# General relativity in microscopic scale

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## Abstract

We use the Modified Reissner-Nordstrom solution to Einstein's field equation for static charged spherical black hole in microscopic scale, by redefine the potential. Therefore, we obtained circular orbits and horizons in atomic domain. Generalization of special relativity in the sense of the model, predicted proton radius, mass defect, bending of light and precession of perihelion. Mass defect used in estimation electron, positron, neutrino and anti-neutrino mass, in addition of proton decaying energy and mass charge equivalent relation.

**Keywords:** Modified Reissner-Nordstrom metric, GR tests, mass charge equivalent.

## Resumen

Usamos la Modificación Reissner-Nordstrom solución de la ecuación de campo de Einstein para la carga estática de los agujeros negros en escala microscópica, por redefinir el potencial. Por lo tanto, obtenemos órbitas circulares y horizontes en el dominio atómico. La Generalización de la relatividad especial en el sentido del modelo predice el radio del protón, defecto de masa, curvatura de luz y la precesión del perihelio. El defecto de masa utilizado en la estimación de electrones, positrones, neutrinos y masa de anti-neutrinos, además de la energía de protones en descomposición y la relación de masa con carga equivalente.

**Palabras clave:** Modificación métrica de Reissner-Nordstrom, pruebas de GR, masa con carga equivalente.

## I. INTRODUCTION

The universality of the wave nature of any physical system keeps the properties of the system when we measured them. According to a principle of scale relativity, the laws of physics should be applied to systems of references whatever their scales. But when the total mass $m$, becomes larger than Planck's mass, its Compton length becomes smaller than Schwarzschild's radius. Thus becoming unmeasureable, since it enters into black hole horizon. On the contrary, for total mass less than Planck' mass, the Schwarzschild's radius becomes smaller than Compton length, where the concept of position loses its physical meaning (Landu and Lifshitz, 1972) [1]. It appears that the notion of Schwarzschild's radius is also loses its physical meaning.

Nottale suggested the fractal nature of space time (Nottale, 1989, 92, 93) [2, 3, 4], attempted to connect the fractal approach to general relativity by considering a simple model of fractal metric as:

$$ds^2 = \zeta^2(t, \Delta t)\left(1 - \frac{\tau}{\Delta t}\right)^2 c^2 dt^2 - dl^2.$$

Where, $\zeta$ is some finite fractional function with $\zeta = 1$ for $\Delta t > \tau$, $\tau = \frac{\hbar}{mc^2}$ is De Broglie time of a system of inertial mass, $m$, and $dl^2$ is a fractal spatial element.

On the other hand, various authors (J. Bekenstein, 1974) [5], (G. 't Hooft, 1985) [6] and (L. Susskind, *et al* 1993) [7] have suggested that the black hole is a normal quantum system with discrete energy levels.

Therefore, it is easy to compare between the Black hole and the nucleus and put forward a model using general relativity metric in fraction space. This could be done by transforming the macroscopic scale constant (universal gravitational constant G) to microscopic scale constant (Planck constant, $\hbar$) through the transformation $\frac{GM}{c^2} \rightarrow \frac{\hbar}{mc}$. This demands the conversion of the effectiveness of macroscopic scale with mass $M$ to microscopic scale with mass $m$.

## II. MICROSCOPIC GENERAL RELATIVITY MODEL (MGR)

In this model we applied the scale transformation to Reissner-Nordstrom metric to obtain the microscopic scale metric.

Reissner-Nordstrom metric in spherical coordinate symmetric is given by:

$$d\tau^2 = -\left(1 - \frac{2GM}{rc^2} + \frac{GQ^2}{r^2c^4}\right)dt^2 + \frac{1}{c^2}\left(1 - \frac{2GM}{rc^2} + GQ2r2c4 - 1dr2 + 1c2r2d\theta2 + sin2\theta d\emptyset2.\right. \quad (1)$$

By substituting $\frac{GM}{c^2} \to \frac{\hbar}{mc}$ (Compton's length) in (1).then, the metric becomes:

$$d\tau^2 = -\left(1 - \frac{2\hbar}{m_scr} + \frac{\hbar Q^2}{4\pi\epsilon c^3 m^2 r^2}\right)dt^2 + \frac{1}{c^2}\left(1 - \frac{2\hbar}{m_scr} + \hbar Q24\pi\epsilon c3mQ2r2 - 1dr2 + 1c2r2d\theta2 + sin2\theta d\emptyset2.\right.$$

We restore $\frac{1}{4\pi\epsilon}$ in the equation above to the electric term. For Simplicity we can rewrite the metric Eq. (1) as follows:

$$d\tau^2 = -\left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)dt^2 + \frac{1}{c^2}\left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)^{-1}dr^2 + \frac{1}{c^2}r^2(d\theta^2 + sin^2\theta d\emptyset^2). \quad (2)$$

Where: $r_S = \frac{2\hbar}{m_sc}$, $r_Q^2 = \frac{\hbar Q^2}{4\pi\epsilon c^3 m_Q^2}$, $m_s$ is the total mass of the black hole and $m_Q$ is the electro-mass.

