Solitons propagation in a self-refractive waveguide

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Abstract

We show a simulation for the free propagation of a Gaussian and multi-spatial soliton beam in a self-refractive waveguide through the Beam Propagation Method (BPM). We observed and conjectured that a Gaussian beam can be decomposed into Solitons by the propagation in a self-refractive media. We observe too that given the presence of nonlinearity these spatial solitons displays also self-bending phenomena.

Keywords: Solitons, photorefractive and Kerr effects, nonlinear waveguides.

Resumen

Mostramos una simulación para la propagación libre de un haz solitónico multi-espacial en una guía de onda auto refractive mediante el Método de Propagacion de Haz (BPM). Observamos y conjeturamos que un haz Gaussiano puede ser descompuesto en Solitones por la propagación en un medio auto-refractivo. Observamos también que dada la presencia de no linearidad estos solitones espaciales muestran también el fenómeno de autoflexión.

Keywords: Solitones, efectos fotorefractivo y de Kerr, guía de ondas no lineal.

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I. INTRODUCTION

The soliton was discovered with the experimental computer aid by the mathematicians Zabusky and Kruskal [1], but researchs on the topic began in the 19th century when Russell observed a big solitary wave in a watercourse near Edinburgh. His observation was reported to the British Association in 1844 [2]. He showed that this solitary wave has many particle properties, e.g., an elastic interaction, from the analogies with particles Zabusky and Kruskal named these waves, solitons.

In 1973 Hasegawa and Tapper [3] proposed that the pulse of the soliton could be useful in optical communications through of the constructive interplay in between nonlinearity and dispersion. They showed that the solitons propagate according to the nonlinear Schroedinger equation (NLS); this equation was solved previously by Zakharov [4] and by Satsuma and Yajima [5] with the inverse scattering method. Seven years later Mollenauer [6] showed experimentally the solitons propagation in an optical fiber.

Recently, optical spatial soliton studies have been taken up [7, 8] due to the increasing need to transfer data in a faster and more efficient way as the optical technologies advance call for. Nowdays a good knowledge is important for the applications in information processing, considering the advantage provided by the nonlinearity in the media. Motivated by the above, we have made the present work.

II. SOLITONS

A. Nonlinear wave equation in a Kerr medium

Nonlinearity is a property of the medium through which the light travels, and one important effect is the self-focusing, producing changes in the refraction index due to charges distribution in the cristal [9]. Although the optical field is smaller than the interatomic field, still focused with a laser, the nonlinearity is weak but observable. The relationship between the polarization vector $\vec{P}$ and the electric field vector $\vec{E}$ can be expressed as [10]:

$$P = \epsilon_0 \chi^1 E + 2\chi^2 E^2 + 4\chi^3 E^3 + \cdots,$$

where in general χ^1, χ^2 and χ^3 are tensors. The nonlinear equation, for a media that does not respond instantaneously to the electric field $\vec{E}$ and nonlinear polarization $\vec{P}_{NL}$, whether $E \to -E$, $P \to -P$, $\chi^{(2)} = 0$, $\chi^{(1)}(t-\tau) = 0$, and $P_{NL}(E) = \frac{\epsilon_0 a_3}{w^2} E^3$, is:

$$\nabla^2 E(x,z,t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} E(x,z,t) == \frac{1}{\epsilon_0 c_0^2}\frac{\partial^2}{\partial t^2}\left(\frac{\epsilon_0 a_3}{w^2} E^3\right). \quad (1)$$

We propose the next harmonic solution to the equation (1)

$$E(x,z,t) = \frac{1}{2}[\psi e^{(i\omega t)} + \psi^* e^{(-i\omega t)}],$$

with $\psi = \psi(x,z)$.

Now we consider a dielectric material block with thickness dz, and inside this block we propose plane-waves forms: $\psi(x,z) = |\psi| e^{(-ik_x x - ik_z z)}$, with $|\psi|$ constant inside of the dielectric.

