Harmonic Distortion in Radio Frequency Signals Introducing Nonlinear Systems

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Abstract

This paper presents an experiment aimed at introducing undergraduate physics students to nonlinear systems and harmonic generation. By analyzing the spectrum of a radio-frequency signal that has been clipped by silicon diodes, students can explore the differences between linear and nonlinear systems. The experiment demonstrates that asymmetric clipping introduces even harmonics, which are typically absent in symmetrically clipped waveforms. Harmonic distortion, characterized by the addition of unwanted frequency components, is quantified using Total Harmonic Distortion (THD), highlighting its dependence on the degree of clipping. Through this hands-on approach, students gain a deeper understanding of the practical implications of harmonic distortion in electronic circuits and its relevance to fields such as radio communication and audio engineering. By bridging the gap between theoretical concepts and practical observations, this experiment makes the study of nonlinear phenomena and harmonic generation accessible to undergraduate physics education, a subject often reserved for research involving high-power lasers.

Keywords: Nonlinear systems, Harmonic generation, Undergraduate physics education.

Resumen

Este artículo presenta un experimento destinado a introducir a estudiantes de física de grado a los sistemas no lineales y la generación de armónicos. Mediante el análisis del espectro de una señal de radiofrecuencia recortada por diodos de silicio, los estudiantes pueden explorar las diferencias entre sistemas lineales y no lineales. El experimento demuestra que el recorte asimétrico introduce armónicos pares, que suelen estar ausentes en formas de onda recortadas simétricamente. La distorsión armónica, caracterizada por la adición de componentes de frecuencia no deseados, se cuantifica mediante la Distorsión Armónica Total (THD), destacando su dependencia del grado de recorte. Mediante este enfoque práctico, los estudiantes adquieren una comprensión más profunda de las implicaciones prácticas de la distorsión armónica en circuitos electrónicos y su relevancia en campos como la radiocomunicación y la ingeniería de audio. Al conectar los conceptos teóricos con las observaciones prácticas, este experimento facilita el estudio de los fenómenos no lineales y la generación de armónicos a la formación de física de grado, un tema a menudo reservado para la investigación con láseres de alta potencia.

Palabras clave: Sistemas no lineales, Generación de armónicos, Educación en física universitaria.

I. INTRODUCTION

The transmission of a signal through a circuit can lead to distortion, which is characterized by a change in the waveform of the signal [1]. This distortion is particularly common when the circuit responds nonlinearly to the input signal, resulting in the generation of harmonics - frequencies that are multiples of the fundamental frequency of the input signal [2, 3, 4, 5]. Understanding the impact of harmonic distortion on radio frequency signals offers a valuable opportunity to introduce students to nonlinear physical systems and the concept of harmonic generation [6, 7, 8, 9, 10].

Nonlinear systems, often represented mathematically by nonlinear differential equations, are prevalent in physics. They encompass a wide range of phenomena, from the motion of an anharmonic pendulum in classical mechanics [11, 12, 13] to the behavior of an LRC circuit in electronics [14]. Unlike linear systems, nonlinear systems often lack analytical solutions, requiring numerical methods for analysis [15, 16, 17].

Harmonic generation, a key characteristic of nonlinear systems, occurs in various physical contexts, such as studying standing waves on strings and acoustic waves in tubes. In electromagnetic waves, harmonic generation arises from the interaction between light and matter, especially in media with nonlinear optical properties [18, 19]. Traditionally, observing harmonic generation has been limited to research laboratories that are equipped with high-power pulsed lasers, making it inaccessible to most undergraduate students.

To address this gap, this paper proposes a novel and accessible experiment that allows students to explore both nonlinear systems and harmonic generation. By utilizing a simple circuit made up of readily available electronic components, a signal generator, and a digital oscilloscope, the experiment demonstrates how clipping a periodic radio-frequency signal- achieved with two silicon diodes - results in harmonic distortion. Additionally, the experiment facilitates a comparison between linear and nonlinear regimes, giving students a hands-on understanding of the distinct behaviors exhibited by these systems.

