GZK cut-off and its modification in presence of Lorentz invariance violation

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Abstract

In this paper we endeavor to indicate the Greisen-Zatsepin-Kuzmin (GZK) cut off by using two interaction gammas with proton and neutron which leads to production of pion. We show that Lorentz invariance violation may change the location of the GZK cutoff and we find that resorting to a special modified dispersion relation it is possible to increase the GZK cut off beyond 10^{20} *eV* and our result is in agreement with the results experimental data. We also discussed on the properties of these models, modified dispersion relations, so that all of these models cannot extend GZK cut off but it may cause it to drop. Also in some of these models, according to their structure, there is a threshold to Lorentz invariance violation. Our analysis indicates by increasing the mass of initial particle in the interaction with CMB couses reducing energy thresholds in GZK equation and vice versa.

Keywords: Cosmic ray protons, Greisen–Zatsepin–Kuzmin limit, Lorentz invariance violation.

Resumen

En este artículo, nos esforzamos por indicar el corte de Greisen-Zatsepin-Kuzmin (GZK) mediante el uso de dos gammas de interacción con protones y neutrones que conducen a la producción de piones. Mostramos que la violación de la invariancia de Lorentz puede cambiar la ubicación del límite de GZK y encontramos que recurriendo a una relación de dispersión modificada especial es posible aumentar el límite de GZK más allá de 1020 *eV* y nuestro resultado está de acuerdo con los resultados de los datos experimentales. También discutimos sobre las propiedades de estos modelos, las relaciones de dispersión modificadas, de modo que todos estos modelos no pueden extender el corte de GZK pero pueden causar que caiga. También en algunos de estos modelos, según su estructura, existe un umbral para la violación de la invariancia de Lorentz. Nuestro análisis indica que al aumentar la masa de la partícula inicial en la interacción con CMB se reducen los umbrales de energía en la ecuación GZK y viceversa.

Palabras clave: Protones de rayos cósmicos, Límite de Greisen-Zatsepin-Kuzmin, Violación de la invariancia de Lorentz.

I. INTRODUCTION

Cosmic rays were discovered over 100 years ago and their origin remains uncertain. They have an energy spectrum that extends from ~ 1 GeV to beyond 10^{20} *eV*, where the rate is less than 1 particle per km^2 per century. Shortly after the discovery of the cosmic microwave background (CMB) in 1965, it was pointed out that the spectrum of cosmic rays should steepen fairly abruptly above about 10^{19} *eV*, provided the sources are distributed uniformly throughout the Universe. This prediction by Greisen and by Zatsepin and Kuz'min, has become known as the GZK cut off [1, 2]. At the GZK cut off energy level, the interaction length (a function of the power spectrum of interacting background photons coupled with the reaction cross section) becomes of order 50 Mpc [3].

The possibility that cosmic rays may interact with the photons of CMB with an energy larger than the GZK cut off has been the topic of discussion [4, 5, 6, 7].

From the experimental point of view has been found a few particles having energy higher than the constraint given by GZK cutoff limit and claimed to be disproving the presence of GZK cutoff or at least for different threshold for that[8, 9]. So, the questions are

1) How can one get definite proof of non-existence GZK cut off?
2) If GZK cutoff doesn't exist, then what would be the reason?

The first question could be answered by observation of a large sample of events at these energies, which is necessary for a final conclusion since the GZK cutoff is a statistical phenomena. Should be mentioned the measurements by Hires [10], Yakutsk [11] and the Pierre Auger Colaboration [12] seem to validate the existence of the GZK cutoff.

For the second question, one explanation can be derived from Lorentz violation(LV). If we do the calculation for the GZK cutoff in Lorentz violated world we will get the modified proton dispersion relation. LV not only may change the location of the GZK cutoff, it may even lift off the existence of the GZK cutoff.

Investigation of the Cherenkov radiation effects on the extend the GZK limit has been pointed out in [13]. There is a another report on the GZK cutoff in presence spontaneous violation of Lorentz invariance [14]. Effect of the modified dispersion relation on the GZK cutoff without a special MDR model reffered to in [15]. Recently modification of the Compton scattering and the GZK cutoff in presence of Lorentz invariance violation has been pointed out in [16].