When an intense beam travels in a nonlinear homogeneous media the refractive index is changed in a nonuniform way, such that the media acts like a guide for its own light, this means, the beam makes its own guide. Whether light intensity has the same spatial distribution in the transverse plane, the beam propagates self-consistently without change in its spatial distribution. In this condition the diffraction is compensated by the nonlinear effect and the beam is confined to its own self-created guide; this self-guided beam is called a soliton. This “self-guided” light in a Kerr optical medium is described by the Helmholtz equation

$$[\nabla^2 + n^2(I)k^2]E = 0, \quad (2)$$

where the refractive index is function of the light intensity (I). Equation (2) can be expressed in the paraxial form. Considering now small nonlinear effects, this means that we can express the refractive index like $n(I) = n + n_2 I$, with $n_2 I << n$, and $I = \frac{|E|^2 n}{2\eta_0}$, then we have

$$\frac{\partial^2 A}{\partial x^2} + \frac{\eta_2}{\eta_0} k^2 |A|^2 A = 2ik\frac{\partial A}{\partial z} . \quad (3)$$

Equation (3) is the Schröedinger nonlinear equation and one solution is [5]

$$A(x,z) = A_0 \operatorname{sech}(\frac{x}{W_0}) \exp(-i\frac{z}{4z_0}) , \quad (4)$$

where W_0 is constant. A_0 satisfies $\frac{n_2 k^2 A_0^2}{2\eta_0} = \frac{1}{W_0^2}$, and $z_0 = \frac{1}{2} k W_0^2$ is called the Rayleigh range [10].

Applying the paraxial approximation we obtain

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{3}{4}\frac{a_3}{c_0^2} k^2 |\psi|^2 \psi = 2ik\frac{\partial \psi}{\partial z} . \quad (5)$$

Finally, comparing equations (3) and (5) we get,

$$\psi = A_0 \operatorname{sech}\left[A_0 \left(\frac{3}{8}\frac{a_3}{c_0^2}\right)^{\frac{1}{2}} x\right]. \quad (6)$$

B. Beam Propagation Method (BPM)

Now the next task is to propagate an electromagnetic field in a guide with length $= L$. Ad hoc, we use the Beam Propagation Method (BPM) [11], this method divides the distance L into n intervals of length Δz and this is further divided in two. Considering now the jth interval; we carry out a free propagation in the first half of Δz; exactly in the central plane we perform a phase correction that mainly depends on the refractive index changes. Finally, in the second half, we carry out another free propagation between the last two planes.

III. RESULTS

Firstly, we show the characteristic of the self-refractive effect and finally the solitons propagation. In our daily life, we can see that a beam of light propagating in a media with constant refraction index disperses gradually as it move away from the source, e.g., the light of a lamp in the field. This method does not work to send information between distant points. To simulate this, we considered a Gaussian beam propagation through a waveguide with length $l = 5.0$, and width $d = 3.0$ (all units measured in length units), refractive index $n1 = 2.5$ and on air substrate $n2 = 1.0$, see Fig. 1. We see in this case (Fig. 1a) that the beam come into the guide normally $(\theta = 0^o)$, in the up to down direction, with wavelength $\lambda = 0.02$, width=1.0 and located in the center of the guide $(x_0 = 0)$. We see that the light is dispersed through the propagation in the waveguide, from now on we show in gray scale the light intensity, associating the white light to the maximum intensity and the black color to the zero intensity. In Fig. 1 b) we show the transversal sections of the electric field intensity $|E|$, and we highlight the small effects.

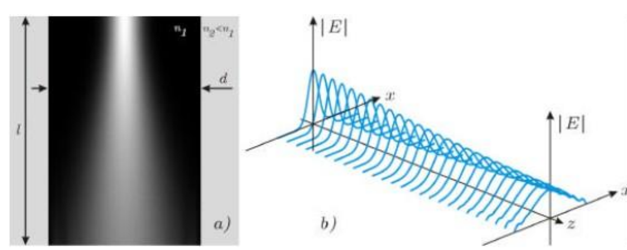

FIGURA 1. a) This graphic represents the laser intensity that propagate through a waveguide with constant refractive index. The plot in b) shows the transversal sections magnitude of the electric field for the propagation of light in the linear media.

