Thermodynamic cycle of average annual rainfall over the globe

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Abstract

Once speaking on the energy budget of the Earth Heinrich Hertz had recognized that evaporation and the consequent precipitation over the globe is achieved through a hypothetical steam engine leading to around one meter of annual rainfall. Here, the sea works as boiler, while the layer of air where clouds are formed plays the role of condenser, 15°C colder than its boiler. Hertz had observed that this steam engine can hardly achieve one-third of the corresponding Carnot efficiency. His saying has been pedagogically demonstrated here to be true by estimating the amount of water-vapor evaporating annually over the sea, followed by its lifting to an altitude where it condenses giving us the anticipated rain.

Keywords: Rainfall, oceans, evaporation, condensation, steam engine, Carnot efficiency.

Resumen

Hablando una vez sobre el presupuesto energético de la Tierra, Heinrich Hertz había reconocido que la evaporación y la consiguiente precipitación sobre el globo se logra a través de una máquina de vapor hipotética que conduce a alrededor de un metro de precipitación anual. Aquí, el mar funciona como caldera, mientras que la capa de aire donde se forman las nubes hace el papel de condensador, 15°C más frío que su caldera. Hertz había observado que esta máquina de vapor difícilmente puede alcanzar un tercio de la eficiencia de Carnot correspondiente. Se ha demostrado pedagógicamente que su dicho es cierto al estimar la cantidad de vapor de agua que se evapora anualmente sobre el mar, seguido de su elevación a una altitud donde se condensa dándonos la lluvia anticipada.

Palabras clave: Lluvia, océanos, evaporación, condensación, máquina de vapor, eficiencia de Carnot.

I. INTRODUCTION

In a classic lecture in 1885 bearing topic "On the energy balance of the Earth", Heinrich Rudolf Hertz [1] had mentioned "… the greater portion of the Earth's surface is indeed covered with water, which evaporates under the influence of the incident solar radiation, and indeed not on a very small scale. Every year, we may assume that on the average a water layer a meter high over the entire Earth is converted into water-vapor…" Also "if we consider the evaporated surface of the sea as the boiler, then we must take as the condenser the layer of the air in which the clouds are formed. The latter layer is cooler than the boiler, and it must be so, for otherwise absolutely no conversion of heat into any kind of work would be possible. The difference [in temperature] is, however, not great, we can hardly assume that the difference between the temperature of the Earth and that of the cloud-covering layer amounts to more than 15 °C on the average. But a steam engine, whose condenser is only 15 °C colder than its boiler can, even with the most perfect design, convert at most about 1⁄20 of the heat provided into useful work".

II. DEVELOPMENT

We know that most perfect designed engine follows the Carnot cycle between heat source temperature T_1 and heat sink temperature T_2; in the situation under consideration temperatures are $T_1 = 288\,°K$, and $T_2 = 273\,°K$ and the efficiency so obtained would be

$$\eta_{Carnot} = 1 - \frac{T_2}{T_1} = 1 - \frac{273}{288} \cong 1/20 = 0.05. \qquad (1)$$

Hertz, further elaborates that "The large sun-machine of which we are speaking, however, is far from actually reaching this theoretically possible maximum; probably it achieves scarcely one-third of it". As per Hertz conjecture the real efficiency η_{Real} will be one-third of Carnot efficiency η_{Carnot}, that is 0.017 wide (1). The students are usually eager at this point to examine the above statement through another expression for efficiency, which is the real efficiency

$$\eta_{Real} = \frac{W}{Q_1}. \qquad (2)$$

The overhead symbols will be discussed in the sequel along with the cycle, which describes the processes of evaporation in the boiler ($T_1 = 288\,°K$) at vapor pressure $P_{288K}^{vapor} = 1704.1\,Pa$ [2] and condensation at an altitude where $T_2 = 273\,°K$ and vapor pressure $P_{273K}^{vapor} = 610.8\,Pa$; the process of phase transitions occurring at both ends having a temperature difference of $15\,°C$. This fact advises that we have to follow the cycle [3] shown in figure 1 which describes it perfectly. The solar radiation provides latent heat Q_1 [4] to evaporate sea water along the path $1 \rightarrow 2$ displayed in the PV diagram portraying the phase transition. The vapor so formed moves up to a lower vapor pressure $\left(P_{273K}^{vapor}\right)$ and a lower temperature (T_2) point 3 [5] where it starts condensing following the path $3 \rightarrow 4$. At the point 4 the condensation process concludes and the rain water dropping to the surface of oceans, thereby beginning the cycle once again as of point 1.