In section II we investigate the influence of cosmic rays energy on scattering. In sections III and IV we test to the GZK cutoff in Lorentz invariant world. Finally, in section V we endeavor to extend the GZK cutoff beyond 10^{20} *eV*, resorting to the modified dispersion relation.

II. INFLUENCE OF COSMIC RAYS ENERGY ON SCATTERING

Let us analyze a proton (mass m_p) photon (energy *k*) scattering. In the center of mass frame, the energy of proton is given by

$$E = \gamma m_p \,, \tag{1}$$

where $\gamma = \left(1 - v^2\right)^{\frac{1}{2}}, C = 1.$ In this frame, the energy of the photon is

$$k' = \gamma k = \frac{Ek}{m_p} \,. \tag{2}$$

Now, if we boost back this photon in the center of mass frame, we get an idea of the influence of the initial energy of proton to the energy loss due to a scattering event

$$k'' = \begin{bmatrix} \gamma & \gamma v & 0 & 0 \\ \gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} k' \\ k'\cos\theta \\ k'\sin\theta \\ 0 \end{bmatrix} = \begin{bmatrix} \gamma k' + \gamma v k' \cos\theta \\ \gamma v k' + \gamma k' \cos\theta \\ k'\sin\theta \\ 0 \end{bmatrix}.$$

As there is no preferred direction, $\cos\theta$ averages to 0 and the energy of photon will be $\gamma k'$ which is equal to $\frac{E^2 k}{\left(m_p\right)^2}$. Then the energy transferred in a scattering is proportional to the square of the energy of proton. As the proton gains more and more energy, the loss in a scattering with a CMB photon will be costlier and costlier.

III. CALCULATION OF THE GZK CUT-OFF

Consider a system with n particles having four momentum vectors $p_1, \dots p_n$. In scattering theory the invariant mass *M* of such an n-particle system is given by

$$M = \left(p_1 + \dots + p_n\right)^2 . \tag{3}$$

We can rewrite this to

$$M = \left(E_1 + \dots + E_n\right)^2 - \left(\vec{p}_1 + \dots + \vec{p}_n\right)^2 . \tag{4}$$

We know that in the center of mass (CM) frame the total linear momentum equals zero, so in the CM frame the invariant mass simplifies to

$$M_{CM} = \left(E_1 + \dots + E_n\right)^2 . \tag{5}$$

So we can conclude the center of mass energy equals $E_{CM} = \sqrt{M_{CM}}$.

The rest energy of a particle equals mc^2 and we have put *c* equal to 1. For producing a particle you need at least an amount of energy equals to its rest energy, so for producing n particles you need at least an energy equal to $m_1 + \dots + m_n$. So based on the (5) the threshold value for M_{CM} for producing particles 1,…, *n* is given by

$$M_{threshold} = \left(\sum_{i=1}^{n} m_i\right)^2 . \tag{6}$$

At first we consider the reaction that takes place, between the cosmic ray protons p^+ and the CMB photons, is given by the following (where π^0 stands for the neutral pion)

$$\gamma + p \to p + \pi \,. \tag{7}$$

The threshold energy for these reactions can be calculated with the help of equation (6),

$$M_{threshold} = \left(m_p + m_\pi\right)^2 . \tag{8}$$

Using the equation (4) we have following inequality

$$m\gamma^2 + m_p^2 + 2E_y E_p - 2\vec{p}_\gamma . \vec{p}_p \geq m_p^2 + m_\pi^2 + 2m_p m_\pi \,.$$

If the angle between the velocity vectors of the photon and the proton is given by 180° and since the term $m_\gamma{}^2$ equals zero, after a little calculation, the threshold energy for the proton is thus

$$E_p \geq \frac{m_\pi^2 + 2m_p m_\pi}{4E_\gamma} \,. \tag{9}$$

If we consider $E_\gamma = 101$ and $m_\pi \approx 135 \frac{MeV}{C^2}$ and $m_p \approx 938 \frac{MeV}{C^2}$, so

$$E_p \geq 602 \times 10^{19} eV \,. \tag{10}$$

So if the proton has an amount of energy that is greater or equal to $602\times10^{19}eV$ it can react with the CMB-photons in the way that is written down in (9).

