Deriving Reactive Power and Apparent Power from RMS Power and Average Power in AC Circuits

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Abstract
Novel equations to calculate the magnitude of Reactive Power and the Apparent Power in AC circuits are derived in this paper. These equations use the RMS Power and the Average Power. PSpice simulations are also done which verify the theoretical equations.

Keywords: RMS Power, Average Power, Reactive Power, Apparent Power, and PSpice simulation of AC Power.

Resumen
En este artículo se derivan nuevas ecuaciones para calcular la magnitud de la potencia reactiva y la potencia aparente en circuitos de CA. Estas ecuaciones utilizan la potencia RMS y la potencia media. También se realizan simulaciones PSpice que verifican las ecuaciones teóricas.

Palabras clave: Potencia RMS, Potencia promedio, Potencia reactiva, Potencia aparente y simulación PSpice de potencia CA.

I. INTRODUCTION

In AC circuits, the concepts of Average Power (W), Reactive Power (VAR) and Apparent Power (VA) are well known (see, e.g., [1] or [2]). However, RMS (Root-Mean-Square) Power (W) is less known and hardly discussed in the literature. In fact, at least one author (see [3]) thinks it has no physical meaning.

In this paper, it will be shown how the Apparent Power and the magnitude of the Reactive Power can be derived by knowing just the RMS Power and the Average Power. Furthermore, PSpice computer simulations will be done to verify the newly derived equations.

II. THEORETICAL BACKGROUND

Consider Figure 1, which shows a sinusoidal voltage source that produces a voltage across a load consisting of a capacitor and the resistor R_1. The sinusoidal voltage across the load is given by

$$v_L(t) = V_m \sin(\omega t + \theta_V) \text{ V}, \quad (1)$$

where V_m (V) is the amplitude of the voltage, $\omega = 2\pi f = 2\pi / T$ (rad/s) is the angular frequency of

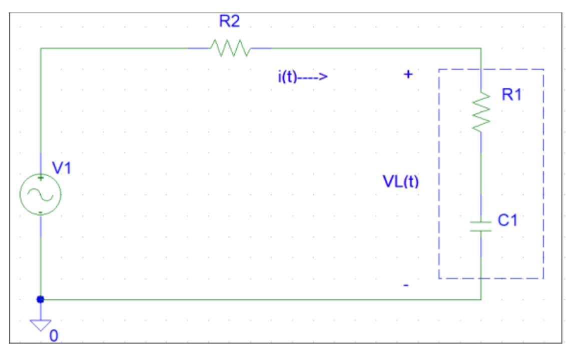

FIGURE 1. AC series circuit with a load capacitor in series with the load resistor, R_1.

the applied voltage, f is the frequency in Hz, T is the period in seconds and θ_V is the phase of the voltage. The corresponding *steady-state* current is given by

$$i(t) = I_m \sin(\omega t + \theta_I) \text{ A}, \quad (2)$$

where I_m (A) is the amplitude of the current and θ_I is the phase of the current.

Recall that the instantaneous power absorbed by the load is given by

$$\begin{aligned}p(t) &= v_L(t)i(t)\\ &= V_m\sin(\omega t+\theta_V)I_m\sin(\omega t+\theta_I)\\ &= \frac{V_mI_m}{2}\cos(\theta_V-\theta_I)-\frac{V_mI_m}{2}\cos(2\omega t+\theta_V+\theta_I)\\ &= P_{AVE}-|S|\cos(2\omega t+\theta_V+\theta_I)\\ &= P_{AVE}-|S|\cos\left(2\omega t+2\theta_I+\theta_V-\theta_I\right)\\ &= P_{AVE}-|S|\cos\left(\theta_V-\theta_I\right)\cos\left(2\omega t+2\theta_I\right)\\ &\quad +|S|\sin\left(\theta_V-\theta_I\right)\sin\left(2\omega t+2\theta_I\right)\\ &= P_{AVE}-P_{AVE}\cos\left(2\omega t+2\theta_I\right)+Q\sin\left(2\omega t+2\theta_I\right),\end{aligned} \quad (3)$$