The scale transformation can be obtained as:

$$GM \to \frac{\hbar c}{m}. \quad (3)$$

There is true singularity in (2) at $m = 0$, mass singularity, which leads to redefining the microscopic scale as the scale of mass in the range, $0 < m_0 < m_{pl}$, where $m_0$ is the rest mass of the microscopic Particle and $m_{pl}$ is Planck mass. In addition, there are two singularities (horizons) at:

$$r_{\pm} = \frac{\hbar}{m_sc}\left(1 \pm \sqrt{1 - \frac{Q^2 m_s^2}{4\pi\epsilon\, m_Q^2\hbar c}}\right). \quad (4)$$

We find that $r_+$ overlaps $r_-$ when, $\frac{Q^2 m_s^2}{4\pi\epsilon\, m_Q^2\hbar c} = 1$. This implies,

$$m_Q = \pm\sqrt{\frac{\hbar}{4\pi\epsilon c^3}}\frac{Qm_sc}{\hbar}. \quad (5)$$

Equation (5) represents threshold electro-mass when, $\epsilon = \epsilon_0$, $m_s = m_{pl}$ and $Q = Q_{pl}$, where, $m_{pl}$ and $Q_{pl}$ are Planck's mass and charge respectively. In this case, we find

$m_Q = m_{pl}$. This is equivalent to the state where, Schwarzschild's radius or Mach's radius equal to Compton length, then the mass should be equal to Planck's mass (threshold mass). Now we can constrain microscopic charge to maximal limit as Planck's charge with radius equal to Compton length. This result can be shown directly by using the electron wave function in the framework of special scale-relativity and below the scale $\lambda$ ($\lambda$ is Compton length), the Planck length, $l_{pl}$, becomes a minimal scale (Nottale-2003) [8], $\acute{\psi} = \exp\left(i\,4\pi\alpha \ln\left(\lambda/l_{pl}\right)\right)\psi$. Hence, $\acute{\psi} = \psi$ when $\lambda = l_{pl}$. Therefore, the expression of electron wave function can be generalized to describe electro-mass.

Solving the geodesic equation for the line element metric (2), one finds the possible circular orbits for massless particles:

$$r_c = \frac{3}{2}\frac{\hbar}{m_sc}\left(1 \pm \sqrt{1 - \frac{2Q^2 m_s^2}{9\pi\epsilon\, m_Q^2\hbar c}}\right) \approx \frac{3}{2}\frac{\hbar}{m_sc}\left(1 \pm 1 - Q2ms29\pi\epsilon mQ2\hbar c\right). \quad (6)$$

For massive particles on has:

$$r_c = \left[\frac{r_Q^2}{r_s} + \frac{L^2}{r_s}\right] \pm \left(\frac{r_Q^2}{r_s} + \frac{L^2}{r_s} - \frac{3}{2}\left(\frac{r_s}{r_Q^2 + L^2}\right)L^2\right). \quad (7)$$

Where, $L$ is the angular momentum per unit mass of the particle.

According to Eq. (7) there are different regimes (stable and unstable) depending on the angular momentum. The unstable orbits for massive particles are located at the radius $r_{c-}$:

$$r_{C-} = \frac{3L^2 r_s}{2(r_Q^2 + L^2)}. \quad (8)$$

The other stable orbits, $r_{c+}$, are located at:

$$r_{c+} = 2\left[\frac{r_Q^2}{r_s} + \frac{L^2}{r_s}\right] - \frac{3L^2 r_s}{2(r_Q^2 + L^2)}. \quad (9)$$

Equation (9), from atomic view point, represents the possible atomic orbits. The smallest radius (Bohr radius) occurs when, ($L = 0$).

In the case of uncharged nucleus, $Q = 0$, the unstable orbit is

$$r_{c-} = \frac{3\hbar}{m_sc}. \quad (10)$$

The stable orbit is

$$r_{c+} = \frac{3\hbar}{m_sc}\left(\frac{4L^2}{3r_s^2} - 1\right). \quad (11)$$

At this limit the stable orbit becomes larger than the unstable one, which approaches $r_s$, (behaves like massless). The orbit doesn't vanish when $Q = 0$. Hence, the existence of the orbit does not depend on electrical force, as well as gravitational force.

## III. GENERALIZATION OF SPECIAL RELATIVITY

In an attempt to generalize the special relativity to includes the gravitational potential, (Mubarak, D. A. and Ali, T., 2003)[9] one has to redefine the transformation $\gamma$ in terms of proper time $\tau$.

As we considered above, in the radial motion, $\theta$ and $\emptyset$ are constants.

