The Wimshurst machine as an electric circuit

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Resumen

Se muestra una forma alternativa de visualizar la operación de una máquina de Wimshurst por medio de un circuito eléctrico equivalente. Este circuito traslada el concepto de la inducción electrostática cruzada presente en los discos de la máquina en términos de elementos de circuito conocidos. Se muestra un análisis sucinto de este circuito.

Palabras clave: Máquina de Wimshurst, generador electrostático, circuito equivalente.

Abstract

It is shown an alternative way to view the operation of the Wimshurst machine using an equivalent electrical circuit. This circuit translates the idea of cross-couple electrostatic induction present in the machine disks in terms of known electric circuit elements. A brief analysis of the circuit is offered**.**

Keywords: Wimshurst machine, electrostatic generator, equivalent circuit.

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I. INTRODUCTION

The Wimshurst machine is one of those devices with extensive presence in all basic electricity laboratories. For the generation of electrostatic charge it is preferred for its simplicity, reliability and elegance. Unlike the Van de Graaff generator, their maintenance problems are lower. Ironically, however, the main problem in the context of teaching is that its operating principle is more difficult to explain than that of the Van de Graaff generator.

The operation of a Wimshurst machine in any of its version is based on two concepts: charge transfer by electrostatic induction and charge storage in a capacitor. A third idea allows the operation of a Wimshurst machine: electrostatic regeneration or, in modern terms, positive electrical feedback.

We are not going to provide an exhaustive description of the functioning of the Wimshurst machine ever since there are many good references thereon [1, 2]. We will just summarize the essential features of its operation and then transfer these to an equivalent electrical circuit that we consider will improve their understanding

II. WIMSHURST MACHINE OPERATION

Even though there are several versions of the Wimshurst machine, we will focus on one of the best known which is shown in figure 1.

The classic Wimshurst machine consists of two nonconductor parallel discs rotating in opposite directions on a common shaft driven by a crank. These discs have attached on their external sides a series of radially oriented metal strips. The inner plate of two Leyden bottles makes contact with the strips through brushes which serve as commutator as the discs turn. (Other models do not use brushes but an inductive comb). From these plates come out two rods whose ends generate high voltage sparks when they are close enough. The outside bottles plates make contact through a grounded wire. Each disc has a crossed metal rod which joins the strips diametrically placed by means of brushes. The rods are named neutralizers rods. This rods looks like an X when looked from the front side. Figure 2 shows an outline from the machine: the front disc with their stabilizer rod Y-Y'; the rod Z-Z' is from the rear disc.

FIGURE 1.The classic Wimshurst machine.

Let's suppose that there is initially some charge $+Q_0$ in a strip from the front disc and other charge $-Q_0$ in the matching strip from the rear disc. When the discs start rotating, the charged strips induce charge with opposite sign upon the front strips crossing their way. When these strips touch the brush from their respective rods, it is neutralized with the charge coming from the strip in the far end from the same disc; at the same time, the corresponding strips from the other disk induce charge over the neutralized strips bringing their polarity reversed and their magnitude amplified. As the discs turn, strips progressively accumulate more and more charge. We then have a cyclical process in which half the strips from each disc alternate their polarity as they touch the ends of their neutralized bars. The right bottle accumulate positive charge as its brush make contact with the right side strips from both discs, and the left bottle accumulate negative charge as its brush make contact with the left side strips from both discs. When the charge density in the output electrodes becomes sufficiently high, a dielectric breakdown is produced and a spark is generated. It should be noted that the charge increase in the Leyden bottles comes from two facts: the steady charge transfer coming from the strips, and the continuous increase in the magnitude of these charges due to the regeneration carried out by the neutralizer bars. So, this is the basic qualitative explanation of the operation of the Wimshurst machine.

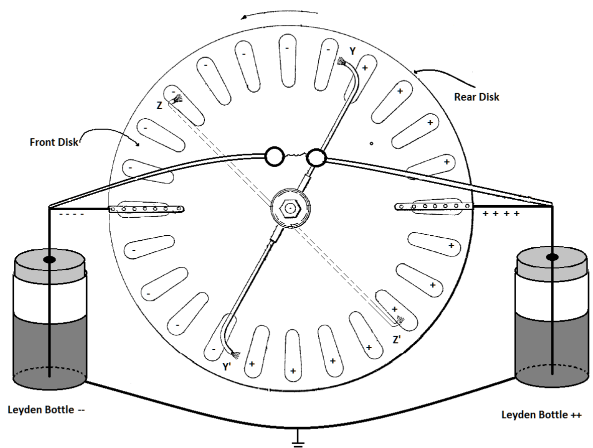

FIGURE 2. Outline of the Wimshurst machine.