To quantify the distortion in the resulting signal, this experiment uses total harmonic distortion (THD), a commonly employed metric in the analysis of sound and radio communication systems [1]. Finally, the relationship between THD and the degree of clipping applied to the input signal is examined. Due to the simplified analysis involved in this experiment, it is well-suited for undergraduate physics students, providing an engaging and intuitive introduction to nonlinear physical systems and the concept of harmonic generation.

II. EXPERIMENTAL SETUP

Figure 1 shows the schematic diagram of the circuit setup. A signal generator produces a 1 kHz sine wave with an amplitude of 5 V, which is carried to a 1 kΩ resistor *R*. Shunt to the ground are two parallel branches, each including a 1N4007 silicon diode in series with a 10 kΩ potentiometer. The diodes are connected in anti-parallel configuration, with their anodes connected to opposite ends of the resistor *R*.

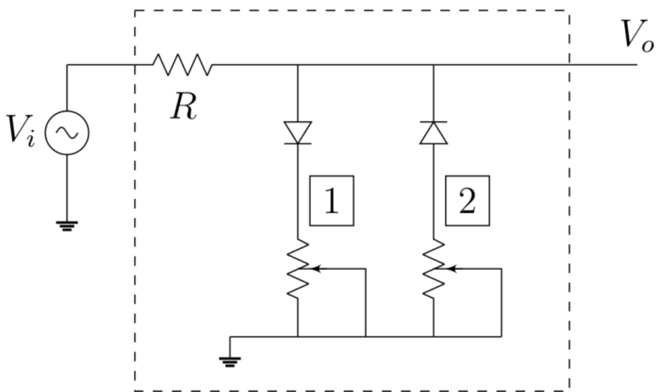


FIGURE 1 . Schematic diagram of the circuit. The periodic input signal V_i is subject to harmonic distortion (nonlinear effect) after passing through the system (circuit enclosed by the dashed line). As a result, the output signal V_o may contain harmonics of the input signal.

By adjusting the potentiometer resistance, one can regulate the flow of current through these parallel branches. Each diode rectifies the oscillating signal, allowing current to pass in only one direction, effectively clipping either the positive or negative segments of the waveform's amplitude. In Figure 1, the diode in branch 1 clips the positive part, while the diode in branch 2 clips the negative part. Consequently, depending on the potentiometer resistance, there may be an asymmetry in the upper and lower parts of the resulting waveform. A digital oscilloscope (POL-15DE, Politerm), with a sampling rate of 1 GS/s and an input impedance of 1 MΩ, is employed to measure the voltage after the electrical shunts, revealing the output signal.

III. CLIPPER MODEL

The circuit shown in Figure 1 exhibits a nonlinear response when the output voltage is not directly proportional to the input voltage. Conversely, in a linear regime, the input and output signals are directly proportional, with the output being a scaled and possibly time-shifted version of the input. To illustrate this, consider an input sinusoidal voltage with amplitude A and frequency f, expressed as $V_i(t) = A\,sin(2\pi f t + \varphi)$, where φ is the phase constant. For a linear response, the output voltage would be:

$$V_o(t) = GA\sin(2\pi f t + \varphi + \theta) = GV_i(t - \tau)$$

where *G* is a frequency-independent constant representing the attenuation (or amplification) of the signal. θ represents the phase shift, which introduces a delay τ between the input and output signals.

