Comparing experimental and theoretical trajectories of water jets: The influence of holes' diameter

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Abstract
One of the most popular errors in physics and science textbooks was the drawing of wrong trajectories of water jets flowing out of three lateral holes made in a container (the jet from the lowest hole has the longest range). The error is still today repeated in many Internet pages. The error was denounced in important pedagogical journals like "The Physics Teacher" and "Physics Education". As correct ones were drawn theoretical trajectories where the longest range has the jet flowing from the middle hole. Noting that some photos in books and Internet pages present jets' patterns that differ from the theoretical trajectories of three jets, it is important to ask: When the experimental jets' trajectories are close to the theoretical trajectories? This article presents the results of two experiments in which the influence holes' diameter on the jets' trajectories was explored. For one-liter bottle, for both experiments, it was found that when the holes' diameter is 1/4 inches the jets' trajectories show theoretically expected horizontal ranges.

Keywords: textbook errors, water jets from three holes, comparing theoretical and experimental ranges of water jets.

Resumen
Uno de los errores más comunes en los libros de texto de física y ciencias era el dibujo de trayectorias erróneas de chorros de agua que salían de tres agujeros laterales hechos en un recipiente (el chorro del agujero más bajo tiene el mayor alcance). El error todavía hoy se repite en muchas páginas de Internet. El error fue denunciado en importantes revistas pedagógicas como "The Physics Teacher" y "Physics Education". Como correctas se dibujaron trayectorias teóricas donde el chorro que sale del agujero central tiene el mayor alcance. Notando que algunas fotos en libros y páginas de Internet presentan patrones de chorros que difieren de las trayectorias teóricas de tres chorros, es importante preguntar: ¿Cuándo las trayectorias de los chorros experimentales se acercan a las trayectorias teóricas? En este artículo se presentan los resultados de dos experimentos en los que se exploró la influencia del diámetro de los agujeros en las trayectorias de los chorros. En el caso de las botellas de un litro, en ambos experimentos se encontró que cuando el diámetro de los orificios es de 1/4 de pulgada, las trayectorias de los chorros muestran rangos horizontales teóricamente esperados.

Palabras clave: Errores de libros de texto, Chorros de agua de tres orificios, Comparación de rangos teóricos y experimentales de chorros de agua.

I. INTRODUCTION

It is known that physics and science textbooks aren't free of errors [1, 2]. Among these errors one is particularly popular: wrong trajectories of water jets flowing out of three lateral holes of a container (Figure 1).

It appeared in 1912 in a German physics textbook [3], and since then was repeated as myth many times [4, 5, 6]. This mythical repetition is especially strange for the books published after 1988 [7, 8, 9, 10, 11, 12], because before that year the error was denounced in the most important pedagogical journals like "*The Physics Teacher*" [13, 14] and "*Physics Education*" [15]. If the authors of science textbooks don't read pedagogical journal, that is much less likely for the authors of Internet pages. For them, allegedly longer horizontal range of the lower jet is a "perfect demonstration" that the water pressure increases with depth. Being so, wrong water jets' trajectories were being repeated many times more (Figures 2 and 3).

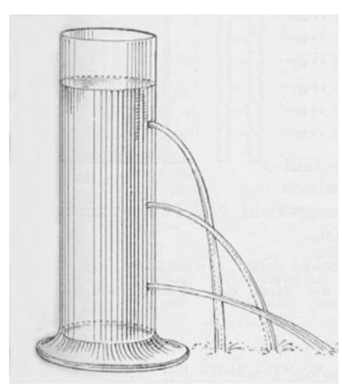

FIGURE 1. Wrong trajectories of three water jets in the textbook [3].

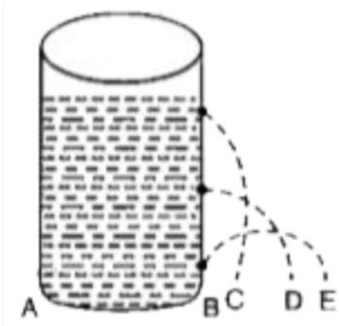

Figure 2. https://www.doubtnut.com/qna/646304968

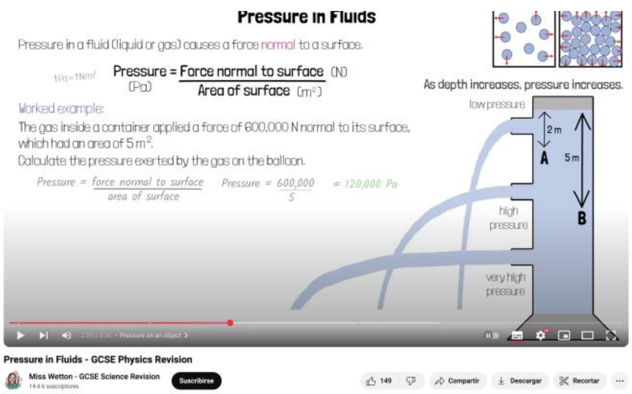

FIGURE 3. https://www.youtube.com/watch?v=IeYZadrSgx4

II. THEORETICAL TRAJECTORIES

One way to have physical insight in the behavior of the water parabolas, better than mythical demonstration "deeper the hole - longer the range", is to apply the laws of the projectile motion on the jets, using the Torricelli's speed of as the initial, horizontal speed. Along this line, some standard end-of-chapter problems in introductory university physics are designed. In such problems students are asked to find out:

(1) The range of the jet on the level of the container's base [16, Problem 66 (a), p. 284], [17, Problem 41, p. 363];

(2) the hole's position for the maximum range of the jet [16, Problem 66 (b), p. 284; Problem 50 (b), p. 459], [17, Problem 46 (c), p. 365)],

or

(3) the hole positions for which the ranges are the same [17, Problem 46 (b), p. 365].

For the lateral hole at the depth h, when the distance between the water surface and the container's bottom is H, the horizontal range is:

$$R = 2\sqrt{h(H - h)}$$

Using calculus, it is easy to show that the hole at the depth $h = H/2$ has the maximum range, equal to H. Any two holes, placed symmetrically regarding to the middle point, would have the same range. If, for example, one is at the depth $H/4$ and the other is at the depth $3H/4$, their ranges would be the same and should be $\sqrt{3}\frac{H}{2}$ (Figure 4).

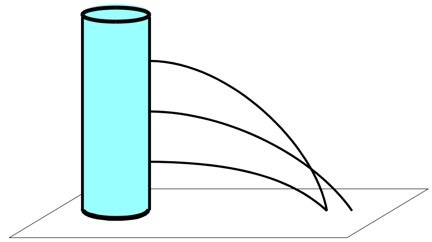

FIGURE 4. Theoretical trajectories of three water jets.

As it will be shown soon, the approach leading to results mentioned above shares a common weakness of the textbook problems. Namely, students are implicitly suggested to model the behavior of the jets if they were ideal, i.e. the Torricelli's formula for the initial velocity is always valid and the motion of the jet is the same as it were an ideal projectile which moves without being disturbed by the air resistance. Needless to say, all complications due to turbulent flow of real jets are simply ignored.

III. REAL TRAJECTORIES

The drawings of wrong water jets trajectories (Figure 1) are showing clearly that authors didn't carry out the experiments. Nevertheless, in some cases, authors of books of experimental fluid mechanics [18, p. 161] or of pedagogical Internet pages provide photographs in which the jet from the lowest hole has the longest range at the level of bottle levels (Figure 5).

FIGURE 5. https://slideplayer.com/slide/8518641/

Sometimes, the photographs present even an enigmatic pattern: five jets flowing out of three holes (Figure 6)!

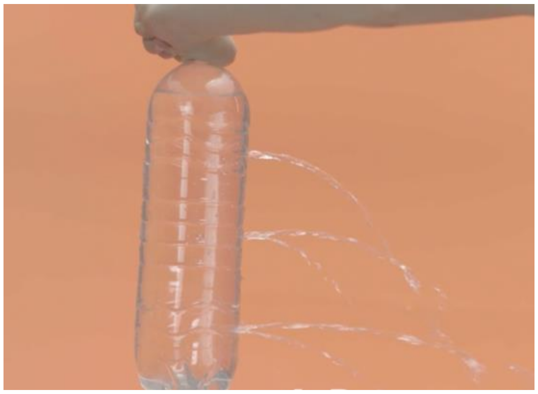

FIGURE 6.
https://www.bbc.co.uk/bitesize/articles/zn2r97h#z99wmbk

Obviously, in these cases the water jets operate in the conditions that don't fit the conditions assumed in the theoretical derivation of the horizontal range formula. So, the important question is: When do the real experimental jets follow their theoretical trajectories?

As it normally happens, the textbooks offer very little or no help. The only condition commonly stated in the derivation of Torricelli's formula is that diameter of the hole should be small in comparison with the diameter of the container [16, p. 267; 17, p. 349].

Examining this condition carefully, one can see that it is posed with the objective to simplify algebra and not to bring some physical insight about the behavior of the jet. Namely, if the hole is much smaller than container's horizontal cross-section, then one can neglect the motion of the water surface.

Pushing this condition to its logical end, one would come to the absurd conclusion that the Torricelli's formula is a correct description of a jet flowing out from a point-like hole [19, Fig. 17.4, p. 214]. But then, it may happen that there is no jet at all.

IV. EXPERIMENT TO FIND OUT HOW THE HOLES' DIAMETER INFLUENCES THE JET RANGES

If the water level is held constant by some device, then the influence of the diameter of the hole on the behavior the jets could be examined not from a mathematical but from physical point of view. As far as we know, Erlich [20, pp. 98-99], is the only author who gave some concrete practical suggestions about how to perform the demonstration with three jets, with the objective to obtain the characteristic (theoretical) pattern given in the Figure 4.