As mentioned before, in a cubic nonlinearity the self-refractive effect appears changing the refraction index function of the light intensity, *i.e.*, $n_g = n_1 + \frac{3}{8}\frac{a_3}{k_0^2}|E|^2$. When the cubic parameter a_3 is negative, the refraction index is lower than n_1 in regions where the intensity $|E| > 0$, particularly is minimum in the center of the guide where the beam intensity is maximum, this suggest that the light disperses more quickly than in a media in which the refraction index is constant, Fig. 2 a) shows this effect. In Fig. 2 a) we have the same parameters conditions but

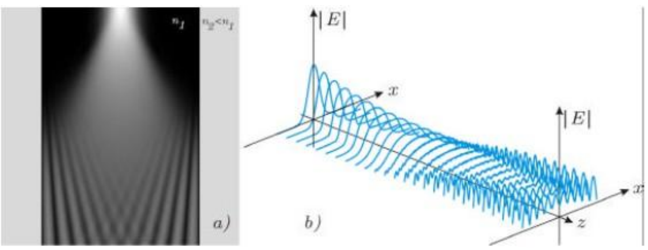

FIGURE 2. a) The gray tones represents the laser intensity propagating in a waveguide with cubic parameter equal to -3000.0. b) Transversal sections of the electric field intensity for the propagation in a self-refractive media with negative cubic parameter.

now a cubic parameter $a_3 = -3000.0$. We observed in this case that the light disperses more quickly than in a linear waveguide, like in a negative lens. Fig. 2 b) shows the transversal sections of the electric field intensity in the same situation that Fig. 2 a), we see in both cases that the light interacts with the border of the waveguide and makes an internal interference pattern. We see in this figure that the light is brought out of focus more quickly than in a linear media. The following questions arises: Does the light remain focused when the cubic parameter is positive?, Could this process continue indefinitely focusing the light in such a way that the width is aproximately zero?.

The answer to the first question is: because of that the maximum refraction index is obtained where the light intensity is maximum, this focuses the light where the intensity is maximum, *i.e.*, the light is self-focusing. For the second question we argued that if this process is repeated successively we could think that, therefore, the light focuses more every time, and at the same time also the refraction index increases, consequently focusing more light and so successively, but this process could not continue indefinitely because of the diffraction is present and the light is dispersed again, see Fig. 3.

We can say that the light propagation in a Kerr media is a consequence of the self-focusing which is generated by the self-refractive effect and the dispersion produced by diffraction; therefore the light cannot be focused to be considered zero-width. In Fig. 3 a) we show how the initial Gaussian beam intensity is self-focused when it propagates through a self-refractive waveguide with cubic parameter $a_3 = 3000.0$. We observe the last transversal sections of the electric field intensity, Fig. 3 b), and we note that there appears relative maximums next to the principal maximum, this confirm our comments about considering that the light focuses to zero-width, because strictly a portion of the light beam is out of the region of focusing.

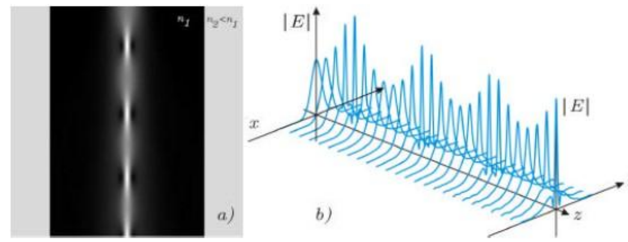

FIGURE 3. a) Graph in gray tones represent the laser intensity propagating through a guide with cubic parameter $a_3 = 3000$. b) Progressive section from the electric field intensity propagating in a self-refractive media with positive cubic parameter.

Consider now a nonlinear waveguide with length $l = 50.0$, $d = 3.0$ and $n_1 = 1.54$ on an air substract. Like in the last case, and we simulate come into the waveguide a Gaussian beam; with an angle $\theta = -5^o$, located now in $x_0 = 1.0$ and changing in every simulation the cubic parameter of the waveguide. When the cubic parameter is zero we have a linear waveguide, and the light interacts with the border of the waveguide making an interference pattern, *i.e.*, a group of dark and bright patterns, and after reflects propagating to the other side. Is evident from our simulation that through this trajectory the beam disperses gradually, see Fig. 4 a).