Now we consider the other reaction; the following process might then occur

$$\gamma+p\rightarrow n+\pi^{+}. \tag{11}$$

The positively charged pion ensures that charge is conserved. According to the square of the total four momentum is Lorentz invariant, any inertial observer will get the same value for it. So we obtain

$$\left(p_p+p_\gamma\right)^2=\left(p_n+p_\pi\right)^2, \tag{12}$$

which can also be written as

$$p_p^2+2p_p.p_\gamma+p_\gamma^2=-\left(m_n+m_\pi\right)^2C^2. \tag{13}$$

We know that if a particle has total energy E and momentum $\vec{p}$, then we may write its four momentum as follows $p=\begin{pmatrix}\frac{E}{C}\\ \vec{p}\end{pmatrix}$ and the square of the four momentum is defined as

$$p.p=p^2=-m^2c^2. \tag{14}$$

Now from above equation we have

$$p_p^2=-m_p^2c^2. \tag{15}$$

Since $p_\gamma^2=0$ and the dot product of the proton and photon four momenta is $p_p.p_\gamma=\frac{-2E_pE_\gamma}{c^2}$, therefore we have finally from (13)

$$E_p=\frac{\left(m_nc^2+m_\pi c^2\right)\ -\left(m_pc^2\right)^2}{4E_\gamma}. \tag{16}$$

The mass of a neutron is $m_n\approx93906\frac{MeV}{C^2}$ and that of a π^+ meson is $m_\pi\approx13906\frac{MeV}{C^2}$. Using the energy of a CMB photon calculated in equation (16) we obtain

$$E_p=3\times10^{20}eV. \tag{17}$$

According to this limit, we do not expect to detect cosmic rays from deep space having energies greater than roughly $10^{20}eV$, because any such cosmic rays particles would scatter off the CMB photons and be lost.

At this point, one may be eager to compare the $602\times10^{19}eV$ GZK cut off to the existence of a $3\times10^{20}eV$ cosmic ray. While it is difficult to reconcile these, it is not impossible, since the GZK cut off just means that the universe has an optical depth of about 50 Mpc to such high energy cosmic rays, not that they cannot exist, if the source of the highest energy cosmic rays are closer than this, there is no contradiction. Whether or not astrophysical sources capable of such acceleration are available in our vicinity is still an open issue in astrophysics. Hence, the GZK cut off will not affect our results, but it remains a puzzle, which has partly motivated the investigation of Lorentz symmetry violation.

Now we are going to obtain the GZK cut off without four momentum, then again consider the reaction (7).

At first we consider energy and momentum conservation in center of mass frame

$$E_p+E_\gamma=E'_p+E_\pi, \tag{18}$$

$$\left|\vec{p}_p\right|+\left(-E_\gamma\right)=\left|\vec{p'}_p\right|+\left|\vec{p'}_\pi\right|. \tag{19}$$

According to the particles are ultra-relativistic and Taylor expand their momenta

$$\left|\vec{p}_p\right|\approx E_p-\frac{m_p^2}{2E_p},\left|\vec{p'}_p\right|\approx E'_p-\frac{m_p^2}{2E'_p},\left|\vec{p'}_\pi\right|\approx E'_\pi-\frac{m_\pi^2}{2E'_\pi}$$

With these simplified momenta, we can solve for the particle energies and obtain two equations

$$\frac{m_p^2}{2E_p}+2E_\gamma=\frac{m_p^2}{2E'_p}+\frac{m_p^2}{2E_\pi}, \tag{20}$$

$$\frac{m_p^2}{{E'_p}^2}=\frac{m_\pi^2}{{E_\pi}^2}. \tag{21}$$

Second equation is derived by the condition that proton and pion have equal velocities. With the (18), we have three equations for three unknowns E_p,E'_p and E_π. By using (18) and (21) we get

$$E_\pi=\frac{-E_\gamma-E_p}{-1\frac{m_p}{m_\pi}}. \tag{22}$$

By the (20) and (21) we get

$$\frac{{m_p}^2}{2E_p}+2E_\gamma=\frac{m_\pi}{2E_\pi}\left(m_p+m_\pi\right)\cdot \tag{23}$$

To remove E_π from above equation it is enough to substitute (22) into (23).