III. THE EQUIVALENT CIRCUIT

It is possible to make a mathematical analysis of the physical behavior of a Wimshurst machine based only on the description of their induction and charging mechanisms already described, but this is not an easy job to do neither to explain [3]. On the other hand, it would seem strange to assume that the Wimshurst machine can be represented as an electrical circuit. No structure can be seen containing neither current nodes nor closed paths; nevertheless, it can be assumed that its elements are capacitors in a periodic switching process.

Since the Leyden bottles in a Wimshurst machine are capacitors which are periodically recharged through the metal strips of the discs and these, in turn, may be considered as capacitors given that they store charges, it would be natural to consider the Wimshurst machine outputs as a capacitor C which is charged periodically by smaller capacitors C_i. The latter refer to the capacitance between each strip and ground.

In order to represent the charge mechanism we will make use of the circuit shown in figure 3. In this, we have a capacitor C with an initial charge Q been recharged by another capacitor C_i which carry a charge Q_0; the recharge –charge transfer- is made through the switch S.

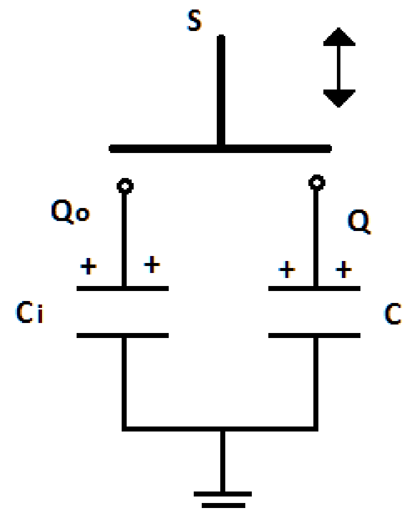

FIGURE 3. Recharging capacitor $\boldsymbol{C}$ from another capacitor $\boldsymbol{C_i}$.

The corresponding capacitor voltages are $\boldsymbol{V=Q/C}$ and $\boldsymbol{V_i=Q_0/C_i}$. When the switch S closes the capacitors they stay in parallel and redistribute their charge. Their common voltage will become:

$$V = \frac{Q+Q_0}{C+C_I}. \tag{1}$$

If $\mathbf{C \gg C_i}$ the voltage increase will be:

$$\Delta V = \frac{Q_O}{C}. \tag{2}$$

If now the switch $\boldsymbol{S}$ opens, capacitor $\boldsymbol{C}$ will be left with a voltage gain given by equation (2). Again, if capacitor $\boldsymbol{C_i}$ is recharged with a similar charge $\boldsymbol{Q_0}$ and the process is repeated closing and opening the switch, the voltage gain will be twice the indicated by equation (2). After repeating this process $\boldsymbol{n}$ times the final voltage increase in capacitor $\boldsymbol{C}$ will be:

$$\Delta V(n) = n\frac{Q_0}{C}. \tag{3}$$

Obviously, the charge gain is nQ_0. Note that if the condition $C \gg C_i$ is not fulfilled the voltage gain may not occur. All would depend on the magnitude of Q_0.

Now, look at the circuit in figure 4 showing the connections between the Leyden bottle, capacitors C_p and C_n, and the metal strips represented by capacitors C_x and C_y. Capacitors C_x are the strips touching C_p via brushes represented by rotary switch S_p; capacitors C_y are the strips touching C_n via brushes represented by rotary switch S_n

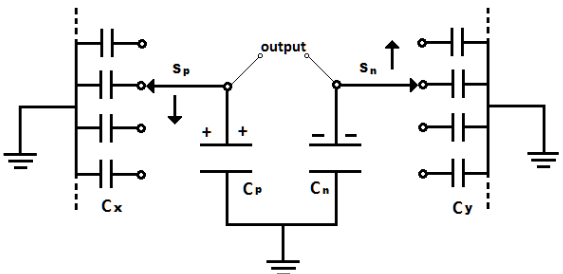

FIGURE 4 Switching process between the metal strips and the Leyden bottles.

This circuit shows that the charging process of C_p and C_n is similar to that shown in figure 3; the only difference is that in this case these capacitors receive charge from multiple capacitors in a switching sequence: C_x in one end and C_y from the other.

Let's suppose that the strips carry a constant charge Q_0, that the disc rotation frequency is F turns per second and that there are m strips per disc; therefore the voltage increase in capacitors C_p and C_n per second, according to equation (3) will be:

$$\Delta V = mF\frac{Q_0}{C}. \tag{4}$$

Which corresponds to an increase in charge of mFQ_0 every second. This increased charge can be viewed as a current injection i taking place in the output capacitors. The current magnitude is

$$i = mFQ_0. \tag{5}$$

In order to include the injection process and the resultant voltage increase in capacitors C_p and C_n we will change the inducer capacitors C_x and C_y and the switch by a current source as can be seen in figure 5.

Taking into account that in a capacitor $i=CdV/dt$, the current source produces a linear incremental voltage provided that mFQ_0 remains constant.

