide this Schema in other two. These Schemas are very different, but both embrace stages of an invariant conduct organization. These stages are presented below:

a. Identification of interacting objects;
b. Identification of the interaction occurring among objects;
c. Application of the interaction law;
d. Description of objects' dynamics.

We observe conducts that are organized in an invariant form: for describing physical objects' dynamics, they need to know the physical law that governs interactions, the interaction occurring and the interacting objects. The students, therefore, *identify relevant objects* for studying them, then *identify the interaction among them*, so they *apply the interaction law for studying objects' dynamics* and finally, *describe how time evolution of the system occurs.*

The third step is epistemologically and ontologically the most important to us, because it is the one that distinguishes the two Schemas. Following this idea we describe both them based in students' conceptualization on the concept of field. We follow presenting both Schemas for *describing interactions among bodies* found in this study:

- To describe interactions without including the concept of field in conceptualization (S.I.1.)

This Schema presents in its' ingredients the *concepts-in-action* of *Force, Potential Energy, Interaction* and *Objects.* It allows students explain situations including *static fields* (the classical electrostatic, gravitational and magnetostatic fields), because in these situations we don't have interaction delay, so everything can be described with Coulomb, Newton or Biot-Savart laws that includes just the concept of Force in the description.

We included in this Schema all explanations that approached the concept of field as a simple mathematical instrument. Students use the concept of Force, once the concept of field is superfluous because of its status of mathematical abstraction. Martin and Solbes [8] found similar results for students' conceptions, but we go a step further once we study this Schema under the light of Greca and Moreira [5] proposal.

We shall also stress that this Schema is an *epistemological obstacle* as stressed by Bachelard [20] for the mastering of situations that involve electromagnetic induction or electromagnetic waves propagation. Therefore, this Schema can be understood as an epistemological obstacle for the mastering of the concept of field in Physics, too. To overcome this difficulty is necessary the construction of a Mental Model for the concept of field.

Example:

Student G:

Goals: to identify objects/ identify interactions/ use interactions law/ describe dynamics;

Knowledge-in-action: ***field*** *is a mathematical artifice* ("The field is a representation of interactions" [1st task – 1st question])/ ***fields*** *and* ***Forces*** *are the same thing* ("The field fills the vaccum/ The vaccum is hollow of matter, but not of interactions" [1st task – 1st question] – so fields and interactions are the same entity)./ $F = \frac{GmM}{r^2}$ *is the gravitational Force exerted on a mass* m *by the mass* M. ("$F_G = ma$/ $\frac{GMm}{r^2} = ma$/ $a = \frac{GM}{r^2}$" [1st task – 2nd question])

Action rules: **If** field and Force are the same thing, **then** the field is superfluous and must be a mathematical artifice./ **If** $F = ma$ and $F = \frac{GmM}{r^2}$, **then** $a = \frac{GM}{r^2}$ (where F is a Force exerted by the earth on an object and a is the object's acceleration).

Inferences: The vaccum is not hollow in matter, but it is filled with interactions./$a = \frac{GM}{r^2}$. (Every object falls with the same acceleration in the earth's surface when we neglect air resistance).

- To describe interactions including the concept of field (S.I.2.);

This Schema has in its composition the following concepts-in-action: *Force*, *Potential Energy*, *Interaction*, *Object* and *field*. It allows students explain the situations involving *Static fields*, as long as the S.I.1. Schema. We include in this Schema all the explanations given using the concept of field. The two major possibilities for this Schema are: the long-term conduct of explaining the interactions using the concept of field (that would be prior knowledge) or attaching a Mental Model for reframing the concept of field in the Schema of interactions (adaptation of a Schema to a situation that has to be interpreted in a different way). So, there is the difference between the third step in organization of the action presented before: the use of the concept of field.

Example:

Student C:

Goals: to identify objects/ identify interactions/ use interactions law/ to identify the field/ to describe dynamics

Knowledge in-action: $\vec{\boldsymbol{g}} = \vec{\boldsymbol{g}}(m_f, \vec{r})$ ("Force of Gravity attracts to everybody with the same gravitational acceleration [$\vec{g}$]. The action of gravity [gravitational field] doesn't depend on the [test] mass" [1st task – 2nd question])/ ***fields*** *exist outside the* ***objects*** ("…the object is dropped down in earth's surface and interacts with it because of the **gravity**. This interaction exists also outside the earth…" [1st task – 4th question)/ ***fields*** *are not substances* ("fields are not substances" [1st task – 1st question])/ *Two* ***objects*** *with the same characteristic interact.*

Action rules: **If** two massive objects interact, **then** they do it through a mediator that exists outside the objects/ **If** the field exert a Force in an object, **then** it is real, even though it doesn't depend on the [test mass].

Inferences: field is real and mediates an interaction ("…we know that [the field] exists and we can observe its interactions, for example, the interactions between masses (gravitational field)" [1st task – 4th question.])/ The field doesn't depend on the test mass ("The action of gravity is the same independently of the [test] mass of the objects"[1st task – 2nd question]).

- Doesn´t present Schemas (S.I.3.):

Students simply do not conceptualize or present vague and disconnected words when trying to master a situation.

Students' Mental Models for framing the concept of field in the explanation of the interaction (M.F.)

We presented the Mental Models constructed by the students for framing the concept of field in the explanation of interactions. Once a great majority of them doesn´t have a prior Schema for the concept of field, they must adapt this Schema constructing a Mental Model. These models differ among them, some describe in a superficial manner field's ontology as, for example, the model M.F.1 (*To describe the ontology of the field in a general way*). Others describe with more detail ontological characteristics attached to the concept of field as the model M.F.2. (*Describe the field as an entity that transfers Energy of a body to another)*, M.F.3. (*to describe the field as possessing Energy*), M.F.4. (*to describe the fields as entities that transfer Energy by colliding the other fields – shock conception)* and M.F.5. (*to describe the field as a mediator of an interaction, as an entity that possesses Energy and that is responsible for applying a Force or changing Energy/Momentum with an object*). Although they look like so different, they share a set of knowledge-in-action presented below:

a. *The* ***field*** *is a function of the* ***distance*** *of the* ***Source Charge*** *to a point in* ***space****;*
b. *The* ***field*** *is a function of the* ***Source Charge****;*
c. *The* ***Force*** *is a function of* ***source*** *and* ***Test Charges****;*
d. *The* ***Force*** *is a function of the* ***distance*** *between* ***Test Charge*** *and* ***Source Charge****;*
e. ***field*** *is attached to* ***Energy****;*
f. ***field*** *mediates an* ***interaction****;*
g. ***Source Charge*** *creates* ***field****;*
h. *The* ***superposition principle*** *is valid for the resultant* ***field*** *generated in the space by more than one* ***Source Charge****.*

We shall stress that we numbered the theorems-in-action and we put in bold letters the concepts-in-action. There is a dialectic relation between them as stresses Vergnaud [16]. We follow characterizing each Mental Model.

- To describe the ontology of the field in a general way (M.F.1.)

This Mental Model doesn't turn explicit or evidences signs of conceptualization on the *mechanism of interaction of the field with particles*. All the very general discourses including the concept of field as an entity existing in space were framed in this model.