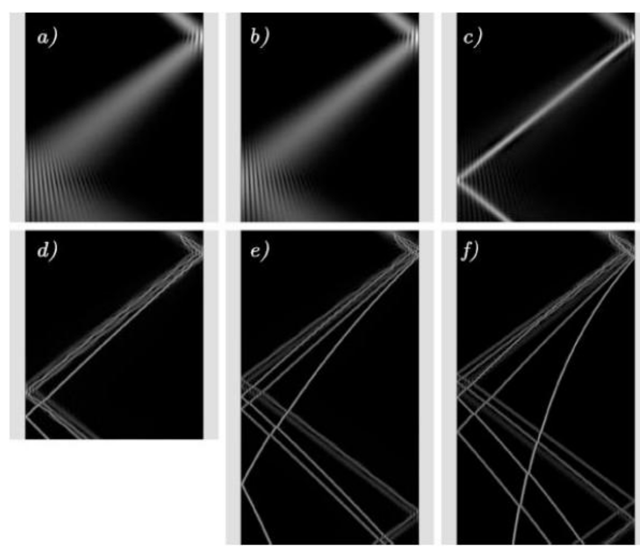

FIGURE 4. Nonlinear cubic effect on the propagation of a Gaussian beam for a_3 equal to: a) 0.0, b) 40.0, c) 400.0, d) 4000.0, e) 6000.0, and f) 8000.0.

Whether we increase the cubic parameter from $a_3 = 40.0$ to 400.0 we see that the light is focuses avoiding dispersion, see Figs. 4 b) and 4 c), respectively. We see again the sequences of self-focusing and dispersion effects, that is a characteristic of a self-refractive media, yet after interaction with the waveguide faces. Now we increase to $a_3 = 4000.0$, we observe that the Gaussian beam is decomposed in thin beams, see Fig. 4 d). These “lumithread” is dispersed in different angles, in apparent violation to the Snell’s law. We observe that these “lumithread” propagates almost without deformation, inclusive after the interaction between them. Whether we increase the parameter from $a_3 = 6000.0$ to 8000.0 (and $l = 75.0$) we see that these lumithread does not propagate in a straight line, moreover, when interact between them they deflect toward the left or right side from the original trajectory, see Figs. 4 e) and 4 f).

The self-bending has been verified in waves-tank, reproducing the historical experiment about solitons from Jhon Scott Russell [2] and C. Y. Gao *et al*. [8], and in nonlinear materials [15, 16]. With this we proved that these “lumithread” observed in the last figure are solitons. We conjecture that a Gaussian beam can be decomposed in solitons by the propagation through a self-refractive media; the definitive proof will be in the possibility that we may have a highly nonlinear media.

The solitons are waves that balance the self-focusing and dispersion produced by a self-refractive media, in this case is a solution of the nonlinear wave equation, and are waves travelling through a nonlinear waveguide without changes in its shape, see Fig. 5, Ref. [13, 14]. It is noteworthy that the width in a soliton depends on the cubic parameter and it does not an independent parameter, like in the Gaussian case.

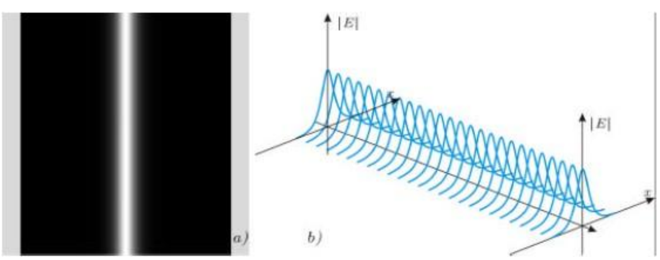

FIGURE 5. a) Gray tones represents the light intensity produced by a soliton propagating through a nonlinear waveguide. b) Transversal sections of the electric field intensity for the soliton propagation.

The soliton formation in a self-refractive waveguide is in according to with the work from Satsuma and Yajima and is called a multi-soliton (*n*-soliton) [5]; and that subsequently Nikolaus and Grischowsky [12] observed experimentally in an optical fiber and found about of fifty of them. Based on the above, in Fig. 6 we show a two-soliton propagation; we observe a self-focusing with relative maximum next to the intense light; this relative maximum produces dispersión performing a periodic propagation, in according with [5, 7].