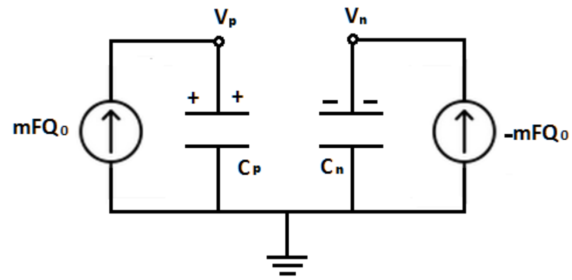

FIGURE 5 Equivalent circuit using current sources.

Remember that Q_0 is an initial charge on any strip whose magnitude change by the induction process. This means that the charge Q_0 from the current sources is not constant but time varying because they interact with the opposite capacitor voltages. In order to include this process in the circuit, we will have to recall that in the induction process, capacitors C_x and C_y interact with each other through their mutual capacitance in a sequential process. The induced charge magnitude depends, among other factors, from the inducer strip's voltage. Since this potential –referred to ground- is approximately equal to the output voltage capacitor (C_p or C_n), we can say that Q_0 in the current sources must be changed to other one proportional to the capacitor voltage. This means that we have a new charge $Q'_0 = kV_C$, where V_C is the voltage capacitor and **k** is a constant which depends of the strip area, the strips separation and other factors. Therefore, the current sources from figure 6 are really ***voltage dependent current sources***. The left source depends from the voltage V_n and the right source depends from voltage V_p. Their current magnitude is thus

$$i = mkFV_C. \tag{6}$$

The final circuit is shown in figure 6.

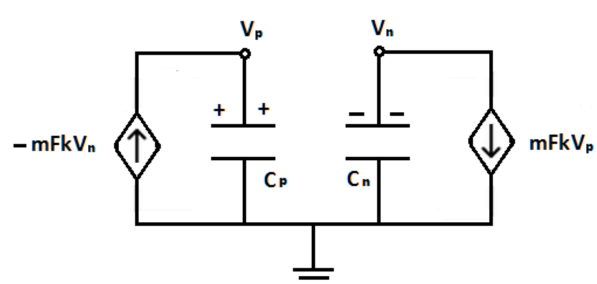

FIGURE 6. Wimshurst machine equivalent circuit.

The negative sign in the left source gives the correct polarity in the capacitor C_p. These dependent current sources clearly illustrate the feedback mechanism present in the Wimshurst machine.

It is possible to foresee that voltages $\boldsymbol{V_p}$ and $\boldsymbol{V_n}$ will increase exponentially if the capacitors have some initial charges. As we have shown, with a constant current source applied to a capacitor the voltage will increase linearly. But if the sources are not constant but increases with time, the voltage increases faster than linear. In fact, if the circuit is solved, it can been show that $\boldsymbol{V_p}$ and $\boldsymbol{V_n}$ grow in an hyperbolic way. Figure 7 shows the circuit solution for $\boldsymbol{V_p}$ with symmetric initial condition and $\boldsymbol{C=1nF}$, $\boldsymbol{V_p(0)=1, V_n(0)=-1v}$ and $\boldsymbol{mFK=10^{-9}}$

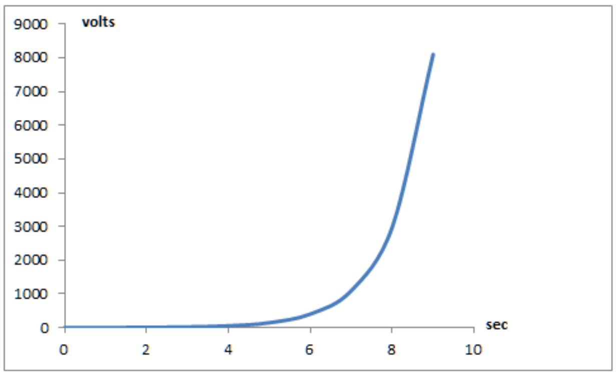

FIGURE 7. The hyperbolic voltage behavior of $\boldsymbol{V_p}$

This solution is

$$V_p(t) = Cosht + Senht \cdot \tag{7}$$

The circuit shown represents only an approximation to the real behavior of the Wimshurst machine. Some secondary facts that have not been considered are:

- The induced charge in a strip is not only due to the front strip but from the adjacent strips.
- In all the capacitors considered there is some leakage charge.
- The strip's capacitances are not really constant.

IV CONCLUSION

The high voltage generation process in a Wimshurst machine involves three interrelated concepts: electrostatic induction, capacitor recharge and electric feedback. When these ideas are translated to an electric circuit containing capacitors, switches and current sources it is possible to improve the comprehension of the operation of this apparatus and facilitate their analysis. The circuit shown meets these characteristics and also allows quantitative assessment of their performance.

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