Example:

Student L:

Goals: to frame the concept of field in the Schema to describe interactions among particles without describing field's ontology

Knowledge-in-action: *The* ***Energy*** *of a* ***physical system*** *is associated to the field and to* ***objects'*** *position in* ***space*** ("[The Energy] is a relation between bodies, therefore, it is associated to the system, to the position of the bodies and to the Gravitational field created by them"[2nd task – 1st question])/ ***Interactions*** *and* ***fields*** *propagate in space* ("There aren't instant interactions in the universe, at least someone has discovered one and I don't know"[2nd task – 2nd question])/ ***field*** *mediates the* ***interactions****, but doesn't depend on it* ("[If all the test masses of the universe disappear] the field [of the Earth] would remain once it doesn't depend on the interaction among bodies, even though it is responsible for it"[2nd task – 3rd question]).

Recursive operations: **If** Energy is a relation between objects, **then** it is associated to the system. **If** it is associated to the system (positions), **then** it must depend on the position of the objects and to the field generated by them./ **If** there aren't instant interactions, the sun needs certain amount of time to interact with earth. **If** field mediates interaction, but doesn't depend on it, **then** it must be independent of the Force.
Inferences: Energy is not located in the objects, neither in the space [Energy is relational] ("Energy is a relation between bodies"[2^{nd} task – 1^{st} question])/Light is associated to an electromagnetic interaction ("[The actual sunlight] is the sunlight emitted by the sun eight minutes ago"[2^{nd} task – 2^{nd} question])./fields are different of Forces ("[If all the test masses of the universe disappear] the Force wouldn't exist… the field would remain"[2^{nd} task – 3^{rd} question])

- Describe the field as an entity that transfers Energy of a body to another (M.F.2.).

This Mental Model presents the field as an entity that exists and drives the Energy from one body to another. It understands the field existing in the region outside the sources and it is just a support for Energy transfer.

Example:

Student B
Goals: to frame the concept of field in the Schema to describe interactions among particles describing the field as a support for Energy transfer.
Knowledge-in-action: *The **field** transfers **Energy** and **momentum** among **objects*** ("Gravitational Potential Energy is in the Bodies, because the field just transports energy"[2^{nd} task- 1^{st} question])/ $d = vt$ is the **distance** elapsed by an electromagnetic wave packet in the space, where v is the velocity of propagation of the wave ("…light has a limited velocity. A star that is far away from the earth doesn't exist anymore, but we still see its light…"[2^{nd} task – 2^{st} question])/ $\vec{F} = \vec{F}(m_p, m_f, \vec{r}_{pf})$ and $\vec{g} = \vec{g}$ ("[If the objects in universe, except the earth, have vanished] it [the force] would vanish, because bodies exert forces mutually… the field would remain, because it belongs to the earth…"[2^{nd} task – 3^{rd} question]).
Recursive operations: **If** the field transfers Energy and momentum and the field is located in the space, **then** the Energy is located in the objects that change Energy mediated by the field./ **If** an electromagnetic wavefront has velocity v, **then** it will take a time interval $t = \frac{d}{v}$ to elapse the distance d./ **If** $\vec{F} = \vec{F}(m_p, m_f, \vec{r}_{pf})$, $\vec{g} = \vec{g}(m_f, \vec{r}_{of})$ and doesn't exist m_p, **then** $\vec{F}$ doesn't exist and $\vec{g}$ still keeps existing.
Inferences: The Energy is located in the Bodies but is transported by the fields ("The energy is in the field, because the field just transports energy…'[2^{nd} task – 1^{st} questions])/ The light seen in the earth and coming from the sun was produced eight minutes ago ("[The sun is the sun emitted eight minutes ago?] Yes"[2^{nd} task – 2^{nd} question])/ The Gravitational Force wouldn't exist, but the Gravitational field would if there is no test mass ("[the gravitational field] would stop existing… the Gravitational field would remain"[2^{nd} task – 3^{rd} question]).

- To describe the field as possessing Energy (M.F.3.).

This Mental Model conceives the Energy located in the field and, therefore, in the space. The Energy is tied to the field and is being propagated in space for mediating an interaction.

Example:

Student H
Goals: to frame the concept of field in the Schema for describing interactions among particles describing the field as possessing the Energy of the system.
Knowledge-in-action: ***Objects** interact with others at distance* ("…interactions occur at distance…"[2^{nd} task – 1^{st} question]), *The **fields** are real entities once they have **Energy** and **momentum*** ("[Energy is located in the field] thinking in the fact that interactions occur at distance, Energy must be in the fields. They possess Energy"[2^{nd} task – 1^{st} question]) */ **Forces** are different of **fields*** ("neglecting gravitational effects that maintain the earth spherical and thinking about it as an uniform mass, doesn't make sense talking about gravitational Force, because I think about Force as an interaction and, without something to interact, Force loses its sense "[2^{nd} task – 3^{rd} question]).
Recursive operations: **If** there is a Force, **then** there must be a mediator/ **If** a particle interacts with another one, **then** this is not made directly/ **If** Force varies, **then** this variation is done by the mediator (field).
Inferences: The field is real ("Energy is localized in the field"[2^{nd} task – 1^{st} question]/ "The field would continue existing because its source remains there [2^{nd} task – 3^{rd} question]).

- To describe the fields as entities that transfer Energy by colliding the other fields – shock conception (M.F.4.).

This model conceives the Energy as located in the field, but it differs from M.F.1. and M.F.3. because these fields collide and this collision gives rise to interactions between the bodies that created the field.
Example:

Student P:
Goals: to frame the concept of field in the Schema to describe interactions among particles describing the fields as colliders that mediate an interaction
Knowledge-in-action: $h = gt^2/2$ ("$h = gt^2/2$"[3^{rd} task – 1^{st} question])./ $g = \frac{GM_T}{(r+h)^2}$ ("$g = \frac{GM_T}{(r+h)^2}$"[3^{rd} task – 1^{st} question])/ m_p is the **test mass**, a very tiny **mass** ("Test

mass is the eraser's mass"[3rd task – 2nd question])/ m_f is the **source mass**, a very huge **mass** ("As a source mass, we use the Earth's mass." [3rd task – 2nd question])./ ***field transports Energy*** ("The energy of the field generated by the Earth affects the field generated by the eraser modifying eraser's angular momentum, making it fall down."[3rd task – 2nd question])./ $g_r = \sum g_j$ (""[3rd task – 3rd question]).

Recursive operations: **If** $h = gt^2/2$, **then** height and gravity acceleration (depending on the earth's mass) are relevant for modelling the object's fall./ **If** one of two objects has a very tiny mass, **then** it must be the test mass./ **If** one of two objects has a very huge mass, **then** it must be a source mass./ **If** two objects interact, **then** their fields collide./ **If** g_E depends on M_E and g_M depends on M_M, **then** as long as $M_E \gg M_M$, $g_E \gg g_M$.

Inferences: Gravity's acceleration and earth's mass are relevant features for modelling the free fall of an object in earth's surface ("We've used as relevant elements: the Earth's mass and constant gravity"[3rd task – 1st question])/ Earth's mass is the source mass and the object's mass is the test mass ("As source mass we used Earth's mass. As test mass we used the eraser's mass"[3rd task – 2nd question])/ The Energy of the Earth's Gravitational field affects the Object's field and, therefore, its' momentum ("The energy of the field generated by the Earth affects the field generated by the eraser modifying eraser's angular momentum, making it fall down."[3rd task – 2nd question])/ The acceleration of free fall of a body in earth taking into account the influence of the moon would be negligible ("The intensity of the field depends on distance, in earth's surface it's [earth's field] much more intense than the field generated by the Moon"[3rd task – 3rd question]).