We did the demonstration twice. In the first demonstration, the trajectories of the jets flowing from a 1-liter bottle were far from the theoretical, if the diameter of the holes was 1/16 inches as suggested by Erlich. The resulting experimental and theoretical ranges, for four different holes' diameters, are shown in the Figure 7. We must say that, due to the dispersion and finite size of the jets, it is difficult to measure their ranges with precision better than 0.5 cm.

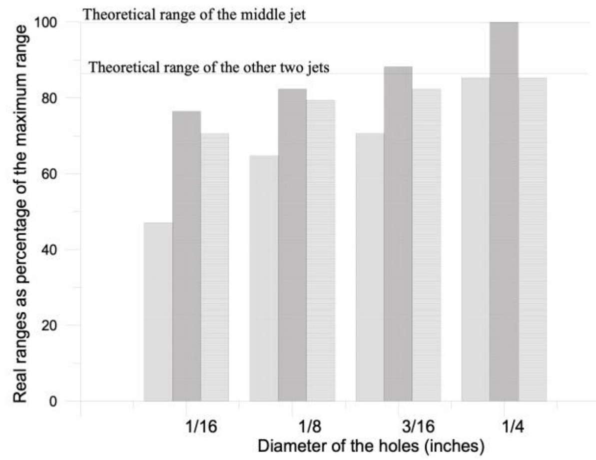

FIGURE 7. Real ranges of the jets flowing out from the holes of different diameters. The first bar corresponds to the hole at the depth H/4, the second bar to the depth H/2, and the third to the depth 3H/4, where H = 17 cm.

As it can be seen from the Figure 7, only jets from the holes with the diameter equal to 1/4 of inch, four times bigger than the value suggested by Erlich, behave approximately as predicted by theory.

In this case, the outflow of water is so fast that some device for controlling the level must be used. One way to do it is to make a horizontal opening, on the side opposite to the one with the holes, and to connect the bottle to a water supply.

For our second measurement a transparent one-liter plastic bottle was used. This time on the bottle were made three holes with six different diameters. With the help of a plastic tube, the water level was kept at 20 cm. Three holes were made at heights of 5 cm, 10 cm and 15 cm above the bottle's bottom. The photos were taken by a high-speed camera Casio-Exilim. The camera can make 300 frames per second. The patterns of water jets flowing from the holes of different diameters can be seen in the six figures below.

FIGURE 8. The jets when the holes' diameter is 0.794 mm (1/32 in or 2/64 in).

FIGURE 9. The jets when the holes' diameter is 1.587 mm (1/16 in or 4/64 in).

FIGURE 10. The jets when the holes' diameter is 1.984 mm (5/64 in or 5/64 in).

FIGURE 11. The jets when the holes' diameter 3.175 mm (1/8 in or 8/64 in).

FIGURE 12. The jets when the holes' diameter is 4.762 mm (3/16 in or 12/64 in).

FIGURE 13. The jets when the holes' diameter is 6.35 mm (1/4 in or 16/64 in).

The obtained patterns are summarized the Figure 14, where blue line corresponds to the jet flowing from the highest hole, green line to the lowest hole and red line to the middle hole.

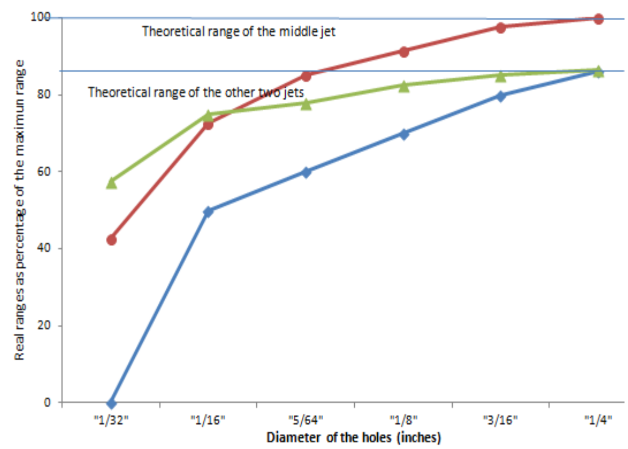

FIGURE 14. The percentage of theoretical ranges obtained for three real jets.

So, in our second experiment the real jets' trajectories are again close to theoretical trajectories if the holes' diameter is 1/4 inches. It is very important to notice that for the smallest diameter (1/32 inches) the water doesn't flow from the upper hole (Figure 8), being stopped by the force of surface tension.

V. CONCLUSION

We are right to reject the mythical trajectories as wrong. But we must accept that finding out the theoretical trajectories is only a math exercise if we don't know when they fit the experimental reality. Contrary to what it is implicitly or explicitly said, it seems that bigger holes do it better than smaller ones.

In addition, it seems to us that we should compare the hole's diameter not with diameter of container but with the depth of the hole. When the jets operate in the conditions in which the range depends on the hole's diameter, then of the two holes with the same diameter, placed symmetrically regarding the middle point, the higher one has a smaller range.

This also suggests that there is a critical value of the diameter (expressed in term of the depth of the highest hole) from which on the range will not be affected by the dimension of the hole.

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