The work so performed

$$W = \left(P_{288K}^{vapor} - P_{273K}^{vapor}\right) \cdot \left(V_{vapor} - V_{water}\right), \tag{3}$$

where V_{water} is the volume of water evaporating annually, while V_{vapor} is the corresponding volume of water vapor through $V_{vapor} = \nu_{vapor} \cdot M_{water}$; here $\nu_{vapor} = 77.97\,m^3 \cdot kg^{-1}$ is the specific volume of water vapor [2] generated per kilogram of evaporated water, and $M_{water} = D \cdot V_{water}$ is the average mass of water evaporating annually; the density of water $D = 10^3\,kg \cdot m^{-3}$. The volume as well as the mass of water evaporating annually, which is around a meter thickness water layer over the sea can be evaluated through [6]

$$V_{water} = \frac{4\pi}{3}\left[R^3 - (R-1)^3\right] \cong 4\pi R^2 = 5.1 \cdot 10^{14}\,m^3 \cdot yr^{-1}; Radius\ of\ Earth\ (R) = 6.371 \cdot 10^6\,m$$

$$M_{water} = D \cdot V_{water} = 10^3 \cdot 5.1 \cdot 10^{14} = 5.1 \cdot 10^{17} kg \cdot yr^{-1} \tag{4}$$

The corresponding volume of water vapor produced annually can also be evaluated as

$$V_{vapor} = \nu_{vapor} \cdot M_{water} = 77.97 \cdot 5.1 \cdot 10^{17} m^3 \cdot yr^{-1} = 3.98 \cdot 10^{19}\,m^3 \cdot yr^{-1} \tag{5}$$

Thus, the work part which is performed annually comes out to be via (3)

$$W = (1704.1 - 610.8) \cdot (3.98 \cdot 10^{19} - 5.10 \cdot 10^{14}) = 4.35 \cdot 10^{22} J \cdot yr^{-1} \tag{6}$$

The denominator part of the expression (2), that is Q_1 can be counted using

$$Q_1 = M_{water} \cdot L = 5.1 \cdot 10^{17} \cdot 2.46 \cdot 10^6 = 1.26 \cdot 10^{24} J \cdot yr^{-1}; Latent\ heat\ (L) = 2.46 \cdot 10^6\,J/kg \tag{7}$$

Thus, the efficiency of evaporation-precipitation cycle appears to be

$$\eta_{Real} = \frac{W}{Q_1} = \frac{4.35 \cdot 10^{22}}{1.26 \cdot 10^{24}} \cong 0.035. \tag{8}$$

This value is, of course, less than the Carnot efficiency estimate (1) in agreement with the second law of thermodynamics, however, it is still twofold compared to Hertz inference of 1.7%. This is so because, in actual atmosphere, it is the mixture of air and water vapor, and not only water vapor as supposed in the calculations here for teaching purpose.

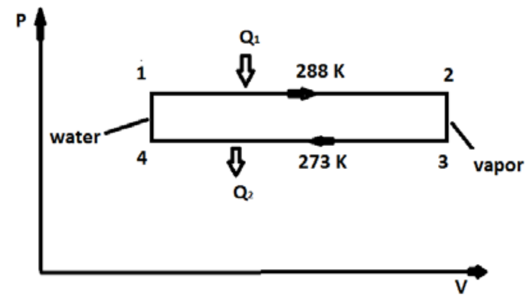

Figure 1. Pressure-volume diagram of the evaporation process (1→2), lifting of vapor (2→3) followed by condensation (3→4) and finally rain (4→1) over the globe. It may be added that the marginal change in the volume of water-vapor when it moves up to the lower vapor pressure point 3 has been ignored.

REFERENCES

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[2] Kuzman, R., *Handbook of Thermodynamic Tables and Charts* (Hemisphere, Washington, D.C., 1976), p. 105

[3] Chernoutsan, A., *The friction and pressure of skating*, Quantum **4**, 25-27 (July/August 1994); https://static.nsta.org/pdfs/QuantumV4N6.pdf

[4] King Hubbert, M., *Outlook for fuel reserves*, Encyclopedia of Energy edited by Daniel N Lapedes (McGraw-Hill, New York, 1976) pp. 11-23; Solar power going to evaporation and precipitation channel has been proposed to be 4.0·〚10〛^16 W; this assessment is consistent with round one meter average annual rainfall over the globe [6].

[5] It may be added that there is a marginal change in the volume of water-vapor when it moves up to the lower vapor pressure point 3; see figure 1.

[6] Agrawal, D. C., *Average annual rainfall over the globe*, The Phys Teach **51**, 540-1 (2013).