- To describe the field based on the idea of interaction of the field with Objects (M.F.5.).

This model understands the Energy as located in the field, but it differs from M.F.1, M.F.3. and M.F.4. because it conceives the field created in the space by Source Charges that create this field for transferring Energy, as Kinect Energy, for Test Charges through the application of a Force (variation of Momentum with time) or through work (variation of Kinect Energy with position). It is the most abstract model presented by students.

Example:

Student D:

Goals: to frame the concept of field in the Schema to describe interactions among particles describing the fields as a mediator of a Force or work exerted on a particle.

Knowledge-in-action: $\vec{g} = \vec{g}(m, \rho)$ ("Earth's data – radius and mass – for determining free fall acceleration."[3rd task – 1st question])/ *Both **objects** and **fields** possess **Energy*** ("The field does work on the eraser transferring energy and momentum to it"[3rd task – 2nd question)/ $F_R = \sum F_j$ ("I would add another Force to the sum of Forces – or resultant force"[3rd task – 2nd question]).

Recursive operations: **If** one needs to describe the field, **then** it's necessary to know its' sources (mass and how it is distributed)/ **If** the body transfers Energy, **the** then it transfers it to the body it interacts with. / **If** an object is to be considered as a secondary source mass, **then** we need to analyze the influence of its field on the test mass, taking into account the Force exerted by this field on the object.

Inferences: The shape of the Earth and its mass are relevant for computing $\vec{g}$ (""Earth's data – radius and mass – for determining free fall acceleration."[3rd task – 1st question])/ The field transfers Energy to an object through exertion of work or Force ("The field does work on the eraser for transferring energy and momentum to it"[3rd task – 2nd question])/ To consider Moon's Gravitational field in an interaction with an object that interacts with Earth's Gravitational field, we must analyze the intensity of the Force exerted by Moon's field on the object ("I would put more Force to the sum of Forces – or resultant force –, I would recalculate the acceleration value of free fall acceleration. However, the difference is so small that wouldn't be easily measured, a thing that makes Gravitational influence of the moon negligeable. " [3rd task – 3rd question]).

We describe in the sequence, the Mental Models of geometrical representation of the field.

- Doesn't present a Mental Model for solving the situation (M.F.6.)

The student doesn't construct a Mental Model for mastering the situation.

Students' model for representing the field geometrically (M.G.)

The models for representing the field geometrically present more uniformity than the one for framing the concept of field in the Schema of describing interactions. A fundamental difference between them is the representation of the field within the sources. We, thus, found two Mental Models

The first Mental Model, labeled M.R.1., is a representation considering the field as existing just outside the sources. It takes into account, sometimes, a theorem-in-action, that classifies the field as just a transferor of Energy (a support for the transfer), developed by the students throughout the tasks and that implies a recursive operation "**if** field transfer Energy between objects, **then** it just exists outside these objects". Some students, by the other hand, believe that because of the existence of the field in vacuum, the field can just exist outside the objects. In the time description we separate the two cases. For now, we're just worried in distinguish those two Mental Models.

The second model, which we labeled M.R.2., is a representation considering the field as existing both in the interior and exterior of the sources. It takes into account, sometimes, for example, the existence of the field in the whole space, including the interior of the sources. It is more adequate and complete than M.R.1.

Both models are attached, however, to a set of knowledge-in-action (concepts-in-action and theorems-in-action, respectively). From these elements, students construct provisory *recursive operations* that allow the inference making with respect to situations. The fundamental goal of these situations is *to represent the field Geometrically* *through field Lines and Arrow Diagrams*. The knowledge-in-action are presented below:

a. *The **field** is a function of the **distance** measured from the **Source Charge** to a point in **space**;*
b. *The **field** is a function of the **Source Charge**;*
c. ***Source Charge** creates **field**;*
*d. A **field vector** is attached to each point in **space** (definition of vector field – a function that associates a vector to each point in space);*
*e. **field** intensity is a function of the **distance** of the **Source Charge** to a point in **space***
*f. The **field** is symmetrical with respect to the **sources**.*

We present below the two Mental Models.

- To represent the field existing just outside the sources (M.G.1.)

This Mental Model represents adequately the field with just one restriction: it does not represent the field inside the sources. We know the field exists all over the space, including the interior of the sources. We just ignore the field within the sources when we approach problems including point particles, once there is a singularity around $r = 0$.
Example:

Student K

Goals: To represent the field Geometrically, considering the existence of the field outside the sources.

Knowledge-in-action: ***field** is a function of points in **space*** (Figure 3 [5^{th} task – 1^{st} ,2^{nd} , 3^{rd} questions])./ *A **field vector** is attached to each point in **space** (definition of vector field – a function that associates a vector to each point in space)* – (Figure 3 [5^{th} task – 1^{st}, 2^{nd} , 3^{rd} questions])./ ***field** intensity is a function of the **distance** of the **Source Charge** to a point in **space*** (Figure 3 [5^{th} task – 1^{st}, 2^{nd} , 3^{rd} questions])./ ***field** transports **Energy*** (from other tasks)

Recursive operations: **If** the field is a function of points in space and to each point there is a vector, **then** one has to draw a vector in each point in space./ **If** one wants to represent the field through field Lines, **then** one must draw a line tangent to the vectors./ **If** the field transports Energy, **then** it just exists outside the sources./ **If** field's intensity diminishes with distance, **then** the arrows' length diminish with distance.
Inferences: Figure 3

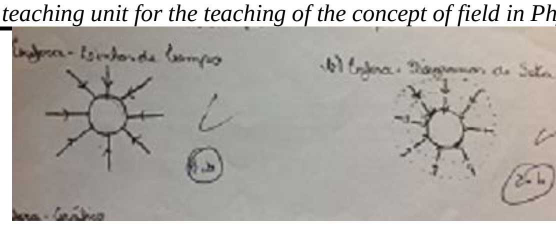

FIGURE 3. Representation of the field of a Sphere.

- To represent the field inside and outside the sources (M.G.2.).

This Mental Model represents adequately the field without apparent restrictions. We know the field is distributed all over the space, including the interior of the sources and this can be associated to a conception of existence of the field all over the space.

Example:

Student E

Goals: To represent the field Geometrically, considering the existence of the field inside and outside the sources.
Knowledge-in-action: ***field** is a function of points in **space*** ("Figure 2"[5^{th} task – 1^{st}, 2^{nd}, 3^{rd} questions]) ***field** is generated by **sources*** ("Figure 2"[5^{th} task – 1^{st}, 2^{nd}, 3^{rd} questions]), ***field** intensity is a function of the **distance** of the **Source Charge** to a point in **space*** ("Figure 2"[5^{th} task – 1^{st}, 2^{nd}, 3^{rd} questions]), The field is symmetrical with respect to the sources ("field"[5^{th} task – 1^{st}, 2^{nd}, 3^{rd} questions]).