where it is well-known that

$$\begin{aligned}P_{AVE} &= \frac{1}{T}\int_0^T p(t)dt=\frac{1}{T}\int_0^T v(t)i(t)dt\\ &= V_{RMS}I_{RMS}\cos(\theta_V-\theta_I)\\ &= |S|\cos(\theta_V-\theta_I),\end{aligned} \quad (4)$$

and

$$\begin{aligned}Q &= \frac{1}{T}\int_0^T v(t)i(t+T/4)dt\\ &= V_{RMS}I_{RMS}\sin(\theta_V-\theta_I)\\ &= |S|\sin(\theta_V-\theta_I).\end{aligned} \quad (5)$$

Please note that P_{AVE} is called the Average or Active Power (W) and Q is called the Reactive Power (VAR).

Furthermore, the Apparent Power $|S|$ (VA) in Eq. (3), Eq. (4) and Eq. (5) is defined to be

$$|S|=\frac{V_mI_m}{2}=\frac{V_mI_m}{\sqrt{2}\sqrt{2}}=V_{RMS}I_{RMS}, \quad (6)$$

where $V_{RMS}=V_m/\sqrt{2}$ is the root-mean-square value of the voltage and $I_{RMS}=I_m/\sqrt{2}$ is the root-mean-square value of the current. Traditionally, the former is measured with a voltmeter and the latter with an ammeter. The product of these two measurements then gives the Apparent Power.

However, in this paper a novel expression will be derived that shows how this Apparent Power can be calculated without the explicit knowledge of both V_{RMS} and I_{RMS}. Indeed, it will be shown that

$$|S|=\sqrt{2P_{RMS}^2-2P_{AVE}^2}, \quad (7)$$

where

$$P_{RMS}=\sqrt{\frac{1}{T}\int_0^T p^2(t)dt}. \quad (8)$$

Please note that Eq. (8) is the RMS value of the instantaneous power.

Rearranging Eq. (7) gives

$$P_{RMS}=\sqrt{\frac{|S|^2}{2}+P_{AVE}^2}. \quad (9)$$

So, Eq. (9) is stating that the RMS Power does have a physical interpretation. This is believed to be a novel finding.

Also in this paper, a novel expression will be given for the Reactive Power Q (VAR). Indeed, it is

$$|Q|=\sqrt{2P_{RMS}^2-3P_{AVE}^2}, \quad (10)$$

which will be derived in the next section below.

II. DERIVATION OF APPARENT POWER AND REACTIVE POWER FROM RMS POWER AND AVE. POWER

In this section, Eq. (7) and Eq. (10) will be derived from Eq. (4) and Eq. (8).

A. Derivation of the Apparent Power

Substituting Eq. (3) into Eq. (8) gives

$$\begin{aligned}P_{RMS}^2 &= \frac{1}{T}\int_0^T\left(P_{AVE}-|S|\cos(2\omega t+\theta_V+\theta_I)\right)^2dt\\ &= Integral_1+Integral_2+Integral_3,\end{aligned}$$

where

$$Integral_1=\frac{1}{T}\int_0^T P_{AVE}^2dt=P_{AVE}^2$$

$$Integral_2=\frac{1}{T}\int_0^T -2P_{AVE}|S|\cos(2\omega t+\theta_V+\theta_I)dt \text{ and}$$

$$Integral_3=\frac{1}{T}\int_0^T |S|^2\cos^2(2\omega t+\theta_V+\theta_I)dt. \quad (11)$$

Using $\omega=2\pi/T$, it is easily shown that $Integral_2=0$ and $Integral_3=|S|^2/2$. Hence, Eq. (11) becomes

$$P_{RMS}^2=P_{AVE}^2+|S|^2/2. \quad (12)$$

Rearranging Eq. (12) and taking the square-root produces Eq. (7).