First, in non-relativistic case: $v \ll c$, the metric in time-like signature is:

$$d\tau^2 = \left(1 - \frac{2\hbar}{m_s cr} + \frac{\hbar Q^2}{4\pi\epsilon c^3 m_Q^2 r^2}\right)dt^2 - \frac{1}{c^2}\left(1 - \frac{2\hbar}{m_s cr} + \hbar Q 2 4\pi\epsilon c3 m Q2 r2 - 1 dr2.\right. \tag{12}$$

Therefore, the transformation is

$$\gamma = \frac{d\tau}{dt} = \sqrt{g_{00} + g_{rr}\frac{v^2}{c^2}}. \tag{13}$$

With, $g_{00} = \left(1 - \frac{2\hbar}{m_s cr} + \frac{\hbar Q^2}{4\pi\epsilon c^3 m_Q^2 r^2}\right)$; $g_{rr} = -\left(1 + \frac{2\hbar}{m_s cr} - \frac{\hbar Q^2}{4\pi\epsilon c^3 m_Q^2 r^2}\right)$

$$\gamma = \frac{d\tau}{dt} = \sqrt{g_{00}} = \sqrt{1 - \frac{2\hbar}{m_s cr} + \frac{\hbar Q^2}{4\pi\epsilon c^3 m_Q^2 r^2}}. \tag{14}$$

$$\gamma^{-1} = 1 + \frac{\hbar}{m_s cr} - \frac{\hbar Q_m^2}{8\pi\epsilon c^3 m_Q^2 r^2}. \tag{15}$$

Second, in relativistic case $v = c$: The metric in space-like signature:

$$g_{00} = -\left(1 - \frac{2\hbar}{m_s cr} + \frac{\hbar Q^2}{4\pi\epsilon c^3 m_Q^2 r^2}\right); g_{rr} = \left(1 + \frac{2\hbar}{m_s cr} - \hbar Q 2 4\pi\epsilon c3 m Q2 r2\right.$$

$$\gamma = \frac{d\tau}{dt} = \sqrt{g_{00} + g_{rr}} = \sqrt{\frac{4\hbar}{m_s cr} - \frac{\hbar Q^2}{2\pi\epsilon c^3 m_Q^2 r^2}}. \tag{16}$$

## IV. SOME APPLICATIONS OF THE MODEL

### A. Mass defect and beta decay

In non-relativistic case the observable mass can be obtained according to Eq. (15)

$$m = m_0\left(1 + \frac{\hbar}{m_s cr} - \frac{\hbar Q^2}{8\pi\epsilon c^3 m_Q^2 r^2}\right). \tag{17}$$

If the proton is assumed to be the nucleus origin, then the observable neutron mass is

$$m_n = m_{n0} + m_{n0}\left(\frac{\hbar}{m_{p0} cr} - \frac{\hbar e^2}{8\pi\epsilon c^3 m_{Qp}^2 r^2}\right). \tag{18}$$

Where $m_{Qp}$, stands for proton electro-mass. $m_{p0}$ and $m_{n0}$ are the proton and neutron rest mass respectively.

According to Eq. (18), the neutron mass does not altered when the second term in R.H.S vanished. This implies that,

$$m_Q = \pm\sqrt{\frac{m_s}{8\pi\epsilon c^2 r}}\, Q. \tag{19}$$

On the other hand, if the neutron is assumed to be the nucleus origin ($Q = 0$), then the observable proton mass is

$$m_p = m_{po} + m_{po}\frac{\hbar}{m_{n0} cr}. \tag{20}$$

Equation (18) and (20) can be formed as:

$$n \to n_0 + n_0\left(\frac{\hbar}{p_0 cr_p} - \frac{\hbar e^2}{8\pi\epsilon c^3 p_0^2 r_p^2}\right). \tag{21}$$

$$p \to p_0 + p_0\frac{\hbar}{n_0 cr_n}. \tag{22}$$

Where $p_0$ refers to proton rest mass and $n_0$ refers to neutron rest mass.

By adding, equation (21) to (22), one finds,

$$n + p \to (n_0 + p_0) + \left(n_0\frac{\hbar}{p_0 cr_p} + p_0\frac{\hbar}{n_0 cr_n}\right) - n_0\frac{\hbar e^2}{8\pi\epsilon c^3 m_{Qp}^2 r_p^2}. \tag{23}$$

On the other hand, beta decay is given by the $\beta^-$ and $\beta^+$ equations,

$$n \to p + e^- + \bar{v}_e. \tag{24}$$

$$p + \frac{E}{c^2} \to n + e^+ + v_e. \tag{25}$$

Adding Eq. (24) to Eq. (25), one has

$$n + p \to (n + p) + (e^- + e^+) + (\bar{v}_e + v_e) - \frac{E}{c^2}. \tag{26}$$

Comparing Eq. (23) with Eq. (26), the equivalent masses for all of neutron, proton, neutrino and anti-neutrino are obtained. In addition, the required energy for proton decaying,

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