Recursive Operations: **If** the field doesn't depend on the distance, **then** it is an uniform (in intensity and direction) field and the arrows must be of the same length. **If** the arrows assume correspondence with the vectors that they represent, **then** the arrows length must be proportional to field's intensity. / **If** an object is symmetrical, **then** the arrows must be symmetrical. **If** the field depends on the distance between source and a point in space and **if** it raises, insofar we approximate source and point, **then** the field depends on the shape of the source. / **If** the field depends on the sources, **then** it must raise its intensity in the neighborhood of the sources and diminish far away from the sources. / **If** the fields exist in space, then they must exist inside the sources.
Inferences: Figure 4

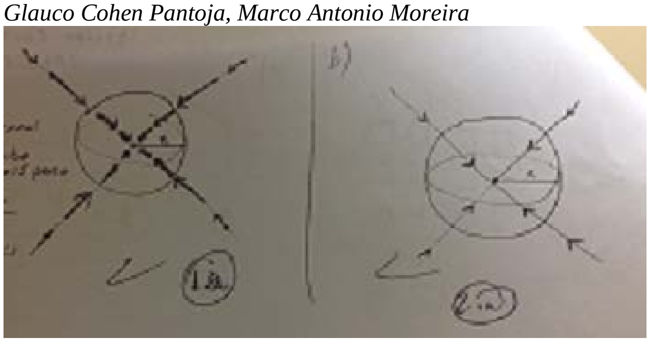

FIGURE 4. Representation of the field of a Sphere.

Students' models to represent the field Mathematically (M.M.)

This Mental Model differ a few from the **M.G.** ones because of a new factor, namely, the introduction of the concepts of Flux e Circulation. When using this Mental Models, at least in the last activity, students didn't evidence their position on the existence of the field in the volume of the sources. We distinguish among four Mental Models below:

- To represent the field Mathematically without using the concepts of Flux and Circulation (M.M.1.)

The model presents the (Gravitational) field as a function of the mass and as depending on the distance from a Source Charge to a point in space; however it mentions the concepts of Flux and Circulation without discussing them.

Example:

Student L

Goals: To represent the field Mathematically without using the concepts of Flux and Circulation.

Knowledge-in-action: $\vec{g} = \vec{g}(m, \vec{r}, \rho)$ ("…a very precise model can take into account planet's morphology [form], density distribution [mass and volume of the source] and tides movement [mass and density]"[6th task – 1st question])/ $\vec{g} \neq \oint \vec{g}.d\vec{l}$ (""[6th task – 1st question])/ $\Phi_G \alpha\, m$ and $\Phi_G \to \Phi_M$ (Gravitational Flux is Mass Flux) ("Mass fluxes, loss of mass, raise of mass/ Flux of mass will be negative"[6th task – 2nd question]).

Recursive operations: **If** the Gravitational Flux depends on the mass and the field depends on the mass, **then** the Gravitational Flux equals mass Flux./ **If** Circulation is global and it relates to the field, **then** it can be null in some places where the field is perpendicular to the closed loop./ **If** we want to identify what influences the field, **then** the mass and its shape are two relevant factors./ **If** $\vec{g}_j(m_j)$ and $m_j \ll M$, **then** $g_j \ll g(M)$.

Inferences: Circulation of the field and the field are not equal ("field's circulation of the field, due to non-homogeneous density and planet's irregularity will be different from zero for a great number of points in space, because $\vec{g}$ and $\hat{t}$"[6th task – 1st question])./ Flux of the Gravitational field is Mass Flux ("mass fluxes [complicate Gravitational field]"[6th task – 2nd question])./ Tiny masses do not influence the resultant Gravitational field when there is a huge mass in the counting ("…remove the tides, neglect small variations in planet's morphology of the planet and density's distribution [will not influence the field]"[6th task – 3rd question]).

- To represent the (Gravitational) field mathematically using the concept of Flux as a normal projection of the field over a Gaussian Surface and the concept of Circulation as a tangent projection over an Amperian Loop (M.M.2.).

This Mental Model represents the (Gravitational) field mathematically as a function both of Source Charge and of the distance between the Source Charge and a point in space, uses the concept of Flux as an indicator of the direction of the field (and considering the mass the source of Gravitational field) and the Circulation as an indicator of a conservative field (if it equals zero), i.e., the one that doesn't present field lines as closed curves.

Example:

Student O

Goals: To represent mathematically the field using the concept of Flux as the normal projection of the field Vectors in a gaussian surface and the concept of Circulation as the tangent projection of the field Vectors in an "amperian" loop.

Knowledge-in-action: $\vec{g} = \vec{g}(\rho, \vec{r})$ ("In Earth's gravitational field modelling we consider that earth possesses the same mass density in all points, the form of Earth's shell as a sphere…" [6th task – 1st question])./ $\Phi < 0$ implies $\vec{g} < 0$ ("$\oint \vec{g}.\hat{n}dS = -4\pi Gm \to \vec{g}.\hat{n} = g cos(\pi) = -g < 0$"[6th task – 1st question])./ $C = 0$ implies g_t (tangent component)./

Recursive operations: **If** we consider the Earth's density as varying, **then** the Model of the Earth's Gravitational field will change./ **If** there are other bodies next to the Earth, **then** the Model of the resultant Gravitational field will change./ **If** the field is anti-parallel to the Gaussian's normal vector, **then** the field is negative./ **If** the field is perpendicular to the "Amperian's" tangent vector, **then** the tangent component is null./ **If** we consider fields generated by sources next to the Earth, **then** we must consider the interaction between them.

Inferences: Flux indicates the direction of the field ("$\Phi < 0$ implies $\vec{g} < 0$ ("$\oint \vec{g}.\hat{n}dS = -4\pi Gm \to \vec{g}.\hat{n} = g cos(\pi) = -g < 0$"[6th task – 1st question])./ Circulation indicates if the field is rotational ("$\oint \vec{g}.\hat{t}dl = 0 \to \vec{g}.\hat{t} = g\, cos\left(\frac{\pi}{2}\right) = 0$ the field doesn't produce rotational force"[6th task – 1st question])./ fields superpose one to the other changing themselves mutually (M.G.4. indicator).

- To represent the field Mathematically using the concept of Flux as an indicator of the direction of the field and the concept of Circulation as an indicator of circular closed field lines (M.M.3.)

This Mental Model represents mathematically the Gravitational field as a function of the mass and of the distance from the source to a point in space. The model also uses the concept of Flux as an indicator of direction of the field (and the mass as the source of the field). It considers Circulation as an indicator of a conservative field, namely, the ones that are not represented by closed lines.

Example:

Student N

Goals: To represent the field Mathematically using the concepts of Flux and Circulation as qualitative indicators for drawing the field.

Knowledge-in-action: $\Phi = \oint \vec{g}.\hat{n}dS = -4\pi G$ (Flux indicates attractive Force) ("observing the equation $\oint \vec{g}.\hat{n}dS = -4\pi G$, we observe that we just have attractive forces"[6th task – 1st question])./ $C = \oint \vec{g}.d\vec{l} = 0$ ("… Due to the equation $C = \oint \vec{g}.d\vec{l} = 0$, we know the field is conservative "[6th task – 1st question])./ $\vec{g}_R = \sum g_j$ ("We can take into account the mass of other planets or systems"[6th taks – 2nd and 3rd questions]).