So we get

$$4E_\gamma{E_p}^2+\left(4{E_\gamma}^2+{m_p}^2-\left(m_p+m_\pi\right)^2\right)E_p+E_\gamma{m_p}^2=0.$$

This equation has two roots with two conditions

$$4E_{\gamma}{}^{2}+m_{p}{}^{2}\geq\left(m_{p}+m_{\pi}\right)^{2}+4E_{\gamma}m_{p}\,,$$
$$4E_{\gamma}{}^{2}+m_{p}{}^{2}\geq\left(m_{p}+m_{\pi}\right)^{2}+4E_{\gamma}m_{p}\,.$$

Therefore

$$E_{p}=\frac{-\left(4E_{\gamma}{}^{2}+m_{p}{}^{2}-\left(m_{p}+m_{\pi}\right)^{2}\right)+\sqrt{\left(4E_{\gamma}{}^{2}+m_{p}{}^{2}-\left(m_{p}+m_{\pi}\right)^{2}\right)^{2}16E_{\gamma}{}^{2}m_{p}{}^{2}}}{8E_{\gamma}}\,.$$

By inserting related values we get

$$E_{p}=1.07\times10^{20}eV\,. \tag{24}$$

And finally

$$E'_{p}=9.45\times10^{19}eV\,. \tag{25}$$

Note that the rate of energy loss of proton is nearly 13% .

IV.DIFFERENT CALCULATIONS OF THE CUT OFF

In this section we will try to answer the following question: what is the maximum energy of a charged particle that can be produced in our galaxy? First consider a source (our galaxy), with radius R, that produces a magnetic field. If a charged particle (let us say a proton) is moving in this magnetic field at a distance r away from the center of the source, it will start to move in circles. If its velocity is $\vec{v}$ and its relativistic mass is given by γm, then the centripetal force, which the proton experiences, can be written as $\frac{\gamma mv^2}{r}$. Since the Lorentz force is the only force exerted on proton, it takes the role of the centripetal force. If we only look at the magnitude of the force and assume $\vec{v}$ perpendicular of $\vec{B}$ then we can write

$$\frac{\gamma mv^{2}}{r}=zevB\,. \tag{26}$$

Obviously the proton's gyroradius *r* of proton cannot be greater than otherwise would fly out of our galaxy. So we have

$$\frac{\gamma mv^{2}}{R}\leq\frac{\gamma mv^{2}}{r}\,. \tag{27}$$

By the definition of momentum $\left(\vec{p}=\gamma m\vec{v}\right)$, we can rewrite this as

$$\left|\vec{p}\right|\leq zeBR\,.$$

Above inequality turns out to be even relativistically correct. Thus the maximum for the magnitude of the momentum is given by

$$p_{\max}=zeBR\,.$$

Since the protons are moving relativistically and c=1 we can conclude that

$$E_{\max}=zeBR\,. \tag{28}$$

A typical value for B in our galaxy is $3\mu G$ and *R* is in order of the size of our galaxy, which is *R*=5pc and *Z*=1 for a proton, so from (28) we have

$$E_{\max}\,\square\,10^{19}eV.$$

Now consider a mental example. Suppose a neutron star falling into a black hole surrounded by low density plasma. The important point in this scenario is to accelerate the star to a speed arbitrarily close to that of light. An observer hovering above the horizon suddenly sees the star rapidly flying by. The changing magnetic field B at the observer's location induces an electric field that accelerates the particles of the surrounding plasma. Acceleration of a given charge lasts for only a brief time, $\Delta t\cong\frac{R}{V}$ where *R* is the radius of the neutron star and *V* is its speed relative to the hovering observer, which near the horizon approaches the speed of light, c. The work W_q done by the induced electric field *E* on a charge *q* is then, roughly,

$$W_{q}\cong qEc\Delta t\cong qER\cong q\frac{\Delta B}{\Delta t}R^{2}.$$

where in the application of Faraday's law we have chosen an Amperian loop in the form of a circle of radius *R*. Taking $\Delta B\cong B$, we get an estimate $W_{q}\cong qBRc$. For iron, with $26\times106\times10^{-19}c$ and the star with $R\cong5\times10^{3}m$, even for a relatively weak magnetic field of $B\cong10^{6}T$ this gives particle energies on the order of $W_{q}\cong1J\cong10^{19}eV$.