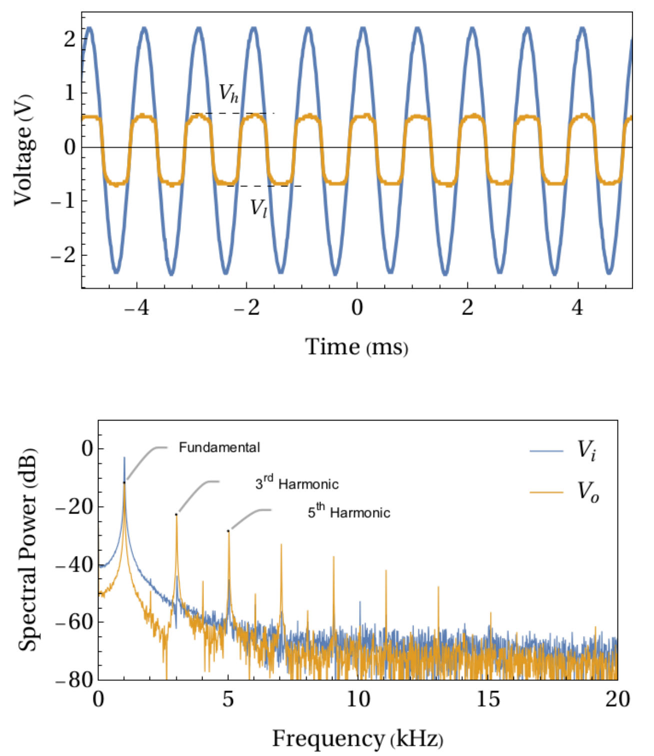


FIGURE 2. Waveforms and their respective Fourier transforms recorded with the oscilloscope. The top panel shows the input sinusoidal signal with 1 kHz frequency and the output signal with clipped amplitude. The amplitudes V_h and V_l are controlled by the potentiometer resistances, affecting the degree of clipping and thus the harmonic content. The bottom panel shows the corresponding frequency spectra, illustrating the presence of harmonics (for example, third and fifth) in the clipped signal, which are absent in the pure sine wave input.

However, for a nonlinear response, the output signal contains not only the scaled and shifted input signal but also additional frequency components that are integer multiples of the fundamental frequency. These additional frequencies are called harmonics, and their presence alters the shape of the waveform. The output signal can thus be expressed as:

$$V_o(t) = GA\sin(2\pi f t + \varphi + \theta) + A_2\sin(2\pi f_2 t + \theta_2) + A_3\sin(2\pi f_3 t + \theta_3) + ,\cdots \quad (1)$$

where $f_n = n\,f$ are the harmonics of the fundamental frequency f. The nonlinear behavior of the diodes in the circuit, specifically the clipping action, is responsible for the generation of these harmonics. The presence of harmonics is clearly visible in the frequency domain, as shown in Figure 2.

In Figure 2, the output voltage V_o is significantly modified compared to the input V_i, deviating from a simple scaled and shifted version. This alteration in waveform is termed harmonic distortion. To quantify the extent of this distortion, the total harmonic distortion (THD) is often used [1]. THD is defined as:

$$\text{THD}\,(\%) = 100\sqrt{\frac{|A_2|^2 + |A_3|^2 + \cdots}{|A|^2}}. \quad (2)$$

THD represents the ratio of the total power in the harmonic components to the power of the fundamental frequency. A pure sine wave has a THD of zero, while a signal with significant harmonic distortion will have a higher THD value. In this experiment, the amplitudes of the harmonics can be directly measured from the peak values in the Fourier transform displayed on the oscilloscope.

IV. RESULTS AND DISCUSSION

Figure 3(a) shows the relationship between the output voltage (V_o) and input voltage (V_i) for both the unclipped signal (no diode influence) and the clipped signal (with diode clipping). In the unclipped case, the relationship is linear, with a slope coefficient of 0.92, indicating a slight attenuation of the input signal. In the clipped case, the relationship deviates from linearity, exhibiting a plateau for input voltages exceeding the clipping threshold. This behavior can be approximated by an error function, which captures the gradual transition from the linear region to the saturated (clipped) region.

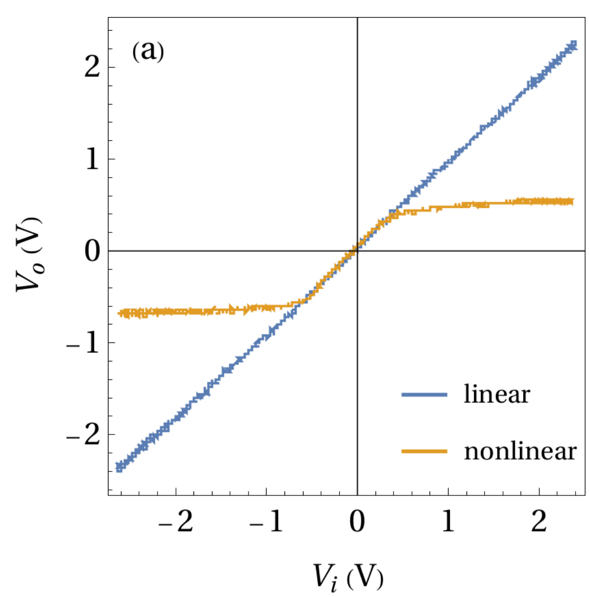
Figure 3(b) shows the variation of THD as a function of the inverse of the clipping voltage V_c, for the symmetric case where $V_c \equiv V_h = V_l$. We can observe that the THD ranges from nearly zero, indicating the absence of distortion, to 33.4% when V_c = 0.67 V. The results show a linear dependence between THD and the clipping voltage, as indicated by the dashed line obtained through linear fitting. This behavior is expected since, according to Eq. (2), the THD rate is proportional to the inverse of the fundamental mode amplitude, while the clipping voltage is directly proportional to the fundamental.