Recursive operations: **If** $\Phi < 0$, **then** Force and field are "attractives"./ **If** $C = 0$, **then** the field doesn't circulate and it is conservative./ **If** $\exists\{\boldsymbol{m_j}\}$, **then** $\vec{\boldsymbol{g}}_R = \sum \vec{g}_j(m_j)$.

Inferences: Φ indicates the differences in the field Lines that come in and that come out of the Gaussian surface ("We can predict that there is more "things" coming in than coming out observing the Equation $\oint \vec{g}.\hat{n}dS = -4\pi G$ and we observe that we just have attractive forces [$\vec{F} = m\vec{g}$] "[6th task – 1st question])./ C indicates if the field is conservative or not (""[6th task – 1st question])./ The model can be complicated if we insert new sources of fields ("We can take into account the mass of other planets or systems"[6th task – 2nd question])./ Not considering masses simplifies the Model ("In a general view, we can adopt just Earth's mass [neglecting others]"[6th task – 3rd question]).

- To represent the field Mathematically identifying the concept of Flux to the concept of field (M.M.4.)

This Mental Model represents the Gravitational field as a function of the mass and of the distance to a point. It uses the concept of Flux as similar to the one of field, because it carries with it the idea of Flux as a function of the mass and of the distance from a point in space to the source.

Example:

Student A

Goals: to represent the field Mathematically identifying the concept of Flux to the concept of field.

Knowledge-in-action: $\vec{g} = \vec{g}(m_f)$ ("The main aspect to be considered is Earth's mass"[6th task – 1st and 2nd questions])./ ***Flux** exerts negative **Force*** ("Analyzing Flux Equation, we conclude that it is negative, because it exerts attraction Forces"[6th task – 1st question])./ $\vec{g}_R = \sum g_j$ ("If we consider the mass of the Moon, besides Earth's mass, we'll have a more complex and precise model… if we sum all gravitational fields, we will have more precision…"[6th task – 2nd question])./ $\vec{g}(\rho)$ ("We can consider, for example, that the earth is completely spherical and homogeneous [obviously depends on the form of the source]"[6th task – 3rd question]).

Recursive operations: **If** the Flux is negative, **then** there is attractive Force. / **If** the mass is tiny, **then** it doesn't contribute to the Force. / **If** we take into account Moon's mass, **then** we must take into account the superposition of the fields.

Inferences: The Flux is directly proportional to Forces (""Analyzing Flux Equation, we conclude that it is negative, because it exerts attraction Forces""[6th task – 1st question])./ We must take into account distinct contributions to the field. ("If we consider the mass of the Moon, besides Earth's mass, we'll have a more complex and precise model… if we sum all gravitational fields, we will have more precision…"[6th task – 2rd question]).

After describing and exemplifying student's internal representations (Schemas and Mental Models) for tackling situations they represent accordingly to these Schemas and Mental Models, we must describe how students' evolved from one conception to another.

D. Frequencies of students' internal representation along tasks

Now we present the time evolution of students' internal representation during the tasks. To each student we attach a letter to keep their identity unknown. To each Schema or Mental Model we associate a number to better fit the table to the article's formatting.

First task

We present below the questions presented in the first task

1. Accordingly to Universal Newtonian Gravitation theory, the Earth, the Moon, the Sun and the planets exert mutual forces without contact and without any material medium among them. The concept of action-at-distance was used to describe this kind of interaction. However, this concept wasn't easily accepted and something called "aether" was invented to serve as a transmission medium for Gravitational Interaction. The aether would be a thin substance that would fill all the space and it would have an extremely small density (this would explain why it couldn't be detected in vacuum). The aether survived during a long period, but it was abandoned. a) Do you remember why? b) Using the concept of field, does the vacuum is really hollow? c) Changingthe question, is the vacuum filled with fields? d) In this case the aether was just changed by the field?Justify.
2. If the Gravitational Force acts upon all bodies proportionally to their masses, why a heavy body doesn't fall faster than a light body?

3. In text-books it is common to represent Electric and Magnetic fields with Lines of Force. a) What is their relation with the field Vector that describes these fields? Do they exist or are just draws of the field? b) Does the field exist or is it a mathematical construction? In fact, we never measure the field directly, but its effect, namely, the Force exerted by it upon a test mass (in the case of Gravitational Force) or upon an electric charge (in the case of the Electromagnetic Force). c) Are these fields real? d) Does it make sense to say that a real quantity cannot be measured? Justify.
4. It is common saying that when we raise an object to a height *h* over Earth's surface, it acquires Potential energy relative to its initial position. a) Is it true that the object really possesses this Energy? b) Is it true that the Earth shares this energy? c) Thinking with the Gravitational field, is it possible to solve this doubt? How? d) Is this Energy modified instantaneously?

- **Schemas (S.I.)**

As expected, most of the students (11 of 18) presented a Schema of describing interactions that doesn't include the concept of field (S.I.1). Even though they've already studied the Gravitational field, they didn't even know how to explain the situation using the concept of field, attributing to it, an abstract/mathematical status only. Five students used the Schema for describing interactions using the concept of field, but in very general way. It is possible that their prior experience with the concept of field may have turned it possible.

TABLE 1. Schemas presented in the resolution of the first task.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1		■		■	■		■	■		■	■	■				■	■	■
2	■		■										■	■	■			
3						■			■									

The number 1 means S.I.1. (interaction-without-field), 2 the number 2 means S.I.2. (To describe interactions including the concept of field in conceptualization) and the number 3 means S.I.3. (No conceptualization).

- **Mental Models (M.F.)**

As also expected, most of the students (13 of 18) didn't present a Mental Model for explaining the action of the field in the interaction. The same five students presented an excessively general Mental Model for explaining the situations presented. This corroborates the absence of the concept of field in the cognitive structures of the majority of the Students [6, 8].

TABLE 2. Mental Models presented in the first task.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	■		■										■	■	■			
2																		
3																		
4																		
5																		
6		■		■	■	■	■	■	■	■	■	■				■	■	■

The label 1 means M.F.1. (To describe the ontology of the field in a general way), label 2 means M.F.2. (To describe the field as an entity that transfers Energy from a body to another), label 3 means M.F.3. (To describe the field as possessing Energy), label 4 means M.F.4. (To describe the fields as entities that transfer Energy by collision to the other fields – shock conception), label 5 means M.F.5. (To describe the field based in the idea of interaction of the field with Objects) and label 6 means M.F.6. (No conceptualization). Similarly to the indexing of Schemas, this rule of coding will be the same until the fourth task. From the fifth task on, the labels will change their meanings and we will sign this change.

Second task

The second task approached the concept of field in a general way, as presented in the sequence in methodology. We presented the most general features of the field and in this class we've emphasized the characteristic of the field as a physical entity that exert Forces and work on particles.

We present below the questions presented in the second task.

1. You must remember of your Gravitation studies that exists a potential Energy associated to the Gravitational Interaction. Departing from studies in Mechanics you must remember that potential Energy is due to the system. Consider the Earth's Gravitational field and the interaction of the Moon with this field. a) What is the Physical System in the problem? b) Which interaction is occurring? c) where is localized the Gravitational Potential Energy? In the Bodies or in the field? Why?
2. Sunlight takes eight minutes to arrive at Earth's surface. When you look at the Sun, are watching it in a version of eight minutes ago? How do you explain this difference?
3. Consider that, for some reason, all planets and stars disappeared from the Universe and just the Earth remained. a) What would you infer about gravitational Force? b) and about the Gravitational field? Justify.