Note that in this section we didn't use CMB photons. Now let's go back to the initial question this section by minor modification: What is the maximum energy of a charged particle that can be produced in our world?

Is it possible to answer this question if we do not use CMB?

If we are aware of the real location of the GZK cut off perhaps one can answer above questions.

V. MODIFICATION OF THE GZK CUT OFF

One of the simplest kinematic frameworks for Lorentz invariance violation is to propose modified dispersion relations(MDR) for particles, while keeping the usual energy-momentum conservation laws. In fact modified dispersion relations may provide a bound for both the energy and the momentum of particles, and such effects have greatly been studied on the fate of Lorentz symmetry at extremely high energy level, as well as quantum gravity phenomenology [17, 18].

Also these modified dispersion relations have been used to describe anomalies in astrophysical phenomena such as the GZK cutoff anomaly[19, 20].

Basically, the structure of MDR's is as following

$$E^2 = p^2 + m^2 \pm f\left(l_p, p, E\right). \tag{29}$$

Which $p = \left|\vec{p}\right|$ and l_p is Planck length. If Lorentz invariance violation is associated with quantum gravity, deviation from ordinary Lorentz invariance should appear at the Planck scale. The function f fcontains all the novel Lorentz invariance violation effects. Note that equation of (29) is only one approach for introducing Lorentz invariance violation [21].

We indicate positive sign in third term make a red-shift in GZK equation but the minus sign create blue-shift. At first we consider form of $E^2 = p^2 + m^2 + f\left(l_p, p, E\right)$, (we call it first model) then using equation of (15) we get

$$p_p{}^2 = -m_p{}^2 c^2 - f\left(l_p, p, E\right). \tag{30}$$

So from (13) we conclude

$$E_p = \frac{\left(m_n c^2 + m_\pi c^2\right)^2 - \left(m_\pi c^2\right)^2}{4E_\gamma} - f\left(l_p, p, E\right). \tag{31}$$

Clearly, equation (31) shows a red-shift in GZK cutoff. But with considering $E^2 = p^2 + m^2 - f\left(l_p, p, E\right)$, (we call it second model) the equation (30) becomes

$$P_p{}^2 = -m_p{}^2 c^2 + f\left(l_p, p, E\right). \tag{32}$$

And the equation (31) becomes

$$E_p = \frac{\left(m_n c^2 + m_\pi c^2\right)^2 - \left(m_\pi c^2\right)^2}{4E_\gamma} + f\left(l_p, p, E\right). \tag{33}$$

Which obviously indicates a blueshift in GZK equation. Now we can conclude that increasing the mass of initial particle in the interaction with CMB couses reducing energy threshold in GZK equation and vice versa.

There are a another distinction between equations of (29). It is enough to compare $m^2 - term$ with statement related to violation, $f\left(l_p, p, E\right)$, in the first model so we will have a threshold shift. While there are no thresholds in the second model. To clearly better it is necessary to investigate different models which thier structure are similar to (29).