A key observation from the frequency spectrum in Figure 2 is the absence of even harmonics in the symmetrically clipped signal. This can be explained by the odd symmetry of the waveform, which means that one half of the waveform is the negative of the other half. Mathematically, this is expressed as $V_o(t) = -V_o(t + T/2)$, where T is the period. The Fourier series representation of such a waveform only contains sine terms with odd multiples of the fundamental frequency, leading to the absence of even harmonics. However, the circuit can be configured to produce asymmetric clipping, where the positive and negative peaks of the waveform are clipped at different levels. This breaks the odd symmetry and introduces even harmonics into the spectrum.

V. CONCLUSIONS

In this study, we demonstrated how a simple circuit using silicon diodes can be employed to clip a sine wave, introducing harmonic distortion and creating a nonlinear system. The experimental results clearly illustrate the difference between linear and nonlinear responses, with Total Harmonic Distortion (THD) increasing as the clipping becomes more pronounced. This linear relationship between THD and the inverse of the clipping voltage provides a quantitative measure of the degree of nonlinearity introduced by the circuit.

This hands-on approach serves as a valuable educational tool for undergraduate physics students, allowing them to directly explore the concepts of nonlinearity, harmonic generation, and signal distortion in a practical setting. By manipulating the circuit parameters and observing the resulting waveforms and frequency spectra, students gain a deeper understanding of these fundamental phenomena. Although this experiment provides a simplified model of harmonic distortion, it serves as an excellent starting point for further investigations. Future work could explore the effects of temperature on diode behavior and harmonic generation, investigate different clipping mechanisms, or extend the analysis to include higher-order harmonics. Additionally, the experiment could be adapted to examine other nonlinear phenomena, such as intermodulation distortion or frequency mixing.



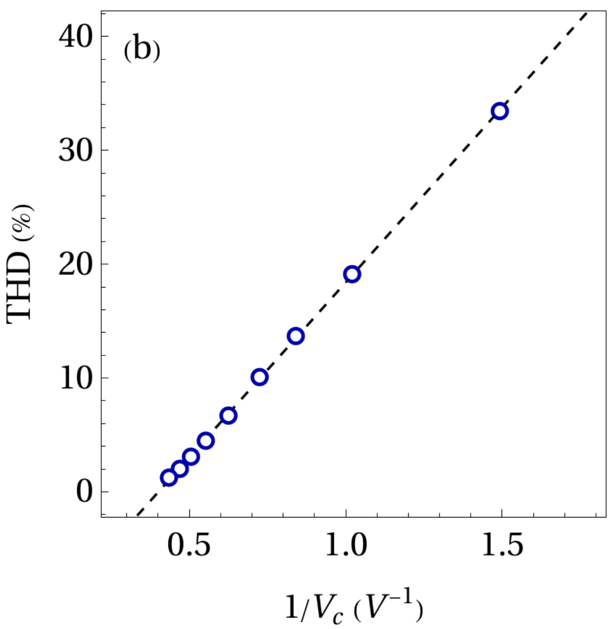


FIGURE 3. (a) Output voltage (V_o) as a function of input voltage (V_i) for the unclipped and clipped signals. (b) Total Harmonic Distortion (THD) as a function of the inverse of the clipping voltage ($1/V_c$) for the symmetric clipping case ($V_c \equiv V_h = V_l$). The dashed line represents a linear fit to the data points.

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