- **Schemas (S.I.)**

After the first class, all students could conceptualize the interactions using the concept of field, but this conceptualization was driven by the construction of a Mental Model whose function was to frame the concept of

Mental Model in a Schema of Interaction. This indicates that our first class and problems facilitated sense making (construction of Schemas or Mental Models for solving situations) of the concept of field for the Students.

Table 3. Schemas used in the second task.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1																		
2	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
3																		

We describe below the complexity of Mental Models developed by the students to Frame the concept of field in conceptualization.

- **Mental Models (M.F.)**

Few students conceptualized in a general way. Seven (of 18) based themselves in the idea that the field carries Energy and developed a Mental Model that puts the field as the deliverer of Energy to other masses that possess this Energy, so they localize Energy just in bodies (a theorem-in-action present in prior conceptualization). Nine students approach the more accepted idea that the field has Energy and transport it to execute a change of Energy with a particle. We shall enlighten that this conception doesn't mention the mechanism of transference of Energy (Force or Work). But this result is interesting, because it shows us the students tried to develop Mental Models to frame the concept of field in the Schema of describing interactions.

TABLE 4. Mental Models constructed in the second task.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1					■							■						
2		■				■					■				■	■	■	■
3	■		■	■			■	■	■	■			■	■				
4																		
5																		
6																		

Third task

We present below the questions presented in the third question.

In this activity you will model an object of mass m (a blackboard eraser, for example) being attracted by the Earth of mass M. Suppose it falls from a height $h = 1{,}70\ m$, departing from rest. Data: $G = 6{,}67 \times 10^{-11} \frac{Nm^2}{kg^2}$, $R_T = 6400km$, $M = 5{,}98 \times 10^{24} kg$.

Based in the system Earth-Eraser, answer:

1. Which elements are relevant to the Model? Which approximations/idealizations were done in the construction of the Model?
2. Which mass is labeled as Source of the Gravitational field? Which mass is labeled as test mass for the interaction? How could you explain Energy/Momentum transfer done by the Gravitational field in this case?
3. Suppose you want to calculate the influence of Moon's Gravitational field in the prior System, namely, Earth-Eraser. Which changes would you adopt in the mathematical model? Which results you expected to be obtained?

In the third task, the questions emphasized modelling of the free fall of a light object (blackboard eraser) in the Earth's surface. We emphasized both interaction of a mass with Gravitational field and the concepts of test mass and source mass.

- **Schemas (S.I.)**

We can see what Vergnaud [16] calls a "throwback". Two students stopped using a Mental Model to conceptualize the field in a situation nearer of conceptualization of concrete objects. But what is an important evidence of Meaningful Learning is the attempt of the students to represent the system using the field theory. 16 students conceptualized the interactions describing them using the concept of field.

TABLE 5. Schemas used in the third task.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	■												■					
2		■	■	■	■	■	■	■	■	■	■	■		■	■	■	■	■
3																		

It is important to stress that the labels for the Schemas are the same presented in the first and second tasks.

- **Mental Models (M.F.)**

We now can see also, the characteristic of plasticity of a Mental Model and understand why we characterized the conceptions involving the concept of field as Mental Models instead of Schemas. Students' conceptions, as we can see, blatantly varied. This change is positive for seven students (six of them changed from M.R.3. to M.R.5. and one from M.R.2. to M.R.5.) that achieved a scientifically accepted level of conceptualization emphasizing the transfer of Energy between Object and field, besides the localization of the Energy in the field. But we observed that four students that used the Mental Model M.R.2. (which attributed the Energy to the Objects) meaningfully developed a more stable structure that understands the Energy in the body and transported by the fields that shock one another in a way the resultant Energy is delivered to the bodies. We can observe that for these students (that used to integrate the same group in the resolution of the problem situations) couldn't overcome the theorem-in-action of "the

Energy is just in the objects" to a more complete "the Kinetic Energy is localized in the Bodies, but the interaction Energy is localized in the field".

TABLE 6. Mental Models constructed in the third task.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1												■						
2					■						■			■				
3						■												
4		■														■	■	■
5			■	■			■	■	■	■					■			
6	■												■					

We shall remember the reader that the labels 4, 5 and 6 are references for M.R.4., M.R.5. and for M.R.6.

Fourth task

In the fourth task we aimed to implement a reconciliatory integration of the content presented. The students had to construct a conceptual map relating interactions and to draw a text presenting the most relevant characteristics of fundamental fields and Forces in nature.

We present below the questions presented in the fourth task.

1. Distinguish among the four interactions of nature with your own words emphasizing the mechanisms of Momentum and Energy transfer for each one.
2. Draw a conceptual map for the concept of field.

- **Schemas (S.I.)**

Again all students presented a Schema including the concept of field. For the two students (students A and M) that returned to show this conception, we can understand their apparent advance due to a situation-scheme effect. Students are somehow "triggered" to use the concept of field when they read or listen to the word "field". The situation mentioned explicitly the concept of field, and this could have facilitated the process.

TABLE 7. Schemas used in the fourth task.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1																		
2	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
3																		

The labels are the same presented in previously.

- **Mental Models (M.F.)**

This is the most difficult case to explain. We had to draw five explanations to understand why learners construct different Mental Models to tackle the situation that is excessively general, a thing that induces a struggle for searching for useful knowledge-in-action to master a situation. The more general the situation is the more it approximates recalling. And recalling can be linked to a reconstruction process, a thing that explains the variation. We describe below the five reasons we've found for this variation:

- Mental Model improvement: students A, L and M. These students presented no conceptions or general or even alternative Mental Models to describe situations and now they present Mental Models that are closer to scientifically accepted explanation. We attribute it as a positive point for Meaningful Learning presented by the students.
- Mental Model stability;
  - Stability (4 → 4): Students B, P, Q, R. The student supports again the colliding Mental Models conception. This show the stability of this conception. That's a negative point for Meaningful Learning presented by students, once the student now presents a more "rigid" Mental Model that was turning gradually into a difficult Mental Model to change.
  - Stability (5 → 5): Students D, H. The student supports again the Mental Model we elected the closest to the scientifically accepted Model. That's a positive point for Meaningful Learning presented by students.
  - Stability (2 → 2): Student E. The student supports a poor model that puts the Energy just in the bodies and uses the field just as a support of Energy transport. This is a negative aspect, once we couldn't perceive evolution in the student's Mental Model.
- Throwback: Students J, O and F. This is a negative point, because this doesn't even show any Meaningful Learning beyond the one presented to include the concept of field in explanation. They presented before an advanced conception and in this task they've just suffered a huge throwback to a Mental Model that even in implicit form doesn't present evidence of complexity in conceptualization.
- Use of Implicit Knowledge: Students G, C, I, N. This is a natural point that show us that a conduct is becoming automatic. Conceptualization starts to be implicit. Students conceptualize the field, but make mentions to the field in a form that brings the theorems-in-action that relate the field to exerted Forces in bodies. It shows us maintenance in the major ideas that are next to the scientifically accepted conception, but implicitly. This is a positive point.
- Consolidation: Student K. This student follow the same path in conceptualization students B, P, Q and R followed. Their initial Mental Model carried

theorems-in-action that attributed Energy just to the bodies and interpreted the field as a mere transferor of Energy. This consolidation occurred when the Student constructed a more complex, but incorrect Mental Model. This is also evidence of Meaningful Learning of an alternative idea.