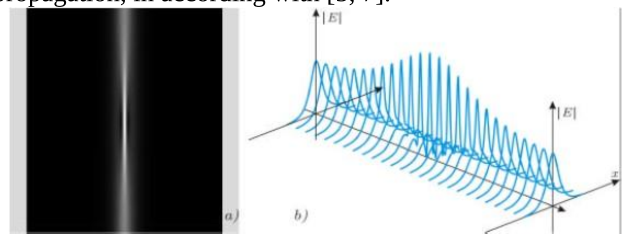

FIGURE 6. a) Gray tones represents the light intensity produced by a two-soliton propagating through a nonlinear waveguide. b) Transversal sections of the electric field intensity for the two-soliton propagation.

Considering now a three-soliton, we observe that the beam is divided in one and after in two beam with relative maximum next to them, producing again a periodic propagation, see Fig. 7. This result is in according with Ref. [10].

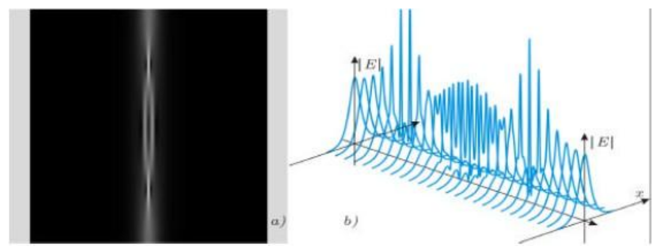

FIGURE 7. a) Gray tones represents the light intensity produced by a three-soliton propagating through a nonlinear waveguide. b) Transversal sections of the electric field intensity for the three-soliton propagation.

Fig. 8 shows the propagation of two solitons with the same phase and width, separated 0.2 and cubic parameter $a_3 = 400.0$, with a lenght $l = 25.0$. We observe that this propagation is periodical and similar to a two-soliton.

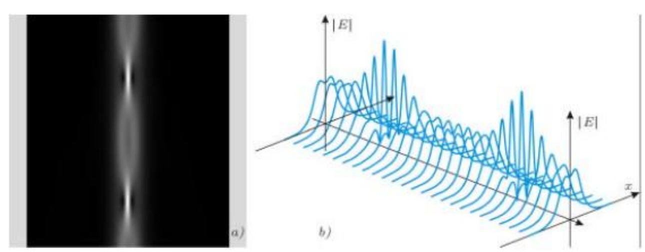

FIGURE 8. a) Gray tones represents the light intensity produced by two solitons propagating very closely and with the same phases through a nonlinear waveguide. b) Transversal sections of the electric field intensity for the solitons propagation in a).

When the phase is different to zero the coupling is of another kind, in the Fig. 9 we show the last case but now with a phase difference of $\theta = \pi/4$ between them. We observe that this propagation has a twist behavior that represents the multi-solitons propagation showed in Fig. 4.

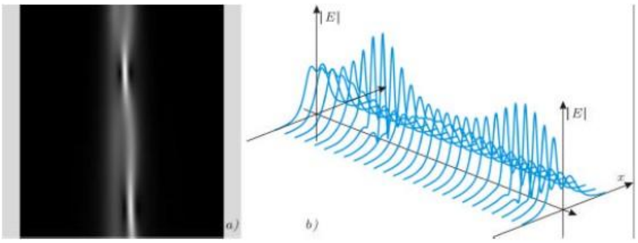

FIGURE 9. a) Gray tones represents the light intensity produced by two solitons propagating very closely and with a phase difference of $\pi/4$ through a nonlinear waveguide. b) Transversal sections of the electric field intensity for the solitons propagation in a).

We simulated the interaction between a soliton and the border of the waveguide; and we observed that the simulation is in according with the experimental result obtained by Rusell [2], the result is not shown.

IV. CONCLUSIONS

We have shown in a numerical way the propagation of different beams, illustrating particularly the multi-spatial solitons propagation in a self-refractive waveguide. Ours results allow us to conjecture and observe that a Gaussian beam can be decomposed into solitons by the propagation in a self-refractive medium.

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