B. Derivation of the Magnitude of the Reactive Power

From Eq. (4), $P_{AVE}==|S|\cos(\theta_V-\theta_I)$ and from Eq. (5), $Q=|S|\sin(\theta_V-\theta_I)$. Hence,

$$|S| = \sqrt{P_{AVE}^2 + Q^2}. \qquad (12)$$

Substituting Eq. (12) into Eq. (9) produces

$$P_{RMS} = \sqrt{\frac{P_{AVE}^2 + Q^2}{2} + P_{AVE}^2}. \qquad (13)$$

Solving Eq. (13) for $|Q|$ produces Eq. (10).

III. VERIFICATION OF THE THEORETICAL EXPRESSIONS WITH PSPICE SIMULATION

To verify the novel theoretical expressions above, PSpice simulations will be performed in this section. (PSpice is electrical/electronic circuit simulation software that is well-known to electrical engineers and physicists. In this paper, PSpice ver. 9.1 is used. To download, please see [4], for example).

The PSpice circuit to simulate the Instantaneous Power (IP) absorbed by the load is shown in Figure 2.

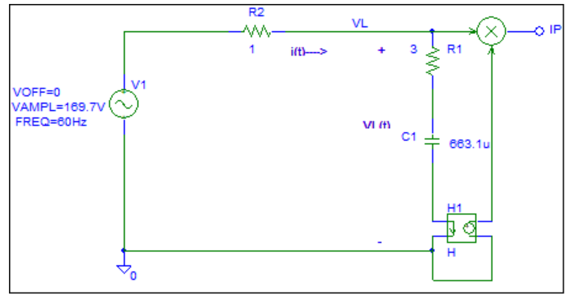

FIGURE 2. PSpice circuit to simulate the instantaneous power (IP) absorbed by the load of Figure 1.

The Instantaneous Power simulated in Figure 2 is applied to the input of Figure 3, where PSpice calculates a modified P_{RMS}^2 in Block A, a modified P_{AVE} in Block D. From these two, the Apparent Power is determined in Block B and the magnitude of the Reactive Power is calculated in Block C.