In summary nine students presented Meaningful Learning processes that were considered as positive, nine presented Meaningful Learning processes that were considered negative. With this result we reconsidered the changing of the task and to change the emphasis of Interactions of Nature to Gravitational Interaction and Electromagnetic Interaction.

TABLE 8. Mental Models constructed in the fourth task.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	■		■			■	■								■			
2					■				■	■								
3																		
4		■									■					■	■	■
5				■				■				■	■	■				
6																		

Fifth task

In this task the students had to represent three kinds of fields (Electric or Gravitational), namely, the Electric field generated by an infinite plate, the Gravitational field generated by a disk of radius a and the Gravitational field generated by a sphere of radius R.

The questions presented in the task a showed below.

1. A very extense retangular plate, eletrically charged with negative charge, generates an Electrical field $\vec{E}$ in the space that behaves accordingly to the following equation $\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{\imath}$. Represent the field through a) field Lines, b) Arrow Diagrams, c) Graphs.
2. The Gravitational field generated by a disk of mass M and radius a in a point (distant of z from its center) on the axis passing by the center of the disk is given by the following expression $\vec{g} = -\frac{2GM}{a^2}\left[1 - \frac{z}{(a^2+z^2)^{\frac{1}{2}}}\right]\hat{k}$. Represent the field with a) field Lines, b) Arrow Diagrams, c) Graphs.
3. A spherically symmetrical planet, with uniformly distributed mass M and radius R, generate the Gravitational field $\vec{g}$ following the form $\vec{g} = -\frac{Gm}{r^2}\hat{r}$ for points in the outside region of the planet and $\vec{g} = -\frac{Gmr}{R^3}\hat{r}$ for points in the inside region of the planet. Represent the field using a) field Lines, b) Arrow Diagrams, c) Graphs.

para pontos no seu interior. Represente o Campo em quest˜ao atrav´es de a) Linhas de Campo, b) Diagramas de Seta, c) Gr´aficos.

- **Mental Models (M.G.)**

We must stress that the field outside the sources is represented correctly by all the students that delivered the task. 17 students out of 18 have represented the field correctly outside the sources, but, the half of them didn't take into account that the field exists inside the source in the case of a volume. When mastering the situation of the field of the Sphere, students present symmetry arguments for justifying the radial form of the field, but do not represent the field inside the sources. By the other hand, in the first task involving geometrical representation of the field, the students could represent them in an appropriate way, at least outside the source. It is necessary to emphasize the role of the field inside the sources; otherwise the students can think that field exists outside the space in which the mass/charge is distributed, but not inside. Another point that must be stressed is the sense making: students could draw fast a Mental Model to represent geometrically the field.

TABLE 9. Mental Models constructed in fifth task.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	■		■	■		■				■	■			■	■			■
2		■			■		■	■	■			■				■	■	
3													■					

Label 1 means M.G.1. (to represent the field existing just outside the sources.), label 2 means M.G.2. (to represent the field inside and outside the sources), label 3 is attached to the students that didn't deliver the task.

Sixth task

In this task, maybe the more complex task presented in the PMTU, we ask the students to Model the Gravitational field of the Earth. In one of the questions we stress that the students have to discuss the role of Flux and Circulation in the mathematical representation of the field.

The questions presented to the students are showed below. Model the Earth's Gravitational field of the Earth departing from Flux and Circulation equations below

$$\oint \vec{g}.\hat{n}ds = -4\pi Gm,$$

and

$$\oint \vec{g}.\hat{t}dl = 0.$$

1. Which are the relevant aspects in Earth's Gravitational field modelling? How can we use

Flux and Circulation equations to predict qualitatively the field?

2. Which aspects can complicate and improve precision of this Model? How do they do that?
3. Which simplifications can be done without compromising model's validity?

- **Mental Models (M.M.)**

Just four students (of 17 that delivered the task) couldn't use the concept of Flux or the concept of Circulation in the description of the field. Two learners used the Mental Model M.M.2 that understands Circulation and Flux as projections of the field in an imaginary closed Curve and Surface. This is an interesting point. In the first class we discuss Circulation and Flux, students could draw a Mental Model that is "aligned" to Scientific accepted knowledge. But what is more interesting is the Model M.M.3. the students that could understand both Circulation and Flux in a more geometrical way using a Mental Model that identifies the Flux as a scalar characteristic associated to field's direction and source (in the case of Gravitational field) and the Circulation as an indicator of closed curved field Lines or "open" lines (in the case of Gravitational field).

It is important to stress that this Mental Model gives to the students a more conceptual view of these concepts and, obviously, can be a much more fruitful Mental Model in mastering situations than understanding the Flux and Circulation just as abstract projections of Vectors in surfaces.

We shall stress that the Mental Model M.M.4. that identifies the Flux or Circulation to the field, which is a very common conception [2], was just found for two students. In general, students can't differentiate easily among Flux, Circulation and field.

We think that this distinction made between M.M.2 (two of 17) and M.M.3. (eight of 17) can be interpreted as an evidence of Meaningful Learning associated to a view that is close to scientific knowledge, once we have Mental Model construction accordingly to what Physics understands as Flux and Circulation. This is important evidence that PMTU could achieve the goals it searched for.

TABLE 10. Mental Models constructed in sixth task.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1		■						■			■	■						
2							■								■			
3			■	■		■			■	■				■		■	■	■
4	■				■													
5													■					

Label 1 means M.M.1. (To represent the field Mathematically without using the concepts of Flux and Circulation.), label 2 means M.M.2. (To represent the field mathematically using the concept of Flux as a normal projection of the field over a Gaussian Surface and the concept of Circulation as tangent projection over an Amperian Loop.), label 3 means M.M.3. (To represent the field Mathematically using the concept of Flux as an indicator of the direction of the field and the concept of Circulation as an indicator of circular closed field lines.), label 4 means M.M.4. (to represent the field Mathematically identifying the concept of Flux to the concept of field.) and label 5 is attached to the students that didn't deliver the task.

V. DISCUSSION

In this work we presented a didactical proposal applied to the teaching of the concept of field in Physics named Potentially Meaningful Teaching Units [3]. We worked with the concept of field because of the lack of research in the teaching of this so general, ubiquitous and important concept, and also because we believe that a general introduction of the concept of field can facilitate the Learning of the Concept of Electromagnetic field.

We presented the PMTU to 18 physics students in Brazil and we analyzed their answers to problems presented to them during instruction. For data analysis we both used Greca and Moreira [18] proposal and Content Analysis [11] to organize the study.

We shall now summarize the results, extend the PMTU for curriculum design, explain how the results answer to research questions, show how this work fits into the literature and we shall point the importance of PMTU for teaching-learning processes.

We presented the Mental Models and Schemas of students related to the situations attached to the concept of field and presented a "time-development" of the changes of representation students took during instruction. We shall now tie things answering to the research questions.