A famous modified dispersion relation has been suggested as following

$$E^2 = p^2 + m^2 + \eta \frac{p^n}{M^{n-2}}. \tag{34}$$

where M^n is the characteristic scale of Lorentz violation[22]. In this model η is a dimensionless factor and $c = 1$. Considering $n = 3, 4$ so that $n < 3$ ruled out by terrestial experiments and at high energies $n \geq 3$ will dominate. Such corrections might only become important at the Planck scale for example energy thresholds for particle reactions can be shifted. If the $p^n - term$ is comparable to the $m^2 - term$ in equation (34), threshold reactions can be significantly shifted, because they are determined by the particle masses. So a threshold shift should appear at $p_{dev} \approx \left(\frac{m^2 M^{n-2}}{\eta}\right)^{\frac{1}{n}}$. If we consider $n = 3$ then amount of p_{dev} for a neutrino, electron and proton are nearly 1 GeV, 10 TeV and 1 PeV respectively. Also by considering $n = 4$ for these particles we obtain 100 TeV, 100 PeV and 3 EeV the same way. We will conclude immediately by increasing the mass of the particle, the energy thresholds increases for Lorentz symmetry violation.

It can simply to be shown that equation (34) leads to an equation similar to (31).

Another MDR which is proposed as following [23]

$$\frac{1}{\eta l_p} \sin\left(\eta l_p E\right) = \sqrt{p^2 + m^2}. \tag{35}$$

When the Planck length $l_p \to 0$ this relation leads to standard dispersion relation. In this relation E and P are the energy and momentum of a particle with mass m respectively, $\hbar = c = 1$, and η is a dimensionless parameter.

By the Taylor expansion of this relation it will be achieved

$$E^2 = p^2 + m^2 + \frac{\eta^4 l_p{}^4 E^4}{3}. \tag{36}$$

This relation also leads to a red-shift same to (31).

A well studied modified dispersion relation based on phenomenological study is [24, 25]

$$E^2 \approx p^2 + m^2 - \eta\left(l_p E\right)^n p^2. \tag{37}$$

Which l_p is Planck length and η is a coefficient of order 1, whose precise value may depend on the specific model, and n, the lowest power of l_p that leads to a nonvanishing contribution, is also model-dependent. This type of dispersion relation is observed in loop quantum gravity [24] with $n=2$. In ref. [26, 27, 28] it has been argued n can be chosen as $n=1$ or $n=2$. In ref. [24] discussed to understand the different physical scenarios with $n=1$ and $n=2$.

By using (37) the equation of (15) changes to

$$p_p{}^2 = -m_p{}^2 c^2 + \eta \left(l_p E \right)^n p^2 . \qquad (38)$$

By the supercedence the equation of (38) in (13) then the equation of (16) becomes to

$$E_p = \frac{\left(m_n c^2 + m_\pi c^2 \right)^2 - \left(m_\pi c^2 \right)^2}{4E_\gamma} + \frac{\eta \left(l_p E \right)^n p^2}{4E_\gamma}, \qquad (39)$$

which can be written

$$E_p = 3\times 10^{20} eV + \frac{\eta \left(l_p E \right)^n p^2}{4E_\gamma} . \qquad (40)$$

Obviously, we have $E_p > 10^{20} eV$. In equation (39), if we neglect l_p and set $l_p = 0$ then we get back the conventional GZK cut off.

Also the equation (9) will become to

$$E_p \geq \frac{m_\pi{}^2 + 2m_p m_\pi}{4E_\gamma} + \frac{\eta \left(l_p E \right)^n p^2}{4E_\gamma} . \qquad (41)$$

The outcome of (40) is in agreement with the results of AGASA [3] and Hayashida [4] and is very interesting in the sense that it can be looked upon as the extended GZK cutoff.

VI. CONCLUSION

We have tried to get the GZK cutoff in different procedures. It's clear that all of these approaches show roughly the same amount of cutoff (near $10^{19} eV$ or $10^{20} eV$). To justify the Paradox of the GZK cutoff we used from Lorentz invariance violation by the special modified dispersion relation. Some of the MDR models have a threshold to Lorentz invariance violation. But the models that we were looking for to extend of the GZK cutoff do not have of this threshold. Also we showed that any model, modified dispersion relation, cannot justify to extension of the GZK cutoff. Based on our conclusion the equation of (40) indicates clearly a blue shift in the GZK cutoff. So it can be concluded model of (37) is compatible with experimental data such as the AGASA and Hayashida. In fact we conclude that by increasing the mass of initial particle in the interaction with CMB make a reducing energy threshold in the GZK equation and vice versa.

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