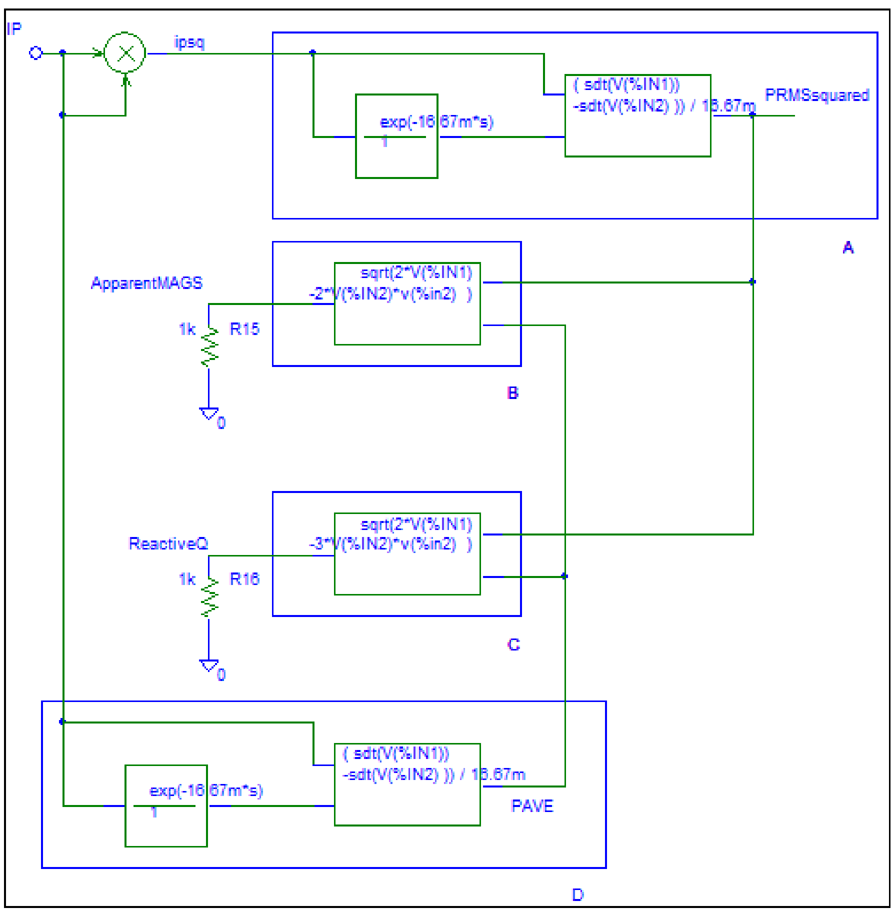

FIGURE 3. PSpice schematic that calculates the square of RMS Power, and the Average Power from the simulated Instantaneous Power of Fig. 2. From these two calculations, the Apparent Power and magnitude of Reactive Power are calculated.

The modified PSpice measurement for the square of the RMS Power is given by

$$P_{RMS}^2(t) = \frac{1}{T}\int_{t-T}^{t} p^2(t)dt. \qquad (14)$$

So, Eq. (14) calculates a moving average of $p^2(t)$ over a period. When the circuit reaches steady-state, Eq. (14) will become a constant.

The modified PSpice measurement for the Average Power is given by

$$P_{AVE}(t) = \frac{1}{T}\int_{t-T}^{t} p(t)dt. \qquad (15)$$

So, Eq. (15) calculates a moving average of $p(t)$ over a period. When the circuit reaches steady-state, Eq. (15) will become a constant.

From Eq. (14) and Eq. (15), the Apparent Power is calculated using the newly derived Eq. (7) and the magnitude of the Reactive Power is calculated using the newly derived Eq. (10).

Two simulations will be done with Figure 2. Both simulations will use a voltage source of $169.7\sin(120\pi t)V$, $R_1 = 3\Omega$ and $R_2 = 1\Omega$. In the first simulation, $C_1 = 663.1\mu F$ which gives a capacitive reactance of $X_C = -4\Omega$. Hence, $Q < 0$. In the second simulation, the capacitor in the load is replaced by an inductor of $L_1 = 10.61mH$. This produces an inductive reactance of $X_L = 4\Omega$. Hence, $Q > 0$.

The simulated values for Average Power, magnitude of Reactive Power and Apparent Power are shown in Figure 4 for the first simulation and in Figure 5 for the second.