Now we shall present an answer to the first question proposed, namely, "*how do representation construction occurs for the concept of fields during the PMTU?*". We found that the PMTU facilitated the execution of processes of Meaningful Learning (sometimes some of these processes took a few students away from scientifically accepted knowledge). Asking in another way: which Meaningful Learning processes are that and how they can be represented? These processes are represented by Schemas when the situation to be mastered is known or by Mental Models when the situation to be mastered is identified to be a new class. As we know these processes occur when there is a deliberate interaction between content to be learned and students' cognitive structure. When students build a Mental Model, they anchor this Mental Model in prior knowledge-in-action (present in old Schemas) and that's why we can both meaningfully understand new situations and master the "old" ones, because we conceptualize when we implement Meaningful Learning Mechanisms.

We could find one class of general Schemas, the one for describing interactions among bodies. This Schema can be divided in two classes distinguished by the use of the concept of field in the mastering of a situation. For most of the students, the concept of field was blatantly new and they had to build a Mental Model for reinterpreting the situations in terms of a field theory.

We could find three classes of Mental Models: Mental Models for framing the concept of field in the Schema of describing interactions; Mental Models for representing geometrically the field and Mental Models for representing mathematically the field. All these Mental Models have classes that ramify understanding of Student's cognition.
The Mental Models for framing the concept of field in the Schema has five possibilities: M.F.1. (to describe the ontology of the field in a general way); M.F.2. (to describe the field as an entity that transfers Energy of a body to another); M.F.3. (to describe the field as possessing Energy); M.F.4. (to describe the fields as entities that transfer Energy by colliding the other fields – shock conception); and M.F.5. (to describe the field based in the idea of interaction of the field with Objects).

The Mental Models for representing geometrically the fields have two classes in our study: M.G.1. (to represent the field existing just outside the sources); and M.G.2. (to represent the field existing inside and outside the sources).

The Mental Models for representing mathematically the field have four classes in our study: M.M.1. (to represent the field Mathematically without using the concepts of Flux and Circulation); M.M.2. (To represent the (Gravitational) field mathematically using the concept of Flux as a normal projection of the field over a Gaussian Surface and the concept of Circulation as tangent projection over an Amperian Loop); M.M.3 (to represent the field Mathematically using the concept of Flux as an indicator of the direction of the field and the concept of Circulation as an indicator of circular closed field lines); and M.M.4 (To represent the field Mathematically identifying the concept of Flux to the concept of field).

We, therefore, found that the proposal of Greca and Moreira [5] gave us mechanisms for representing externally students' cognitive actions (internal representation) and based on this, we think we answered to the first research question.

If we draw students' conceptualization throughout time we can see students progressively (through students' advances and throwbacks) master the concept of field facing difficulties. Great part of the students overcame the difficulties presented in the end of the unit. Other students overcame the difficulties, but achieve a Mental Model that is different from the scientifically accepted Mental Models. With respect to the assimilation Schemas, the students ended up conceptualizing interactions using the concept of field. These models are plastic with exception of the Mental Models built by four students that seemed to be stable ones associated to the common conception of colliding fields. This can be seen as a "shy" evidence of an initial construction of a Schema adaptation. Students also could draw Mental Models for representing correctly the fields outside the sources, but just half of them could be aware of the existence of the field inside the sources.

We can also observe that initial conceptualization including the concepts of Flux and Circulation was achieved, but in a very phenomenological way. This study, which is part of a larger one, aims to identify if students can understand the concepts of Flux and Circulation of Electric and Magnetic fields in both *static* and *dynamic* situations. Few students confound Flux and Circulation with the field, but also few students could achieve mathematical understanding of the concepts. But we should note that this PMTU takes just six classes of 100 minutes.

We, therefore, answer to the second question stating that PMTU can help students to build Mental Models and to use their Schemas. It is of great importance for learning, because accordingly to the theoretical framework, both in Vergnaud's, Ausubel's and Johnson-Laird's theories Meaningful Learning processes involves the deliberate interaction between new knowledge with prior knowledge. Construction and use of Schemas and Mental Models are two blatant evidences of that. Besides, we can see that great part of the students achieved, in a short time, conceptions that are close to scientifically accepted knowledge.

The third question is a delicate one, but we draw an answer to it. We do not have sufficient data to state with certainty if these Mental Models will, actually, perish or turn into stable Schemas, but we have initial evidence that this establishing is in its way of occurring. In three tasks some students repeated the same organization of conduct represented in Mental Models. More light will be shed in this question when we bring the data of the PMTU on the concept of Electric field. We, therefore, have just initial evidence of construction of new Schemas. We answer to the third question stating that Greca and Moreira [5] proposal is fruitful in identifying students internal representations and to plan teaching, but we need further research to give a more convincing answer to establishment of new Schemas departing from constant construction of Mental Models.

We can also frame our work in the context of the other studies that are associated to the concept of field. In comparison to Llancacqueo et al. [6], we've developed a new didactical unity and we investigated student's conceptions using the Greca and Moreira [5] proposal. We took into account some details of their approach as, for example, the using of situations to achieve sense-making of the concepts for the students.

We built our PMTU taking into account some of the premises of Martin and Solbes [8], but we think we based our interpretation in a more explanatory learning theory. We understood not also the conceptions students had (knowledge-in-action), but we represented thought operations implemented by the students. We corroborate their teaching orientations, but we bring other relevant questions for the study, as for example, how does conceptualization occurs and the Meaningful Learning patterns that can occur during instruction.

We based also in the Pocovi & Finley [9] and Krapas & da Silva [10] studies in didactical-historical-epistemological mistakes committed by Physics text-books and we constructed a PMTU that took this into account. We, thus, bring important features for discussion of the concept of field during the instruction. A basic text for discussing this content is in its way of being finished.

After discussing how our work fits into the literature, we'd like to discuss two didactical direct implications of this study: the importance of the PMTU for Meaningful Learning and for Curriculum Design.

What is the importance of PMTU for Meaningful Learning? Ausubel [4] brings for discussion this so important concept and gives two conditions for Meaningful Learning: a potentially Meaningful Learning material and students' intentionality of learning. The PMTU is a form of presenting students with potentially meaningful material. In other words, these didactical unities facilitate Meaningful Learning, but they depend on students' intentionality on Learning. If the student doesn't want to relate prior knowledge with new knowledge, the thread of Meaningful Learning is cut down. That's a complicated factor to approach, because lack of intentionality can be attached to different reasons of distinct natures: biological as, for example, fatigue or laziness; cultural as, for example, students' family pressure for grades; psychological as, for example, bad attitude towards the subject. In this study we could see that the great majority of students were capable of developing mechanisms of Meaningful Learning.

What is the importance of PMTU for Curriculum Design? Designing a curriculum is very hard task that involves lots of specialists in Education and in Physics, requires time and perseverance to enumerate important topics for teaching, taking into account the goals for teaching knowledge, etc. How can the PMTU facilitate curriculum design? The first feature we can highlight is the facility the PMTU gives to us of selecting general topics that can be used to be organized following the principles of PMTU as, for example, *situations give rise to concepts*, *progressive differentiation*, *integrative reconciliation*, *consolidation*, *sequential organization*, etc. We could, for example, start a first course in Physics discussing the topic of *Energy and momentum transfers and interactions.* We could dedicate a PMTU for *Quantum Mechanics*, *Waves* or *Electromagnetism* as we've been doing the last years. PMTUs give us the possibility of thinking in a Physics Curriculum Reform.

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