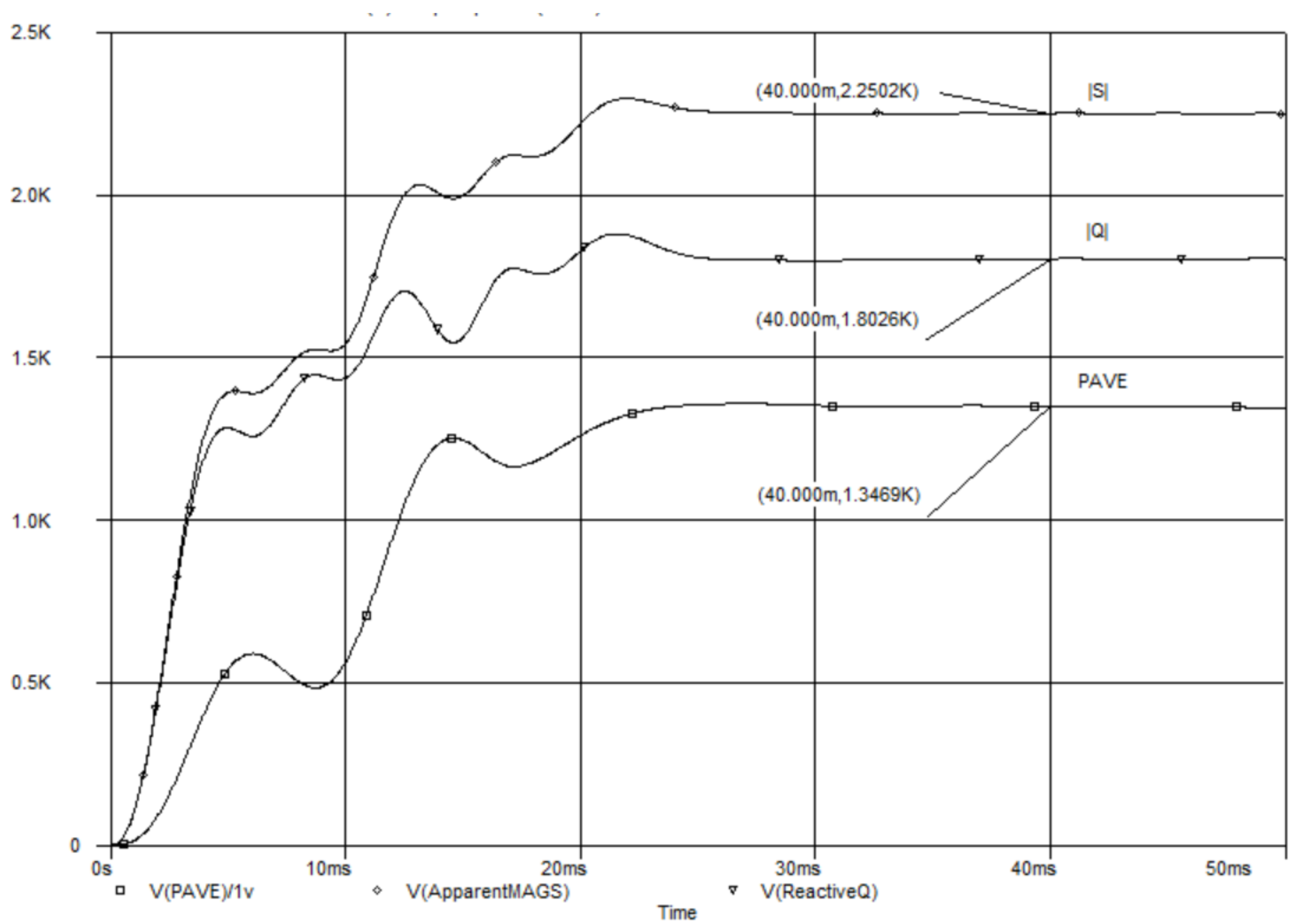

FIGURE 4. PSpice simulated power values for Figure 2 with the capacitor in the load.

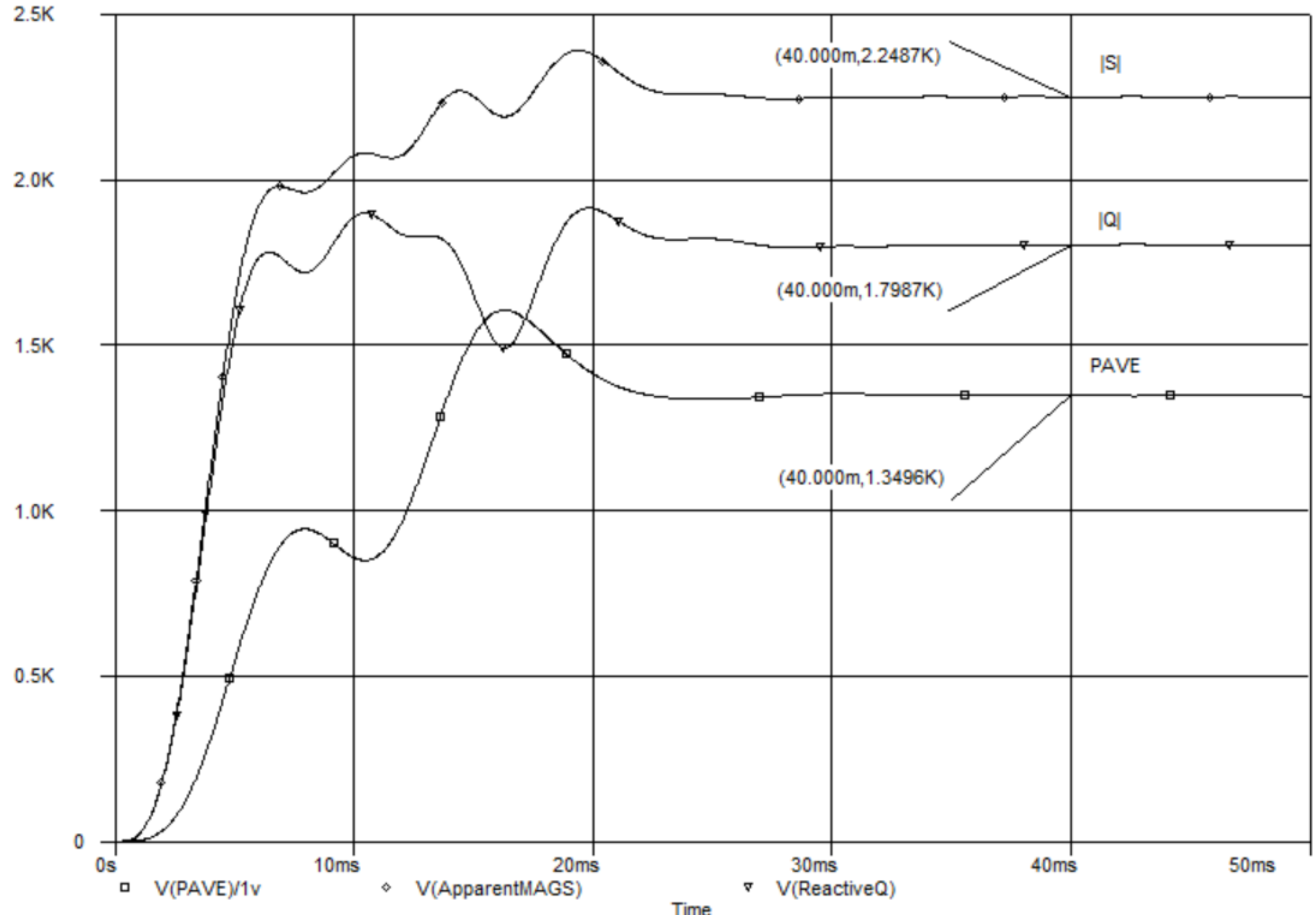

FIGURE 5. PSpice simulated power values for Figure 2 with the inductor in the load.

Close inspection of Figure 4 and Figure 5 reveals that the corresponding steady-state values are very close to each other: however, the initial values are not. However, it is the steady-state values that are of concern.

Table I below compares the steady-state simulated values with their theoretical values for the first simulation.

Table I. Comparison between theoretical and simulated values when the load has the capacitor.

Power	Theory	Simulation 1 (Figure 4)
rage Power (kW)	1.3500	1.3469
, of Reactive Power (kVAR)	1.8000	1.8026
arent Power (kVA)	2.2500	2.2502

Table II below compares the steady-state simulated values with their theoretical values for the second simulation.

Table II. Comparison between theoretical and simulated values when the load has the inductor.

Power	Theory	Simulation 2 (Figure 5)
rage Power (kW)	1.3500	1.3496
, of Reactive Power (kVAR)	1.8000	1.7987
arent Power (kVA)	2.2500	2.2487

As can be seen from Table I and Table II, there is excellent agreement between the theoretical values and the simulated values that uses the newly derived equations that compute the magnitude of the Reactive Power and the Apparent Power from the RMS Power and the Average Power.

IV. CONCLUSIONS

It has been thought that the RMS Power had very little role to play in AC power calculations. However, in this paper, it has been demonstrated that the magnitude of Reactive Power and the Apparent Power can be calculated with just the RMS Power and Average Power using newly derived equations. Furthermore, PSpice simulations supported the theoretical calculations.

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